MAT 201 Test 3, Fall, 2017 Name_____
100 Points Covers Chapter 3

You know the drill. And remember to circle final answers.

1. Let $f(x)=3x^4+4x^3-30x^2+36x$ for the following problems:

   a. (10 pts) Sketch the graph of $f(x)$. Show all extreme points and inflection points. I expect to see a *y*-intercept, but I'm not worried about *x*-intercepts, as long as they're in the right general location.

   b. (10 pts) Find the maximum and minimum of $f(x)$ on the interval $[-4,4]$.

2. (10 pts) Confirm that the hypotheses of the Mean Value Theorem hold for $f(x)=x^3-2x^2+5x-1$ on $[-2,2]$, and find the c that is promised in the conclusion of the theorem.

3. (10 pts) Find all local extremes of $g(x)=3\tan(x)-4x$ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Show all intercepts, extremes, asymptotes and inflection points.

4. (10 pts) Sketch the graph of a *continuous* function g that has all the properties given:

   $g(-10)=-5,\ g(5)=6,\ g(10)=-2$,
   $g'(-10)=0,\ g'(5)$ is undefined, $g'(10)=0$
   $g'(x)>0$ on $(-\infty,-10)\cup(-10,5)\cup(10,\infty)$ and $g'(x)<0$ on $(5,10)$
   $g''(x)>0$ on $(-10,5)\cup(5,\infty)$ and $g''(x)<0$ on $(-\infty,-10)$

   This function has a pointy spot.

5. Evaluate the following limits.

   a. (10 pts) $\lim_{x\to\infty}\left(\sqrt{16x^2-5x+11}-4x\right)$

   b. (10 pts) $\lim_{x\to\infty}\left(\sqrt{16x^2-5x+11}+4x\right)$

6. (10 pts) You don't need to graph $R(x)=\dfrac{3x^3-14x^2+23x-10}{x^2-x-2}$, here, but I do want to see its asymptotes.
   Hint: This function has no holes. This problem requires no calculus.

7. (10 pts) Minimize the vertical distance between $g(x)=x^2-2x-8$ and $h(x)=2x^2-3x+15$.

8. (10 pts) Use the curve at the bottom of Page 2 or one like it, to show me the derivation of Newton's Method.

Bonus Answer up to 2 Bonus questions.

Bonus 1 (10 pts) Finish sketching the graph of $R(x)$ from Problem #6. Hint: One of $R(x)$'s x-intercepts is $\left(\frac{2}{3},0\right)$.

Bonus 2 (10 pts) Use a differential to approximate the error in the area of a disc of radius 3 cm, if the error in measuring the radius is up to 0.01 cm.

Bonus 3 (10 pts) Use the tangent line to approximate $\sqrt{97}$.

Bonus 4 (10 pts) Find $\frac{dy}{dx}$ if $x^2+3xy+y^2=11$. Then find an equation of the tangent line to the curve at $(2,1)$.