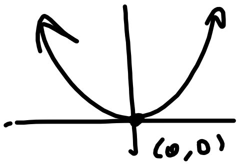
Essential Funcs

$f(x) = x^n \begin{cases} \nearrow x^{2n} \\ \searrow x^{2n+1} \end{cases}$ $(n \in \mathbb{N})$

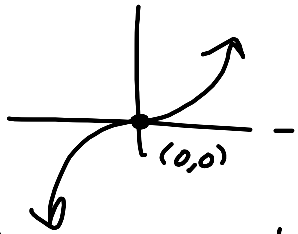
$\vdots$ 9 7 5

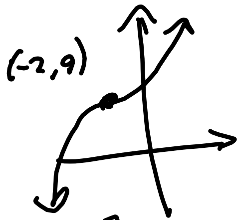
$7(3x+6)^3 + 9$

x^{2n}



x^{2n+1}



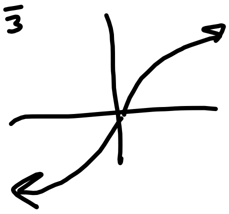


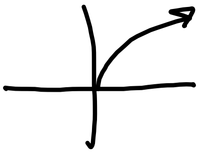
$0 < \frac{a}{b} < 1$

$x^{\frac{odd}{odd}}$ $\frac{3}{5}, \frac{7}{9} \dots$

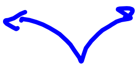
$x^{\frac{odd}{even}}$ $\frac{1}{2}, \frac{7}{12}$

$x^{\frac{1}{3}}$

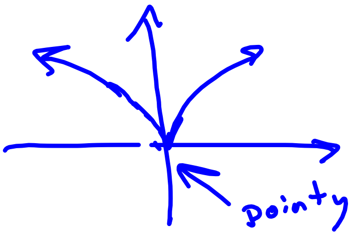




$x^{\frac{1}{2}}$

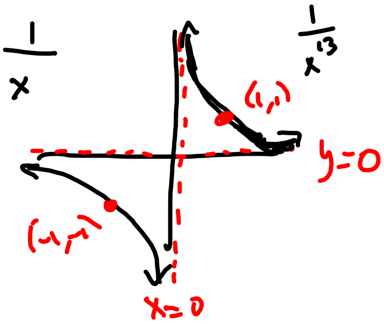


$x^{\frac{even}{odd}}$ $x^{\frac{2}{5}}$



$\frac{1}{x^n} \begin{cases} \nearrow \frac{1}{x^{2n}} \\ \searrow \frac{1}{x^{2n+1}} \end{cases}$

$\frac{1}{x}$ $\frac{1}{x^{13}}$ $\frac{1}{x^{2n+1}}$

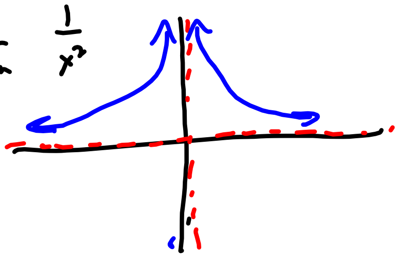


$\frac{1}{.001} = 1000$

$\frac{1}{.0001} = 10{,}000$

$\frac{1}{1000} = .001$

$\frac{1}{x^{2n}}$ $\frac{1}{x^2}$



$F(x) = \dfrac{x^2-3x-4}{x^2+2x-3} = \dfrac{(x-4)(x+1)}{(x+3)(x-1)}$

$-3, 1, -1, 4$
$-3, -1, 1, 4$

$D = \mathbb{R} \setminus \{-3, 1\}$

Holes? No!

Vertical Asymptotes

$\boxed{VA: \; x=-3, \; x=1}$

Zeros: $x = -1, 4$

x-int: $\boxed{(-1,0), \; (4,0)}$

$\dfrac{(x-4)(x+1)}{(x+3)(x-1)}$

Horizontal Asymptotes

HA: $x \to \pm\infty$

$\boxed{y = 1}$

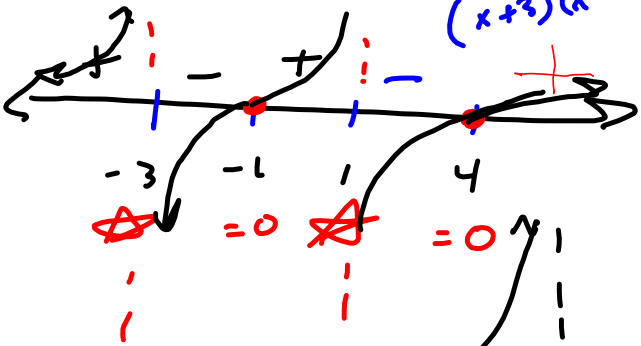
$\dfrac{x^2}{x^2}$

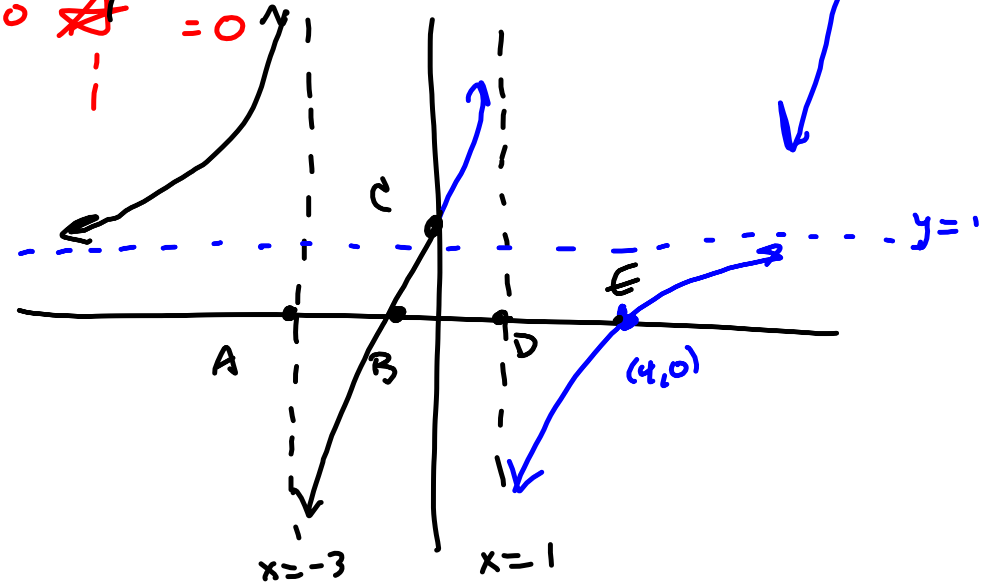
Same $\Rightarrow$ Focus on big stuff

$\dfrac{x^2}{x^2} = 1 \xrightarrow{x \to \pm\infty} 1$

$$\frac{(x-4)(x+1)}{(x+3)(x-1)}$$

$$F(0)=\frac{(-4)(1)}{(3)(-1)}=\frac{4}{3}$$





A= (-3,0)

B = (-1, 0)

C = (0, $\frac{4}{3}$)

D = (1,0)

E = (4,0)

RATIONAL functions.

$$\frac{3x^2+197x-500}{2x^2-\pi x+75.1} \xrightarrow{|x|\to\infty}$$

$$\frac{3x^2}{2x^2}=\frac{3}{2} \rightarrow y=\frac{3}{2} \text{ is H.A.}$$

Proper

$$\frac{3x^2+7}{5x^3-9}: \boxed{\frac{3x^2}{5x^3}}$$ $3x^2$ ← smaller, $5x^3$ ← Bigger

$$\frac{3x^2}{5x^3}=\frac{3}{5x} \xrightarrow{|x|\to\infty} 0$$

$y=0$ is H.A.

$$\frac{5x^3-9}{3x^2+7}: \frac{5x^3}{3x^2}$$ $5x^3$ ← Bigger, $3x^2$ ← smaller

O.A.: oblique Asymptote, requiring long division

$$\frac{5x}{3} \rightarrow y=\frac{5x}{3}=\text{O.A.}$$

$$3x^2+7 \overline{\big)\, 5x^3+0x^2+0x-9}$$

$$-\left(5x^3 \quad +\frac{35x}{3}\right)$$

$$-\frac{35x}{3}-9$$

$$\frac{5x^3}{3x^2}=\frac{5x}{3}$$

$= \dfrac{(x-4)(x+1)\,(x-7)}{(x+3)(x-1)\,(x-7)}$ has a hole @ $(7, F(7))$, where

$F(x) = \dfrac{(x-4)(x+1)}{(x+3)(x-1)}$

$F(7) = \dfrac{(3)(8)}{(10)(6)} = \dfrac{(1)(4)}{(5)(2)} = \dfrac{2}{5} \rightsquigarrow \left(7, \dfrac{2}{5}\right)$ is hole.