

Survey old tests.

From test 3 #3, which I never got to ask you.

$\frac{dy}{dx}$ by limit def'n

$\int_a^b f(x) dx$ by limit def'n

Prove a linear limit (Quadratic Limit is

Bonus)

Prove $\lim_{x \rightarrow 2} 3x - 7 = 1$

Let $\epsilon > 0$ be given. Define $\delta = \frac{1}{3}$. Then

$$0 < |x - 2| < \delta \implies$$

$$|3x - 7 - 1| = |3x - 6| = 3|x - 2| < 3 \cdot \delta = 3 \cdot \frac{\epsilon}{3} = \epsilon \quad \square$$

Bonus

$\lim_{x \rightarrow 2} (x^2 + 4x + 5) = 17$. Assume $\delta \leq 1$

Scratch.

$$x^2 + 4x + 5 = 17 = x^2 + 4x - 12 = \frac{(x+6)(x-2)}{1} < \epsilon$$

$$|x+6||x-2| < \delta |x+6|$$

Need a bound,

$$\delta \leq 1 \implies$$

$$1 < x < 3$$

$$7 < x+6 < 9$$

$$\implies |x+6| < 9$$

$$\text{So, } |x+6||x-2| < 9\delta < \epsilon$$

Let $\epsilon > 0$. Define $\delta = \min\left\{1, \frac{\epsilon}{9}\right\}$. Then $0 < |x - 2| < \delta$

$$\implies |x^2 + 4x + 5 - 17| = |x^2 + 4x - 12| = |x+6||x-2|$$

$$< 9\delta \leq 9 \cdot \frac{\epsilon}{9} = \epsilon \quad \square$$

x^2

u-subst. f. the f. v. m

$$\int x^3 (3x-1)^7 dx$$

$$u = 3x-1$$

$$du = 3 dx$$

$$dx = \frac{du}{3}$$

$$3x-1 = u$$

$$3x = u+1$$

$$x = \frac{u+1}{3}$$

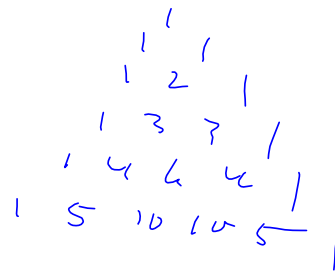
$$= \frac{1}{3^3} \int \underbrace{(u^3 + 3u^2 + 3u + 1)}_x u^7 \frac{du}{3}$$

$$x^3 = \left(\frac{u+1}{3}\right)^3 =$$

$$\frac{u^3 + 3u^2 + 3u + 1}{3^3}$$

$$= \frac{1}{3^4} \int (u^{10} + 3u^9 + 3u^8 + u^7) du$$

$$= \frac{1}{3^4} \left[\frac{u^{11}}{11} + \frac{3u^{10}}{10} + \frac{3u^9}{9} + \frac{u^8}{8} \right] + C$$



POI CHECK

$$\left[\frac{1}{3^4} \left[\frac{(3x-1)^{11}}{11} + \frac{3(3x-1)^{10}}{10} + \frac{(3x-1)^9}{3} + \frac{(3x-1)^8}{8} \right] \right] + C$$

$$= \frac{(3x-1)^8}{3^4} \left[\frac{(3x-1)^3}{11} + \frac{3(3x-1)^2}{10} + \frac{3x-1}{3} + \frac{1}{8} \right] + C$$

$$\int \frac{x^2}{(3x-1)^{\frac{5}{3}}} dx$$

$$u = 3x-1$$

$$du = 3dx$$

$$dx = \frac{du}{3}$$

$$x = \frac{u+1}{3}$$

$$x^2 = \frac{u^2 + 2u + 1}{3^2}$$

Alex

$$= \frac{1}{9} \int (u^2 + 2u + 1) \left(u^{\frac{5}{3}}\right) \frac{du}{3}$$

$$= \frac{1}{3^3} \int \left(u^{\frac{19}{3}} + 2\left(u^{\frac{12}{3}}\right) + u^{\frac{5}{3}}\right) du$$

$$= \frac{1}{3^3} \left[\frac{7}{26} u^{\frac{26}{3}} + 2 \left(\frac{7}{19} u^{\frac{19}{3}} \right) + \frac{7}{12} u^{\frac{12}{3}} \right] + C$$

= etc. Sub $u = 3x-1 = u$.

$$256 = 16^2 = \left(\frac{1}{2}\right)^2 = 2^{10}$$

$$\frac{1}{256} = \frac{1}{2^8} \text{ is more modular.}$$

$$\frac{1}{(x-6)^2(x+5)^3}$$

$$3 \quad \frac{1}{3} \quad *$$

$$-3 \quad *$$

Poly's with $+$ & $*$ → FIELDS $\mathbb{R}, \mathbb{C}, \mathbb{Q}$
 RINGS Like a field, only division is trickier.
 GROUPS Integers with "+"

$$x^2 + 2x - 1$$

$$- (x^2 + 2x - 1)$$

MVT on a cubic $\exists c \in (a, b) \exists$
 $f'(c) = \frac{f(b) - f(a)}{b - a}$

confirm the hypotheses.

IVT } Continuity
 EVT }

Friday Test.

Shell, Washer.

work

MVT I

MVT I on a quadratic

$$c \in (a, b) \exists \frac{1}{b-a} \int_a^b f(x) dx = f(c)$$