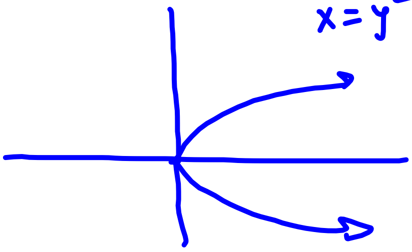
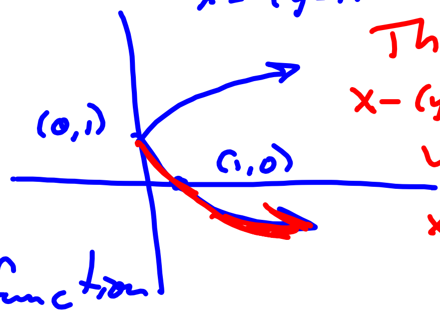
§1.1 #53

Bottom half of the parabola $x+(y-1)^2=0$

$x=y^2$



$x=(y-1)^2$



This is $x-(y-1)^2=0$

want $x+(y-1)^2=0$

write as a function of x.

$$(y-1)^2=-x$$

$$\sqrt{(y-1)^2}=\sqrt{-x}$$

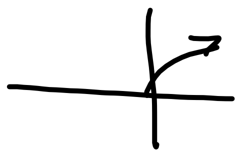
$$|y-1|=\sqrt{-x}$$

$$y-1=\pm\sqrt{-x}$$

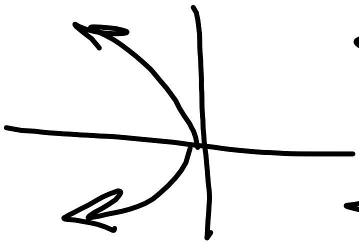
$$y=1\pm\sqrt{-x}$$

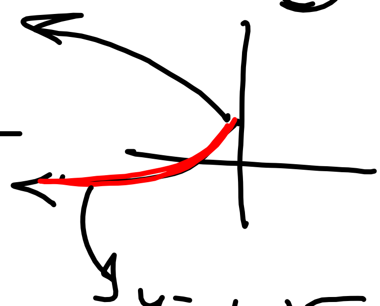
$$x+(y-1)^2=0$$

$$x=-(y-1)^2$$



1. $x=y^2$
2. $x=-y^2$
3. $x=-(y-1)^2$





$y=1-\sqrt{-x}$

57–61 Find a formula for the described function and state its domain.

57. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.

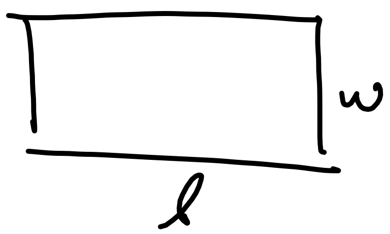
$$2l + 2w = 20$$

$$l + w = 10$$

$$l = 10 - w$$

$$\text{Area} = lw$$

$$= \boxed{(10-w)w = A(w)}$$



l = length (in m)

w = width

31–37 Find the domain of the function. & Sketch

31. $f(x) = \dfrac{x+4}{x^2-9}$ **32.** $f(x) = \dfrac{2x^3-5}{x^2+x-6}$

#31 Need $x^2-9 \neq 0$

$x^2 = 9$

$x = \pm 3$

$(x+3)(x-3) = 0$

$x = \pm 3$

$\mathcal{D} = \{x \mid x \neq \pm 3\} = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$

$= \mathbb{R} \setminus \{\pm 3\}$

31 $g(x) = \dfrac{x+4}{\sqrt{x^2-9}}$ Need $\sqrt{x^2-9} \neq 0$

negative stuff

0

AND

$x^2-9 \geq 0$

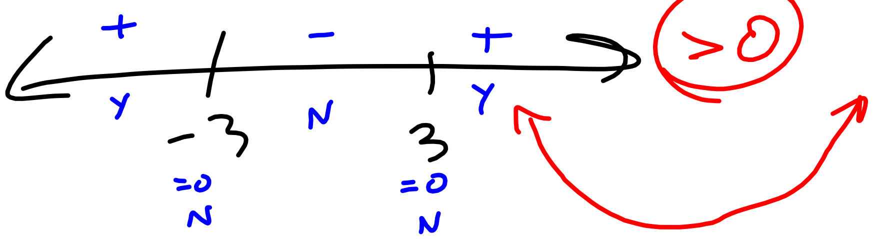
$x^2-9 \neq 0$

$x^2-9 > 0$

Parabola

$x^2-9 = (x-3)(x+3) = 0 \Rightarrow x = \pm 3$

>0



$(-3$ $3)$

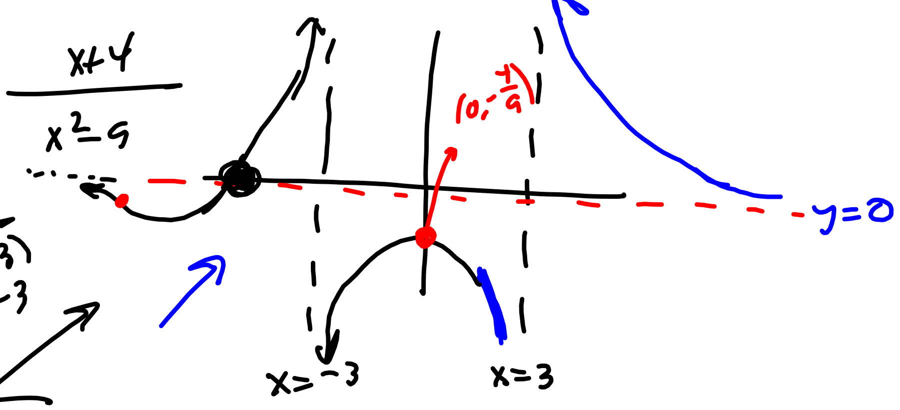
-3 3

$= (-\infty,-3) \cup (3,\infty)$

+ − +

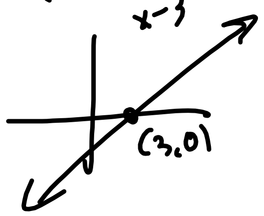
-4 -3 3

$=0$



$\dfrac{x+4}{(x-3)(x+3)}$

$x-3$

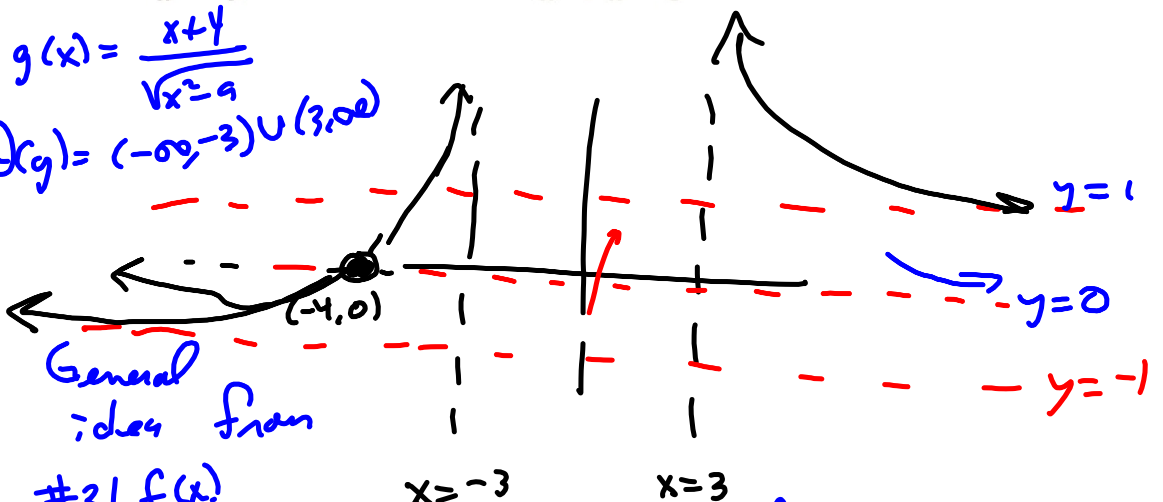


31–37 Find the domain of the function. & Sketch

31. $f(x) = \dfrac{x+4}{x^2-9}$ **32.** $f(x) = \dfrac{2x^3-5}{x^2+x-6}$

$g(x) = \dfrac{x+4}{\sqrt{x^2-9}}$

$\mathcal{D}(g) = (-\infty,-3) \cup (3,\infty)$



What about the horizontal asymptote we stole from f(x)?

Recall $f(x) = \dfrac{x+4}{x^2-9} \xrightarrow{x \to \pm\infty} 0$

because $x^2-9 > x+4$, eventually.

Now $\dfrac{x+4}{\sqrt{x^2-9}}$, the $x+4$ & $\sqrt{x^2-9}$ are close to the same size for large x.

$$\frac{x+4}{\sqrt{x^2\left(1-\frac{9}{x^2}\right)}} = \frac{x+4}{|x|\sqrt{1-\frac{9}{x^2}}} \xrightarrow{x \to +\infty} 1$$

$$\frac{x+4}{|x|\sqrt{1-\frac{9}{x^2}}} \xrightarrow{x>0} \frac{x+4}{x\sqrt{1-\frac{9}{x^2}}}$$

$$= \frac{x\left(1+\frac{4}{x}\right)}{x\sqrt{1-\frac{9}{x^2}}} = \frac{1+\frac{4}{x}}{\sqrt{1-\frac{9}{x^2}}} \xrightarrow{x \to +\infty} 1$$

$|-3|$

$= -(-3)$

$= 3$

$$\frac{x+4}{|x|\sqrt{1-\frac{9}{x^2}}} \xrightarrow{x<0} \frac{x\left(1+\frac{4}{x}\right)}{-x\sqrt{1-\frac{9}{x^2}}}$$

$$= \frac{1+\frac{4}{x}}{-\sqrt{1-\frac{9}{x^2}}} \xrightarrow{x \to -\infty} -1$$