MAT 201 Test 4, Fall, 2017 Name______________________

100 Points Covers Chapter 4

You know the drill. And remember to circle final answers.

1. (20 pts) Use the limit definition of the definite integral to evaluate $\int_0^4 (x^2 + 3x)dx$.

2. (10 pts) Show that $\int_0^1 x^2 dx = \int_0^1 (1 - \sqrt{y})dy$ by evaluating each, separately.

3. (10 pts) Evaluate the definite integral: $\int_0^{\frac{\pi}{2}} |\, 2\sin(x) - 1 \,| dx$.

4. Evaluate the indefinite integrals:

   a. (10 pts) $\int (3x+1)^4 \, dx$

   b. (10 pts) $\int (3x+1)^4 x^2 \, dx$

   c. (10 pts) $\int \sec^4(x)\tan(x)dx$

5. (10 pts) Evaluate the definite integral $\int_0^{\frac{\pi}{4}} \sec^4(x)\tan(x)dx$

6. Suppose I'm pacing back and forth, thinking my usual deep thoughts, and my rate of speed is given by $r(t)$, in feet per second. Tell me what the following integrals represent:

   a. (5 pts) $\int_0^{3600} |r(t)| dt$

   b. (5 pts) $\int_0^{3600} r(t) \, dt$

7. Perform the indicated differentiation:

   a. (5 pts) $\frac{d}{dx}\int_0^x \frac{\sin(3t)}{t^2+4} dt$

   b. (5 pts) $\frac{d}{dx}\int_{x^2}^{\cos(x)} \frac{\sin(3t)}{t^2+4} dt$

See back page for bonus!

Bonus Section

Bonus 1 (5 pts) What would the x_k be in #1, if we were integrating over $[2,4]$, instead of $[0,4]$?

Bonus 2 (5 pts) Explain, with the help of some pictures, what's going on with the two integrals in #3.

Bonus 3 (5 pts) Evaluate the definite integral $\int_{\frac{\pi}{4}}^{\frac{2\pi}{3}} \left|\csc\theta\cot\theta\right| d\theta$

Bonus 4 (5 pts) Find an upper and lower bound for $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin(x)\,dx$, without evaluating the integral itself.

Bonus 5 (5 pts) Confirm that the hypotheses of the Mean Value Theorem hold for $f(x)=x^3-2x^2+5x-1$ on $[0,3]$, and find the c that is promised in the conclusion of the theorem.

Bonus 6 (5 pts) Compute the derivative of $f(x)=\sqrt{3x}$ by the limit definition.

Bonus 7 (5 pts) Use the tangent line to approximate $\cos(33^0)$.

Bonus 8 (5 pts) Find $\frac{dy}{dx}$ if $x^2-3xy+y^2=1$. Then find an equation of the tangent line to the curve at $(1,3)$.

Bonus 9 (5 pts) Explain, using the diagram, below, how Newton's Method takes us from our first guess, x_1, to our second guess, x_2. Then write the general recursion for Newton's Method.