Survey old tests.

From Test 3 #3, which I never got to ask you.

$\frac{dy}{dx}$ by limit defn

$\int_a^b f(x)\,dx$ by limit defn.

Prove a linear limit (Quadratic Limit in Bonus)

Prove $\lim_{x \to 2} 3x-7=1$

Let $\varepsilon>0$ be given. Define $\delta = \frac{\varepsilon}{3}$. Then

$0<|x-2|<\delta \Rightarrow$

$|3x-7-(-1)| = |3x-6| = 3|x-2| < 3\cdot\delta = 3\cdot\frac{\varepsilon}{3} = \varepsilon$ ∎

Bonus

$\lim_{x\to 2}(x^2+4x+5) = 17$. Assume $\delta \le 1$

Scratch.

$x^2+4x+5-17 = x^2+4x-12 = (x+6)\underbrace{(x-2)}_{<\delta}$ want $< \varepsilon$

$|x+6||x-2| < \delta|x+6|$

Need a bound,

$\delta \le 1 \Rightarrow$

$1 < x < 3$

$7 < x+6 < 9$

$\Rightarrow |x+6| < 9$

So, $|x+6||x-2| < 9\delta < \varepsilon$

Let $\varepsilon>0$. Define $\delta = \min\{1, \frac{\varepsilon}{9}\}$. Then $0<|x-2|<\delta$

$\Rightarrow |x^2+4x+5-17| = |x^2+4x-12| = |x+6||x-2|$

$< 9\delta \le 9\cdot\frac{\varepsilon}{9} = \varepsilon$ ∎

x^2

u-substitution

$$\int x^3 (3x-1)^7\,dx$$

$u = 3x-1$

$du = 3\,dx$

$dx = \frac{du}{3}$

$3x-1 = u$

$3x = u+1$

$x = \frac{u+1}{3}$

$$= \frac{1}{3^3}\int \underline{(u^3+3u^2+3u+1)\,u^7}\,\frac{du}{3}$$

$$x^3 = \left(\frac{u+1}{3}\right)^3 = \frac{u^3+3u^2+3u+1}{3^3}$$

$$= \frac{1}{3^4}\int \left(u^{10}+3u^9+3u^8+u^7\right)du$$

$$= \frac{1}{3^4}\left[\frac{u^{11}}{11}+\frac{3u^{10}}{10}+\frac{3u^9}{9}+\frac{u^8}{8}\right]+C$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

POI FECK

$$\boxed{= \frac{1}{3^4}\left[\frac{(3x-1)^{11}}{11}+\frac{3(3x-1)^{10}}{10}+\frac{(3x-1)^9}{3}+\frac{(3x-1)^8}{8}\right]+C}$$

$$= \frac{(3x-1)^8}{3^4}\left[\frac{(3x-1)^3}{11}+\frac{3(3x-1)^2}{10}+\frac{3x-1}{3}+\frac{1}{8}\right]+C$$

$$\int \frac{x^2}{(3x-1)^{-5/7}}\,dx$$

$u = 3x-1$
$du = 3\,dx$
$dx = \frac{du}{3}$

$x = \frac{u+1}{3}$
$x^2 = \frac{u^2+2u+1}{3^2}$

Alex

$$= \frac{1}{9}\int (u^2+2u+1)(u^{\frac{5}{7}})\frac{du}{3}$$

$\frac{5}{7} + \frac{14}{7}$

$$= \frac{1}{3^3}\int \left(u^{\frac{19}{7}} + 2(u^{\frac{12}{7}}) + u^{\frac{5}{7}}\right) du$$

$\frac{5}{7} + \frac{7}{7}$

$$= \frac{1}{3^3}\left[\frac{7}{26}u^{\frac{26}{7}} + 2\left(\frac{7}{19}u^{\frac{19}{7}}\right) + \frac{7}{12}u^{\frac{12}{7}}\right] + C$$

= etc. Sub in $3x-1 = u$.

$256 = 16^2 = (2^4)^2 = 2^8$

$\frac{1}{256} = \frac{1}{2^8}$ is more modular.

$$\frac{1}{(x-6)^2(x+5)^3}$$

$3 \quad \frac{1}{3} \quad \times$

$-3 \quad \times$

Poly's with $+ \ \& \ \times \longrightarrow$

FIELDS $\mathbb{R}, \mathbb{C}, \mathbb{Q}$

RINGS Like a field, only division is trickier.

GROUPS Integers with "+"

$x^2 + 2x - 1$

$-(x^2 + 2x - 1)$

MVT on a cubic $\exists\, c \in (a,b) \ni$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Confirm the hypotheses.

$\left.\begin{matrix} \text{IVT} \\ \text{EVT} \end{matrix}\right\}$ Continuity

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Friday Test.

Shell, Washer.

Work

MVTI

MVTI on a quadratic

$c \in (a,b) \ni \frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$