201 E4 FALL, 2017

(1) $\displaystyle\int_0^4 (x^2+3x)\,dx$ $\qquad \Delta x = \frac{b-a}{n} = \frac{4}{n}$

$x_k = a + k\Delta x = 0 + \frac{4k}{n}$

$$\sum f(x_k)\,\Delta x = \sum\left(\left(\frac{4k}{n}\right)^2 + 3\left(\frac{4k}{n}\right)\right)\left(\frac{4}{n}\right)$$

$$= \frac{4}{n}\sum\left(\frac{16k^2}{n^2} + \frac{12k}{n}\right) = \frac{4}{n}\cdot\frac{16}{n^2}\sum k^2 + \frac{4}{n}\cdot\frac{12}{n}\sum k$$

$$= \frac{64}{n^3}\cdot\left(\frac{n^3}{3} + \cdots\right) + \frac{48}{n^2}\left(\frac{n^2}{2} + \cdots\right) \xrightarrow{n\to\infty}$$

$$\frac{64}{3} + \frac{48}{2} = \frac{64+72}{3} = \boxed{\frac{136}{3}}$$

$\begin{array}{r} \overset{1}{2}4 \\ 3 \\ \hline 72 \end{array}$

Check: $\displaystyle\int_0^4 (x^2+3x)\,dx = \left[\frac{1}{3}x^3 + \frac{3}{2}x^2\right]_0^4$

$= \frac{1}{3}(64) + \frac{3}{2}(16) = \frac{64}{3} + 24$ ✓

(2)

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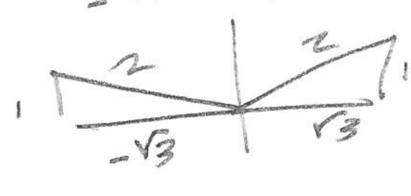
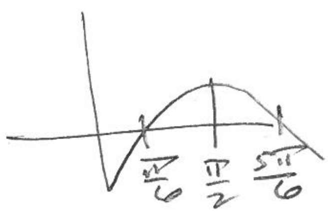
(2) $\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big]_0^1 = \frac{1}{3}$

$\int_0^1 (1-\sqrt{y})\,dy = \left[y - \frac{2}{3}y^{\frac{3}{2}}\right]_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$

(3) $\int_0^{\frac{\pi}{2}} (2\sin(x) - 1)\,dx$

$\sin x = \frac{1}{2}$





$= -\int_0^{\frac{\pi}{6}} (2\sin(x) - 1)\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2\sin(x) - 1)\,dx$

$= -\Big[2(-\cos(x)) - x\Big]_0^{\frac{\pi}{6}} + \Big[2(-\cos(x)) - x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= 2\cos\left(\frac{\pi}{6}\right) + \frac{\pi}{6} - [2\cos(0) + 0]$

$-2\cos\left(\frac{\pi}{2}\right) - \frac{\pi}{2} - \left[-2\cos\left(\frac{\pi}{6}\right) - \frac{\pi}{6}\right]$

$= 2\frac{\sqrt{3}}{2} + \frac{\pi}{6} - [2] - 2(0) - \frac{\pi}{2} + \frac{2\sqrt{3}}{2} + \frac{\pi}{6}$

$= \sqrt{3} + \frac{\pi}{6} - 2 - \frac{\pi}{2} + \sqrt{3} + \frac{\pi}{6} = \boxed{2\sqrt{3} + \frac{\pi}{6} - 2}$

$\frac{\pi}{2} - \frac{\pi}{3} = \frac{(3-2)\pi}{6} = \frac{\pi}{6}$

$\approx \boxed{.940502840}$

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4 a

$$\int (3x+1)^4 \, dx \qquad u = 3x+1, \quad du = 3\,dx$$

$$= \frac{1}{3}\int u^4 \cdot 3\,dx = \frac{1}{3}\int u^4\,du = \frac{1}{3}\cdot\frac{1}{5}u^5 + C$$

$$= \boxed{\frac{1}{15}(3x+1)^5 + C}$$

b

$$\int (3x+1)^4 x^2 \, dx \qquad u = 3x+1, \quad du = 3\,dx$$

$$3x+1 = u$$
$$3x = u-1$$
$$x = \frac{u-1}{3}$$
$$x^2 = \frac{u^2-2u+1}{9}$$

$$= \frac{1}{3}\int\left(\frac{u^2-2u+1}{9}\right)\left(u^4\right)3\,dx$$

$$= \frac{1}{27}\int u^4\left(u^2-2u+1\right)du = \frac{1}{27}\int\left(u^6-2u^5+u^4\right)du$$

$$= \frac{1}{27}\left[\frac{1}{7}u^7 - \frac{1}{3}u^6 + \frac{1}{5}u^5\right] + C$$

$$= \boxed{\frac{1}{189}(3x+1)^7 - \frac{1}{81}(3x+1)^6 + \frac{1}{135}(3x+1)^5 + C}$$

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4 c $\int \sec^4(x)\tan(x)\,dx$

$= \int \sec^3(x)\left(\sec(x)\tan(x)\right)dx$

$= \int u^3\,du = \frac{1}{4}u^4 + C = \boxed{\frac{1}{4}\left(\sec^4(x)\right) + C}$

5 $\int_0^{\frac{\pi}{4}} \sec^4(x)\tan(x)\,dx = \frac{1}{4}\sec^4(x)\Big]_0^{\frac{\pi}{4}}$

$= \frac{1}{4}\sec^4\left(\frac{\pi}{4}\right) - \frac{1}{4}\sec^4(0)$

$= \frac{1}{4}\left(\sqrt{2}\right)^4 - \frac{1}{4} = \frac{1}{4}(4) - \frac{1}{4} = \boxed{\frac{3}{4}}$

6 a $\int_0^{3600} |r(t)|\,dt$ = total distance (in feet) covered by pacing, in one hour.

b $\int_0^{3600} r(t)\,dt$ = Net change in position, after all the pacing. in units of feet.

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(7) (a) $\frac{d}{dx}\int_0^x \frac{\sin(3t)}{t^2+4}\,dt = \boxed{\frac{\sin(3x)}{x^2+4}}$

(b) $\frac{d}{dx}\int_{x^2}^{\cos(x)} \frac{\sin(3t)}{t^2+4}\,dt$

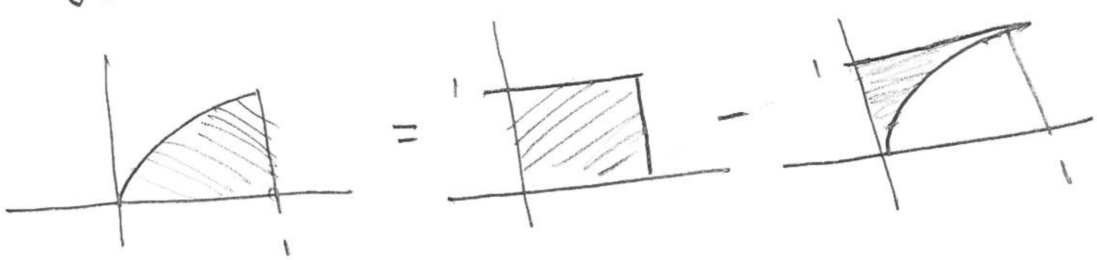
$$= \frac{d}{dx}\left[-\int_0^{x^2}\frac{\sin(3t)}{t^2+4}\,dt + \int_0^{\cos x}\frac{\sin(3t)}{t^2+4}\,dt\right]$$

$$= \boxed{\left(-\frac{\sin(3x^2)}{x^4+4}\right)(2x) + \left(\frac{\sin(3\cos(x))}{\cos^2(x)+4}\right)(-\sin(x))}$$

B

B1 $\frac{4-2}{n} = \frac{2}{n} = \Delta x$, $x_k = a + k\Delta x = \boxed{2 + \frac{2k}{n} = x_k}$

B2 $\int_0^1 \sqrt{x}\,dx = \int_0^1 1\,dy - \int_0^1 y^2\,dy$



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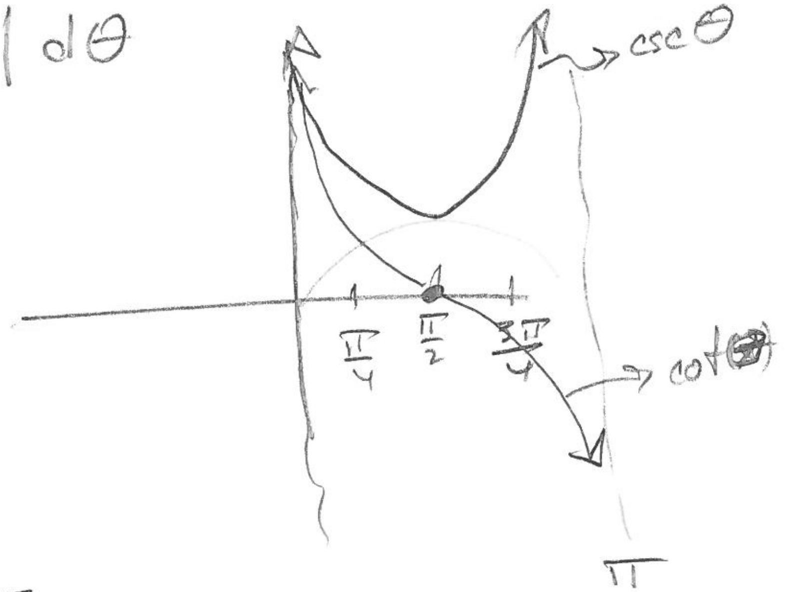
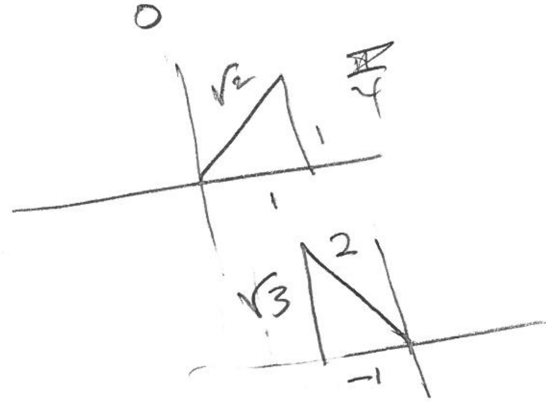
B3

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} |\csc\theta \cot\theta|\, d\theta$$



$$= \int_{\frac{\pi}{4}}^{\frac{2\pi}{3}} \csc\theta \cot\theta \, d\theta$$

$$- \int_{\frac{\pi}{2}} \csc\theta \cot\theta \, d\theta$$

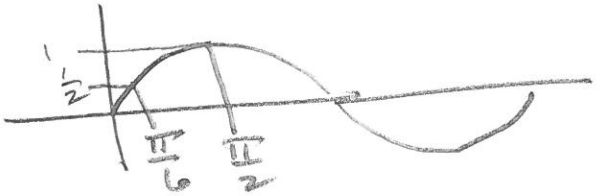


$$= -\csc\theta \Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - (-\csc\theta)\Big]_{\frac{\pi}{2}}^{\frac{2\pi}{3}}$$

$$= -1 + \sqrt{2} + \left[\frac{2}{\sqrt{3}} - 1\right]$$

$$= \boxed{\sqrt{2} - 2 + \frac{2}{\sqrt{3}}}$$

B4



$\frac{1}{2} \le \sin x \le 1$ on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

$b - a = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

So, $m(b-a) \le \int_a^b f \le M(b-a)$

$\frac{1}{2}\left(\frac{\pi}{3}\right) \le \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin(x)\, dx \le 1\left(\frac{\pi}{3}\right)$

So $\boxed{\frac{\pi}{6} \le \int \le \frac{\pi}{3}}$

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B5 f is polynomial, so cnt^s & differentiable $\forall x \in \mathbb{R}$, in particular,

f cnt^s on $[0, 3]$ &

f difb^l on $(0, 3)$

$$\frac{f(b) - f(a)}{b-a}$$

$$f'(x) = 3x^2 - 4x + 5$$

$= ?$

$f(0) = -1$

$f(3)$:

$$\begin{array}{r|rrrr} 3 & 1 & -2 & 5 & -1 \\ & & 3 & 3 & 24 \\ \hline & 1 & 1 & 8 & 23 \end{array}$$

$$\therefore \text{MAV} = \frac{f(b) - f(a)}{b-a} = \frac{23-(-1)}{3-0} = \frac{24}{3} = 8$$

$f'(x) \overset{\text{SET}}{=} 8 \rightarrow$

$3x^2 - 4x + 5 = 8 \rightarrow$

$$\begin{array}{r|l} 2 & 52 \\ 2 & 26 \\ & 13 \end{array}$$

$3x^2 - 4x - 3 = 0$

$b^2 - 4ac = (-4)^2 - 4(3)(-3)$

$= 16 + 36 = 52$

$$x = \frac{4 \pm 2\sqrt{13}}{2(3)} = \frac{2 \pm \sqrt{13}}{3} \rightarrow$$

$$\boxed{c = \frac{2 + \sqrt{13}}{3}}$$

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B6 $f(x) = \sqrt{3x} \Rightarrow$

$$\frac{f(x+h) - f(x)}{h} = \left(\frac{\sqrt{3(x+h)} - \sqrt{3x}}{h}\right)\left(\frac{\sqrt{3(x+h)} + \sqrt{3x}}{\sqrt{3(x+h)} + \sqrt{3x}}\right)$$

$$= \frac{3x + 3h - 3x}{h\left[\sqrt{3x+3h} + \sqrt{3x}\right]} = \frac{3\cancel{h}}{\cancel{h}\left[\sqrt{3x+3h} + \sqrt{3x}\right]}$$

$$\xrightarrow{h \to 0} \frac{3}{\sqrt{3x} + \sqrt{3x}} = \boxed{\frac{3}{2\sqrt{x}}}$$

Check $(3x)^{\frac{1}{2}} = y \Rightarrow$ ✓

$$y' = \frac{1}{2}(3x)^{-\frac{1}{2}}(3) = \frac{3}{2\sqrt{3x}}$$

$x_0 = 30^\circ = \frac{\pi}{6}$

B7 $\cos 33^\circ = \cos\left(\frac{\pi}{6} + \frac{3\pi}{180}\right)$

$f(x) = \cos(x)$

$f'(x) = -\sin(x)$

$$L(x) = f'(x_0)(x - x_0) + f(x_0)$$

$$= -\sin\left(\frac{\pi}{6}\right)\left(\frac{3\pi}{180}\right) + \cos\left(\frac{\pi}{6}\right)$$

$$= -\frac{1}{2}\left(\frac{\pi}{60}\right) + \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3}}{2} - \frac{\pi}{120}} \approx \cos(33^\circ)$$

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B8 $x^2-3xy+y^2=1 \Longrightarrow$

$2x-3y-3xy'+2yy'=0 \Longrightarrow$

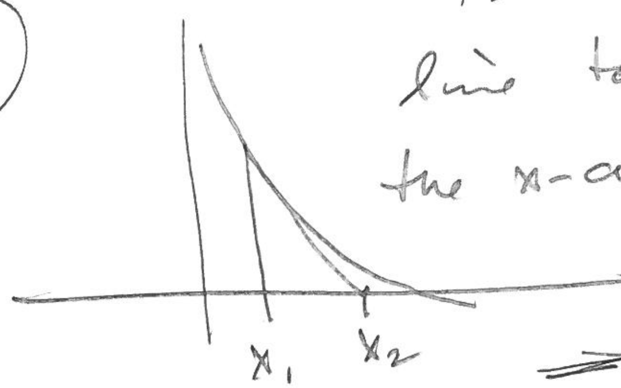
$(2y-3x)y' = 3y-2x \Longrightarrow$

$y' = \frac{3y-2x}{2y-3x} \Longrightarrow$

$y'\Big|_{(1,3)} = \frac{3(3)-2(1)}{2(3)-3(1)} = \frac{9-2}{6-3} = \frac{7}{3} = m_{tan}$

$$\boxed{y = \frac{7}{3}(x-1)+3}$$

B9



x_2 is where the tangent line to f @ $x=x_1$ meets the x-axis:

$y = f'(x_1)(x-x_1)+f(x_1) \overset{SET}{=} 0$

$\Longrightarrow f'(x_1)x - f'(x_1)x_1 = -f(x_1)$

$\Longrightarrow f'(x_1)x = f'(x_1)x_1 - f(x_1)$

$\Longrightarrow x = x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$