Recall rationalizing denominators

$$\frac{3}{1+\sqrt{2}} = \left(\frac{3}{1+\sqrt{2}}\right)\left(\frac{1-\sqrt{2}}{1-\sqrt{2}}\right) = \frac{3-3\sqrt{2}}{1^2-\sqrt{2}^2} = \frac{3-\sqrt{2}}{1-2} = \frac{3-\sqrt{2}}{-1}$$

Exploiting $(a+b)(a-b) = a^2 - b^2$

$$\boxed{\neq \sqrt{2} - 3}$$

Let $f(x) = \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$. We find $\lim_{h \to 0} f(x)$:

(Note: $\dfrac{\sqrt{x+0} - \sqrt{x}}{0} = \dfrac{0}{0}$?! Indeterminate form.)

The workaround:

$$\left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right) = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\underbrace{\sqrt{x+h}\,\sqrt{x+h}}_{x+h} \; \boxed{+\sqrt{x+h}\sqrt{x} - \sqrt{x}\sqrt{x+h}} \; \underbrace{-\sqrt{x}\sqrt{x}}_{-x}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}} \xrightarrow{h \to 0} \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}} = \lim_{h \to 0} f(x)}$$

Flip side: $x \to \infty$

$$f(x) = \frac{\sqrt{x^2+9} - x}{}$$

Will return

Flatulence of the

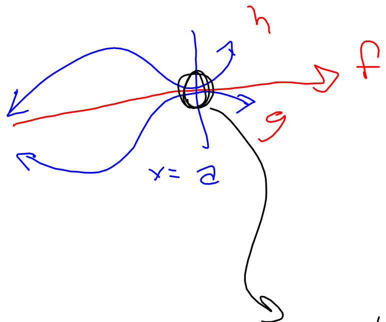
Mind.

Squeeze Theorem (Special, for pathological situations)

If $g(x) \le f(x) \le h(x)$

and $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$

then $\lim_{x \to a} f(x) = L$



The point (a, L)

Consider $\alpha(x) = \sin\left(\frac{\pi}{x}\right)$

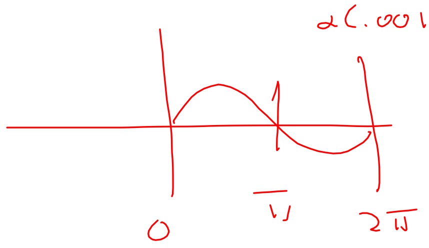
Intuitively, you might plug in $x = .1, .01, x = .001, \ldots$

$\lim_{x \to 0} \alpha(x)$

$\alpha(.1) = \sin\left(\frac{\pi}{.1}\right) = \sin(10\pi) = 0$

$\alpha(.01) = \ldots = \sin(100\pi) = 0$

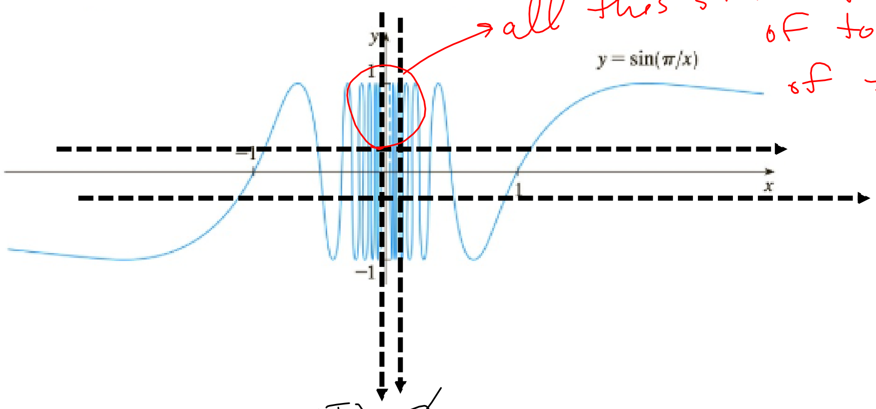
$\alpha(.001) = \ldots = \sin(1000\pi) = 0$



But, $\lim_{x \to 0} \alpha(x) \nexists$

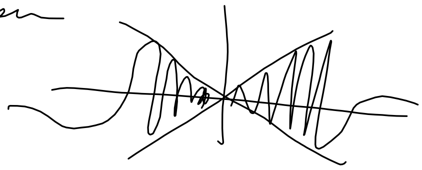
So just plugging in #s is not always sufficient, and may mislead.

all this stuff poking out of top & bottom of the tub (window)



So $\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right) \nexists$

Bringin' home the squeeze theorem



consider $f(x) = x \cdot \sin\left(\frac{\pi}{x}\right)$

since $-1 \le \sin\left(\frac{\pi}{x}\right) \le 1$

$-x \le x \sin\left(\frac{\pi}{x}\right) \le x$ if $x \ge 0$

(If $x < 0$ $-x \ge x \sin\left(\frac{\pi}{x}\right) \ge x$)

$\lim_{x \to 0} x = 0$ ∴ $\underset{\downarrow \atop 0}{-x} \le x \sin\left(\frac{\pi}{x}\right) \le \underset{\downarrow \atop 0}{x}$

So $x \sin\left(\frac{\pi}{x}\right) \xrightarrow{x \to 0} 0$

Limit Laws : What you hope & expect.

If $\lim\limits_{x\to c} f(x) = K$ and $\lim\limits_{x\to c} g(x) = L$ Then

$\lim (f \pm g) = \lim f \pm \lim g = K \pm L$

$\lim (fg) = KL$

$\lim \frac{f}{g} = \frac{K}{L}$ (if $L \neq 0$)

$\lim (f^n) = K^n$

→ Just because $\lim (f+g)$ exists doesn't necessarily mean $\lim f$ & $\lim g$ exist, separately.

Consider $f(x) = x$ & $g(x) = -x$

Then $\lim\limits_{x\to\infty} f(x) = \infty$ (NOT a real #)

$\lim\limits_{x\to\infty} g(x) = -\infty$ (" " " ")

$f+g = x + (-x) = 0$ &

$\lim\limits_{x\to\infty} (f+g) = 0 = \lim\limits_{x\to-\infty} (f+g)$

Close to done thru §1.6.