

You know the drill. And remember to circle final answers.

1. Let $f(x) = x^3 + 3x^2 - 9x - 27$ for the following problems:
 - a. (10 pts) Sketch the graph of $f(x)$. Show all extreme points and inflection points. I *also* expect to see the x -intercept(s) and y -intercept. This one is cooked up to factor by grouping (like the previous version of the test was supposed to have!).
 - b. (10 pts) Find the maximum and minimum of $f(x)$ on the interval $[0, 4]$.
2. (10 pts) Confirm that the hypotheses of the Mean Value Theorem hold for $f(x) = x^3 + 3x^2 - 9x - 27$ on $[0, 4]$, and find the c that is promised in the conclusion of the theorem.
3. (10 pts) Find all local extremes of $g(x) = 3\cot(x) + 4x$ in the interval $(0, \pi)$.
4. (10 pts) Sketch the graph of a *continuous* function g that has all the properties given:
 $g(0) = 4, g(1) = 2, g(2) = 1, g(3) = 0, g(4) = 2,$
 $g'(1) = g'(3) = 0,$
 $g'(x) < 0$ on $(-\infty, 1) \cup (1, 3),$
 $g'(x) > 0$ on $(3, \infty)$
 $g''(2) = g''(4) = 0$
 $g''(x) > 0$ on $(0, 1) \cup (2, 4),$
 $g''(x) < 0$ on $(-\infty, 0) \cup (1, 2) \cup (4, \infty)$
 $\lim_{|x| \rightarrow \infty} g(x) = 5$
5. Evaluate the following limits. (Take heed: x is approaching $-\infty$ in this version!)
 - a. (10 pts) $\lim_{x \rightarrow -\infty} \left(\sqrt{16x^2 - 5x + 11} - 4x \right)$
 - b. (10 pts) $\lim_{x \rightarrow -\infty} \left(\sqrt{16x^2 - 5x + 11} + 4x \right)$
6. (10 pts) You don't need to graph $R(x) = \frac{2x^3 - 5x^2 - x + 6}{x^2 - 4x + 3}$, here, but I do want to see its asymptotes.
Hint: This function has no holes. This problem requires no calculus.
7. (10 pts) Minimize the vertical distance between $g(x) = 2x^2 + 5x + 2$ and $h(x) = -x^2 + 17x - 17$.

8. (10 pts) Derive the recursion formula for Newton's method and use the figure, below to illustrate how x_2 is obtained from x_1 .

Bonus Answer up to 2 Bonus questions.

Bonus 1 (10 pts) Finish sketching the graph of $R(x)$ from Problem #6. Hint: One of $R(x)$'s x -intercepts is $(2,0)$.

Bonus 2 (10 pts) Use a differential to estimate how much paint it takes to cover a sphere of radius 5 m with a coat of paint that is 0.05 cm thick.

Bonus 3 (10 pts) Use the tangent line to approximate $\sqrt{104}$.

Bonus 4 (10 pts) Find $\frac{dy}{dx}$ if $x^2 + 3xy + y^2 = 11$. Then find an equation of the tangent line to the curve at $(2,1)$.

