MAT 201
100 Points

Test 5, Fall, 2017
Covers Chapter 5 (and before)

Name KEY

You know the drill. And remember to circle final answers.

1. (20 pts) Sketch and find the area of the region bounded by $f(x) = 4\sqrt{2x}$ and $g(x) = 2x^2$.

2. Sketch the solid and then write the integral for the volume of the solid of revolution obtained by revolving the region bounded by $f(x) = 4\sqrt{2x}$ and $g(x) = 2x^2$ about the y-axis, using...

   a. (10 pts) ... washers (slices) and
   b. (10 pts) ... cylindrical shells (probably easier).

3. (20 pts) A 4-kg bucket, holding 20 kg of water is hoisted up from a 30-meter-deep well on a chain that has linear density .5 kg/m. How much work is done lifting the bucket of water up to the top of the well? (Use 9.8 m/s^2 for the acceleration due to gravity.)

4. (20 pts) Find the average value, f_{AVG}, of $f(x) = 3x^2 - 2x + 5$ on the interval $[1,6]$. Then find all $c \in (1,6)$ such that $f(c) = f_{AVG}$

5. (10 pts) If $g(x) = \int_0^{\sin(x)} \left(\pi t^2 + 17t - \cos(t)\right) dt$, what is $g'(x)$?

6. Suppose x and y are related to one another by the equation $x^2 - 4x\sin(y) - y^3 = 81$.

   a. (5 pts) Find $\dfrac{dy}{dx}$.
   b. (5 pts) Based on part a., find an equation of the tangent line to the curve at the point $(3,2)$.

Bonus Section Work up to 3 of the following problems, for up to 30 extra points.

Bonus 1 (10 pts) Use the limit definition of the derivative to find $f'(x)$ for $f(x) = 2x^2 + 5x$.

Bonus 2 (10 pts) Prove that $\lim\limits_{x \to 3} (5x - 4) = 11$, using the formal definition of the limit.

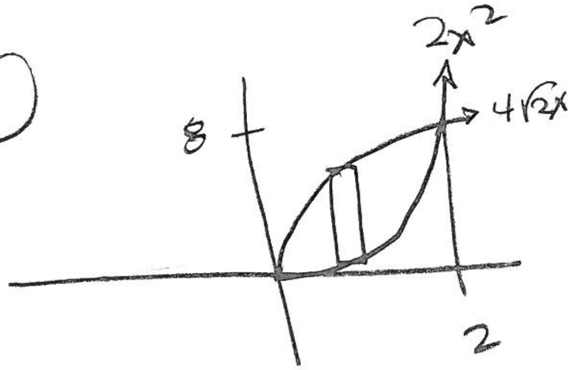
Bonus 3 (10 pts) Evaluate the definite integral $\int_0^{\pi} |2\cos(x) - 1| \, d\theta$

Bonus 4 (10 pts) Find all extreme values of $f(x) = \sin(2x) - x$ on $[0, 2\pi)$.

Bonus 5 (10 pts) Use the tangent line to approximate $\sqrt[3]{30}$.

201 E5

1



$f = g$ 5

$\int_a^b f-g$ setup 10

$\int_a^b f-g$ eval 5

$$\text{Area} = \int_a^b f-g = \int_0^2 \left(4\sqrt{2x} - 2x^2\right) dx$$

$$= 4\int_0^2 (2x)^{\frac{1}{2}} dx - 2\int_0^2 x^2 dx = 4\int_{0=x}^{2=x} u^{1/2}\left(\frac{du}{2}\right) - 2\cdot \frac{x^3}{3}\Bigg]_0^2$$

$u = 2x$

$du = 2dx$

$dx = \frac{du}{2}$

$$= 2\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^2 - \frac{2}{3}\left[8-0\right]$$

$$= \frac{4}{3}\left[(2x)^{\frac{3}{2}}\right]_0^2 - \frac{16}{3} = \frac{4}{3}\left[4^{\frac{3}{2}} - 0\right] - \frac{16}{3}$$

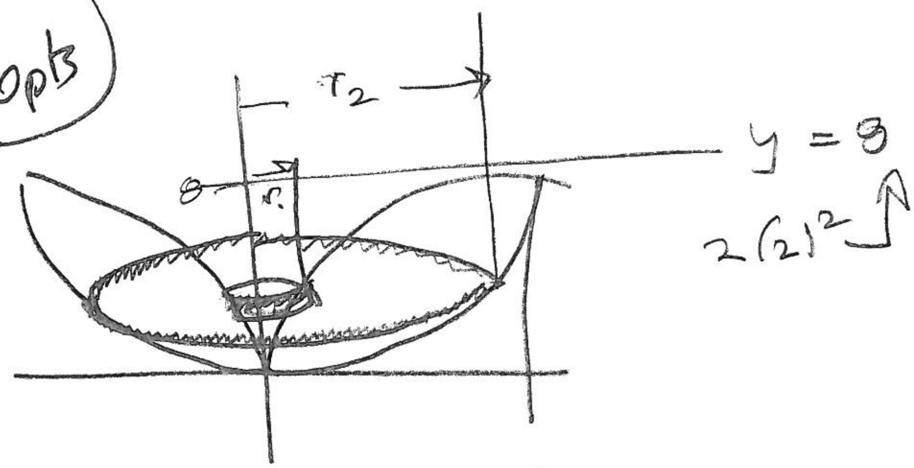
20 pts

$$= \frac{4}{3}\left[8\right] - \frac{16}{3} = \frac{32-16}{3} = \boxed{\frac{16}{3}}$$

2

2

10pts



$$\pi \int_0^8 \left((r_2)^2 - (r_1)^2\right) dy$$

$r_2 = x$ from $y = 2x^2$

so $x^2 = \frac{1}{2} y$

$x = \pm \sqrt{\frac{y}{2}}$

Take the positive

$r_1 = x$ from $y = 4\sqrt{2x}$

$\frac{y}{4} = \sqrt{2x}$

$\frac{y^2}{16} = 2x$

$\frac{y^2}{32} = x$

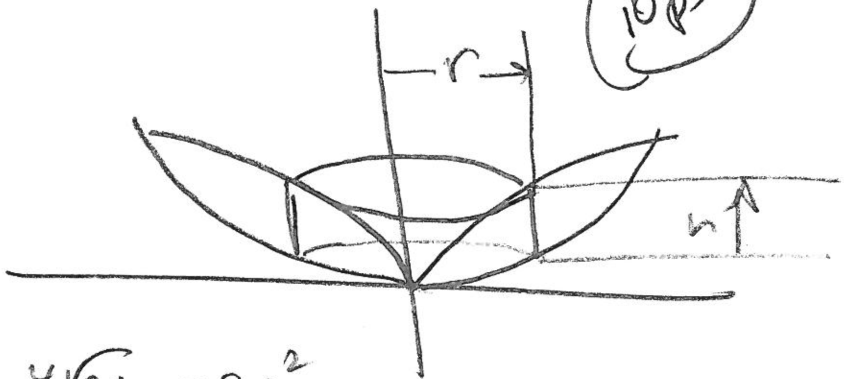
$$= \boxed{\pi \int_0^8 \left(\left(\sqrt{\frac{y}{2}}\right)^2 - \left(\frac{y^2}{32}\right)^2\right) dy}$$

≈ 30.159289

201 ES

2 b

10 pts



$r = x$, $h = 4\sqrt{2x} - 2x^2$

$$\text{Volume} = 2\pi \int_0^2 x \left(4\sqrt{2x} - 2x^2\right) dx \approx$$

30.159286

201 E5

20pts

(3) 4 kg bucket, 20 kg water, 30-m well.
chain has density .5 kg/m
work done lifting bucket & water thru 30 m.

$(4\text{kg})(9.8)(30) = \boxed{1176 \text{ J for the bucket}}$

$(20\text{ kg})(9.8)(30) = \boxed{5880 \text{ J for the water}}$

for the chain, we know

$(.5 \text{ kg/m})(30 \text{ m})(9.8) = 147 \text{ N @ bottom}$

& 0 kg @ top, so

$(0, 147)$

$(30, 0)$

$m = -\frac{147}{30} \frac{N}{m}$

Bucket – 5pts
Water – 5pts
Chain – Model – 7pts
Eval – 3pts

$F = -\frac{147}{30}(y - 0) + 147$

$= -\frac{147}{30}y + 147$

work done $\int_0^{30} \left(-\frac{147}{30}y + 147\right) dy$

2205 + 5880 + 1176

$= -\frac{147}{30}\left[\frac{y^2}{2}\right]_0^{30} + \left[147y\right]_0^{30}$

$-2205 + 4410 = \boxed{2205 \text{ J}}$ chain

TOTAL WORK: $\boxed{9261 \text{ J}}$ FINAL ANS

201 E5

(3) 4 Kg Bucket, 20 Kg Water, 30-m well,

Rope/chain is $\frac{.5\text{ Kg}}{\text{m}}$

Bucket + Water

$$(24\text{ Kg})(9.8\ \text{m}/\text{s}^2)(30\text{ m}) = \boxed{7056\text{ J.}}$$

$(1176\text{ J} + 5880\text{ J})$

Chain: $(.5\text{ kg}/\text{m})(9.8\ \text{m}/\text{s}^2)(0\text{ m})$ @ Top

$= 0\text{N}$ @ Top

$(.5\ \frac{\text{kg}}{\text{m}})(9.8\ \text{m}/\text{s}^2)(30\text{ m}) = 147\text{ N}$ @ Bottom

$$\Rightarrow \left.\begin{aligned}(y_1, F_1) &= (0, 147)\\ (y_2, F_2) &= (30, 0)\end{aligned}\right\} m = \frac{0-147}{30-0} = -\frac{147}{30}$$

$$\rightarrow F = -\frac{147}{30}(y-0) + 147$$

$$= -4.9y + 147\text{ N}$$

$$\text{Work} = \int_0^{30} (-4.9y + 147)\,dy = \left[-\frac{4.9}{2}y^2 + 147y\right]_0^{30}$$

$$= \left[-2.45y^2 + 147y\right]_0^{30} = -2.45(900) + 147(30)$$

Chain $\rightarrow$

$= -2205 + 4410 = \boxed{2205\text{ J}}$

$7056 + 2205 = \boxed{9261\text{ J TOTAL}}$

201 E5

(4) $f(x) = 3x^2 - 2x + 5 \longrightarrow f_{AVG}$ on $[1,6]$ 13

20pts

$$\frac{1}{6-1}\int_1^6 (3x^2 - 2x + 5)\,dx = \frac{1}{5}\left[x^3 - x^2 + 5x\right]_1^6$$

$$= \frac{1}{5}\left[\left(6^3 - 6^2 + 5(6)\right) - \left(1^3 - 1^2 + 5(1)\right)\right]$$

$$= \frac{1}{5}\left[216 - 36 + 30 - (5)\right]$$

$$= \frac{1}{5}\left[205\right] = \boxed{41 = f_{AVG}}$$

$3x^2 - 2x + 5 = 41$

$3x^2 - 2x - 36 = 0$

$a = 3, b = -2, c = -36$

$b^2 - 4ac = (-2)^2 - 4(3)(-36)$

$= 4 + 432 = 436$

$2 \,|\, 436$

$2 \,|\, 218$

109

$$x = \frac{4 \pm 2\sqrt{109}}{2(3)} = \frac{2 \pm \sqrt{109}}{3}$$

$$\longrightarrow \boxed{c = \frac{2 + \sqrt{109}}{3} \approx 3.813435503}$$

(5) $$\boxed{g'(x) = \left(\pi \sin^2(x) + 17\sin(x) - \cos(\sin(x))\right)\left(\cos(x)\right)}$$

10pts

2201 E5

(6) $x^2 - 4x\sin(y) - y^3 = 81 \longrightarrow$

(a) $2x - 4\sin(y) - 4x\cos(y)\,y' - 3y^2 y' = 0$

$\left(-4x\cos(y) - 3y^2\right) y' = 4\sin(y) - 2x$

$$\boxed{y' = \frac{4\sin(y) - 2x}{-4x\cos(y) - 3y^2}}$$

(b) $\rightarrow y'\Big|_{\substack{x=3\\y=0}} = \dfrac{4\sin(0) - 2(3)}{-4(3)(\cos(0)) - 3(0)^2}$

$= \dfrac{-6}{-12} = \dfrac{1}{2}$

$$\boxed{y = \tfrac{1}{2}(x - 3) + 0}$$

201 E5

B1 $\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2+5(x+h)-(2x^2+5x)}{h}$

$= \frac{2(x^2+2xh+h^2)+5x+5h-2x^2-5x}{h}$

$= \frac{2x^2+4xh+2h^2+5h-2x^2}{h} = \frac{4xh+2h^2+5h}{h}$

$= 4x+2h+5 \xrightarrow{h\to 0} \boxed{4x+5}$

$(h \neq 0)$

B2 $\lim_{x\to 3}(5x-4)=11$

Proof
Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{5}$. Then
$0 < |x-3| < \delta \Rightarrow |(5x-4)-11| = |5x-15|$

$= 5|x-3| < 5\delta = 5\cdot\frac{\varepsilon}{5} = \varepsilon$ ▨

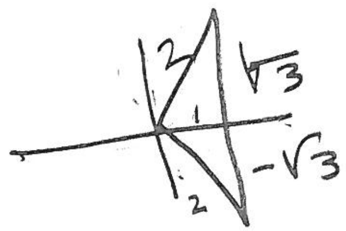
201 E5

B3 $\displaystyle\int_0^{\pi} |2\cos x - 1|\, dx$

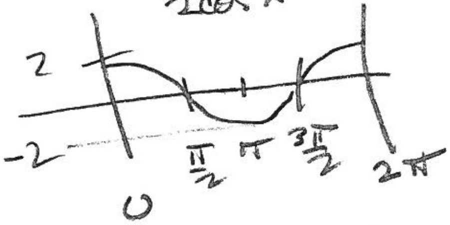
$\frac{\pi}{3}, \frac{5\pi}{3}$

$2\cos x - 1 = 0$

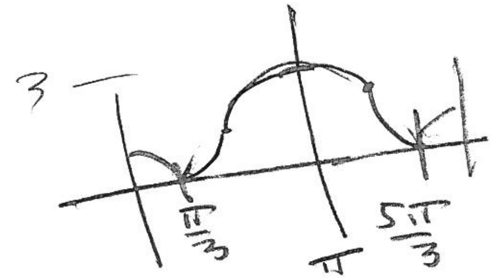
$\cos x = \frac{1}{2}$



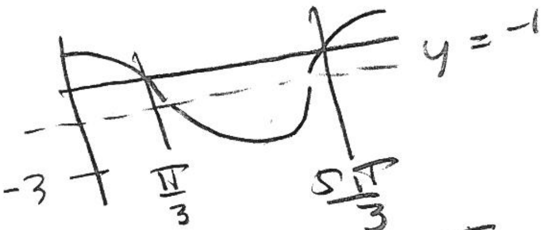
2cos x



|2cos x - 1|



2cos x - 1



$$\int_0^{\pi} |2\cos x - 1|\, dx = \int_0^{\frac{\pi}{3}} (2\cos x - 1)\, dx - \int_{\frac{\pi}{3}}^{\pi} (2\cos x - 1)\, dx$$

$$= \Big[2\sin x - x\Big]_0^{\frac{\pi}{3}} - \Big[2\sin x - x\Big]_{\frac{\pi}{3}}^{\pi}$$

$$= 2\sin\left(\frac{\pi}{3}\right) - \frac{\pi}{3} - 0 - \left[2\sin\pi - \pi - \left(2\sin\left(\frac{\pi}{3}\right) - \frac{\pi}{3}\right)\right]$$

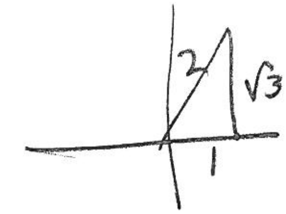
$$= 2\cdot\frac{\sqrt{3}}{2} - \frac{\pi}{3} - \left[0 - \pi - \left(2\cdot\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right)\right]$$

$$= \sqrt{3} - \frac{\pi}{3} + \pi + \sqrt{3} - \frac{\pi}{3} = \boxed{2\sqrt{3} + \frac{\pi}{3} \approx 4.51299167}$$

201 E5

B4 $f(x) = \sin(2x) - x$ on $[0, 2\pi)$

$f'(x) = 2\cos(2x) - 1 \overset{SET}{=} 0$ ⟶ ⟶



$\cos(2x) = \frac{1}{2}$ $\quad x \in [0, 2\pi)$

$2x \in [0, 4\pi)$

$f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) - \frac{\pi}{6}$

$= \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

$2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

$f\left(\frac{5\pi}{6}\right) = \sin\left(\frac{5\pi}{3}\right) - \frac{5\pi}{6}$

$= -\frac{\sqrt{3}}{2} - \frac{5\pi}{6}$

$f\left(\frac{7\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{7\pi}{6}, \; f\left(\frac{11\pi}{6}\right) = -\frac{\sqrt{3}}{2} - \frac{11\pi}{6}$

B5 $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}, \; x_1 = 27, \; f(x_1) = 3$

want $x = 30$

B4 $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$ MAX

$\left(\frac{5\pi}{6}, -\frac{\sqrt{3}}{2} - \frac{5\pi}{6}\right)$ MIN

$\left(\frac{7\pi}{6}, \frac{\sqrt{3}}{2} - \frac{7\pi}{6}\right)$ MAX

$\left(\frac{11\pi}{6}, -\frac{\sqrt{3}}{2} - \frac{11\pi}{6}\right)$ MIN

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f'(x_1) = \frac{1}{3}(27)^{-\frac{2}{3}} = \frac{1}{3}(3)^{-2} = \frac{1}{27}$

So $f(x) \approx L(x) = \frac{1}{27}(x - 27) + 3$

$\rightarrow f(30) = \frac{1}{27}(30 - 27) + 3 = \frac{3}{27} + 3 = \frac{1}{9} + 3$

$= \frac{28}{9} = \boxed{3.\overline{1} \approx \sqrt[3]{30}}$

ACTUAL = 3.107232506