201 E3 FALL '17

1 a 10pts $f(x) = 3x^4 + 4x^3 - 30x^2 + 36x$

$\longrightarrow f'(x) = 12x^3 + 12x^2 - 60x + 36$ SET 0

Guess $x = 1$:

For those who didn't re-take.

$$\begin{array}{c|cccc} 1 & 12 & 12 & -60 & 36 \\ & & 12 & 24 & -36 \\ \hline & 12 & 24 & -36 & 0 \end{array}$$

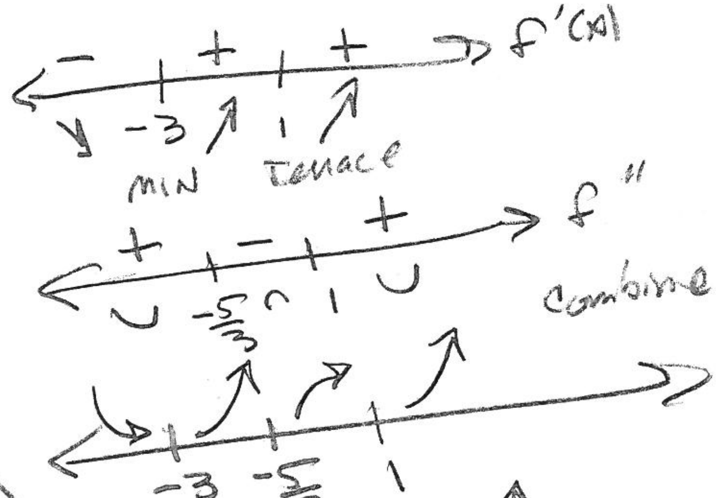
Yes! $(x-1)(12x^2 + 24x - 36)$

$12x^2 + 24x - 36 = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

So, $f'(x) = 12(x-1)^2(x+3)$



$f''(x) = 36x^2 + 24x - 60$

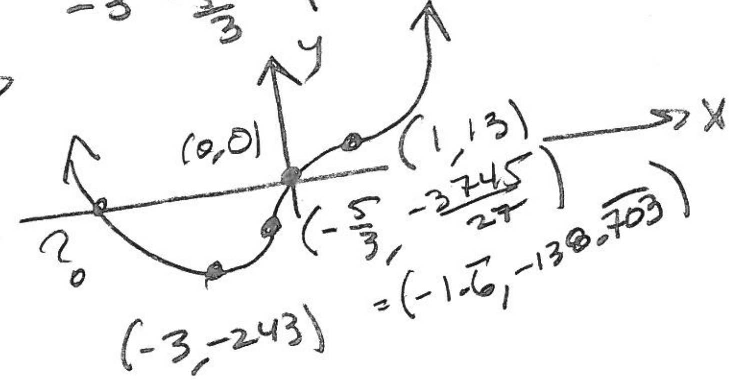
$= 12(3x^2 + 2x - 5) = 0$

$\Rightarrow 3x^2 + 5x - 3x - 5$

$= x(3x+5) - 1(3x+5)$

$= (3x+5)(x-1) = 0 \longrightarrow$

$x \in \{-\frac{5}{3}, 1\}$



201 E3M

1a cntd

$$\begin{array}{r|rrrrr} -3 & 3 & 4 & -30 & 36 & 0 \\ & & -9 & 15 & 45 & -243 \\ \hline & 3 & -5 & -15 & 81 & -243 \end{array} = f(-3)$$

$$\begin{array}{r|rrrrr} -\frac{5}{3} & 3 & 4 & -30 & 36 & 0 \\ & & -5 & \frac{5}{3} & \frac{425}{9} & -\frac{3745}{27} \\ \hline & 3 & -1 & -\frac{85}{3} & \frac{749}{9} & -\frac{3745}{27} \end{array} = f(-\tfrac{5}{3})$$

$$= -138.\overline{703}$$

$$\begin{array}{r|rrrrr} 1 & 3 & 4 & -30 & 36 & 0 \\ & & 3 & 7 & -23 & 13 \\ \hline & 3 & 7 & -23 & 13 & 13 \end{array} = f(1)$$

201 E3M

(b) Max & min on $[-4,4]$:

$$\begin{array}{r|rrrrr} -4 & 3 & 4 & -30 & 36 & 0 \\ & & -12 & 32 & -8 & -112 \\ \hline & 3 & -8 & 2 & 28 & \boxed{-112 = f(-4)} \end{array}$$

$$\begin{array}{r|rrrrr} 4 & 3 & 4 & -30 & 36 & 0 \\ & & 12 & 64 & 136 & 688 \\ \hline & 3 & 16 & 34 & 172 & \boxed{688 = f(4) \text{ MAX}} \end{array}$$

$\boxed{f(-3) = -243 \text{ MIN}}$

10pts

$f(1)$ Terrace

2 10pts

f is poly $\Longrightarrow$ cont^s on $[-2,2]$

& difb^l on $(-2,2)$

$$\frac{f(2)-f(-2)}{2-(-2)} = m_{AV} = \frac{9-(-27)}{4} = \frac{36}{4} \boxed{= 9 = m_{AVG}}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 5 & -1 \\ & & 2 & 0 & 10 \\ \hline & 1 & 0 & 5 & 9 = f(2) \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & 5 & -1 \\ & & -2 & 8 & -26 \\ \hline & 1 & -4 & 13 & -27 = f(-2) \end{array}$$

$f'(x) = 3x^2 - 4x + 5 \overset{SET}{=} 9$

$\Longrightarrow 3x^2 - 4x - 4 = 0$

$3x^2 - 6x + 2x - 4 = 0$

$3x(x-2) + 2(x-2) = 0$

$\Longrightarrow 3x(x-2) + 2(x-2) = 0$

$(x-2)(3x+2) = 0$

$x \in \{-\frac{2}{3}, \cancel{2}\}$

$\Longrightarrow \boxed{c = -\frac{2}{3}}$

201 E3M

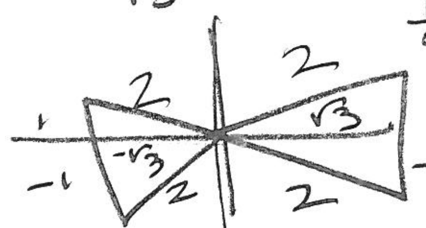
3. $g(x) = 3\tan x - 4x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

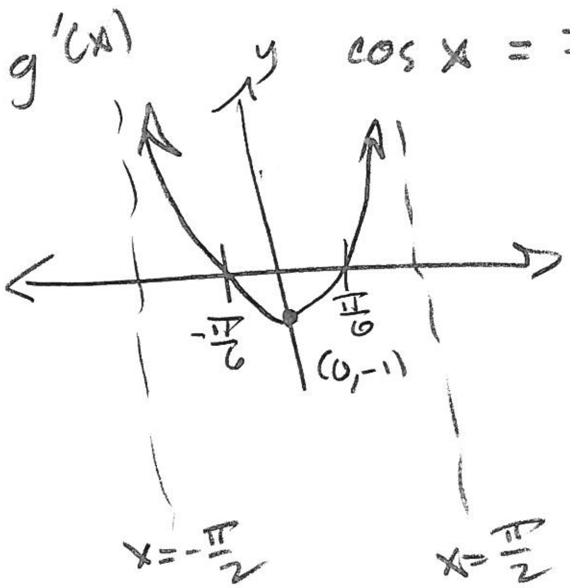
$\to g'(x) = 3\sec^2 x - 4 \overset{\text{set}}{=} 0 \to$

$\sec^2 x = \frac{4}{3} \to$

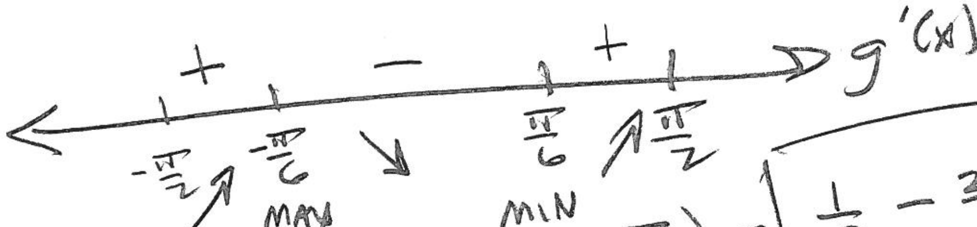
$\sec x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}} \to$ $\frac{\pi}{6}, -\frac{\pi}{6}$

$\cos x = \pm\frac{\sqrt{3}}{2}$





10pts

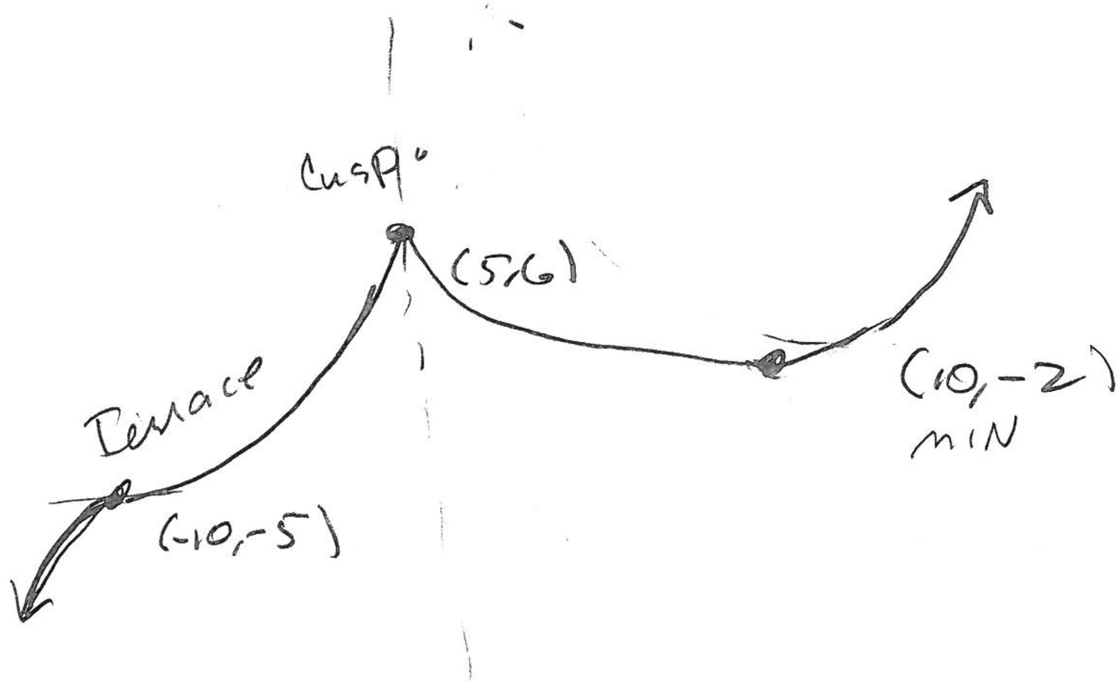


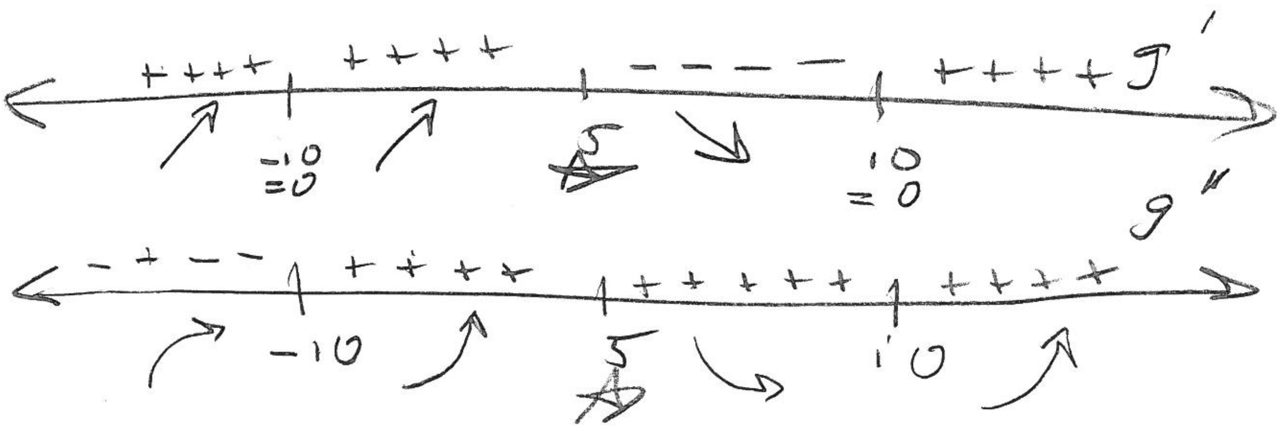
$g\left(\frac{\pi}{6}\right) = 3\tan\left(\frac{\pi}{6}\right) - 4\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{\sqrt{3}} - \frac{2\pi}{3} \text{ MIN } x=\frac{\pi}{6}}$

$g\left(-\frac{\pi}{6}\right) = -g\left(\frac{\pi}{6}\right) = \boxed{\frac{2\pi}{3} - \frac{1}{\sqrt{3}} \text{ MAX @ } x = -\frac{\pi}{6}}$

201 E3 M

(4)





.201 E3

5 a 10 pts $\lim_{x\to\infty}\left(\sqrt{16x^2-5x+11}-4x\right)$

$$\frac{\sqrt{16x^2+5x-11}-4x}{1}\cdot\frac{\sqrt{16x^2-5x+11}+4x}{\sqrt{16x^2-5x+11}+4x}$$

$$=\frac{16x^2-5x+11-16x^2}{\sqrt{16x^2}\sqrt{1-\frac{5}{16x}+\frac{11}{16x^2}}+4x}$$

$$=\frac{-5x+11}{|4x|\sqrt{1-\frac{5}{16x}+\frac{11}{16x^2}}+4x}$$

$x\to\infty \Rightarrow$

$|4x|=4x$, b/c $x>0$

$$=\frac{-5x+11}{4x\sqrt{1-\frac{5}{16x}+\frac{11}{16x^2}}+4x}$$

$$=\frac{x\left(-5+\frac{11}{x}\right)}{4x\left(\sqrt{1-\frac{5}{16x}+\frac{11}{16x^2}}+1\right)}$$

$$=\frac{-5+\frac{11}{x}}{4\left(\sqrt{1-\frac{5}{16x}+\frac{11}{16x^2}}+1\right)}\xrightarrow{x\to\infty}\frac{-5}{4(1+1)}=\boxed{-\frac{5}{8}}$$

5b $\lim_{x\to\infty}\left(\sqrt{16x^2-5x+11}+4x\right)=4\infty+4\infty$

$=8\infty=\boxed{\infty}$

10 pts

201 E3

6 10pts

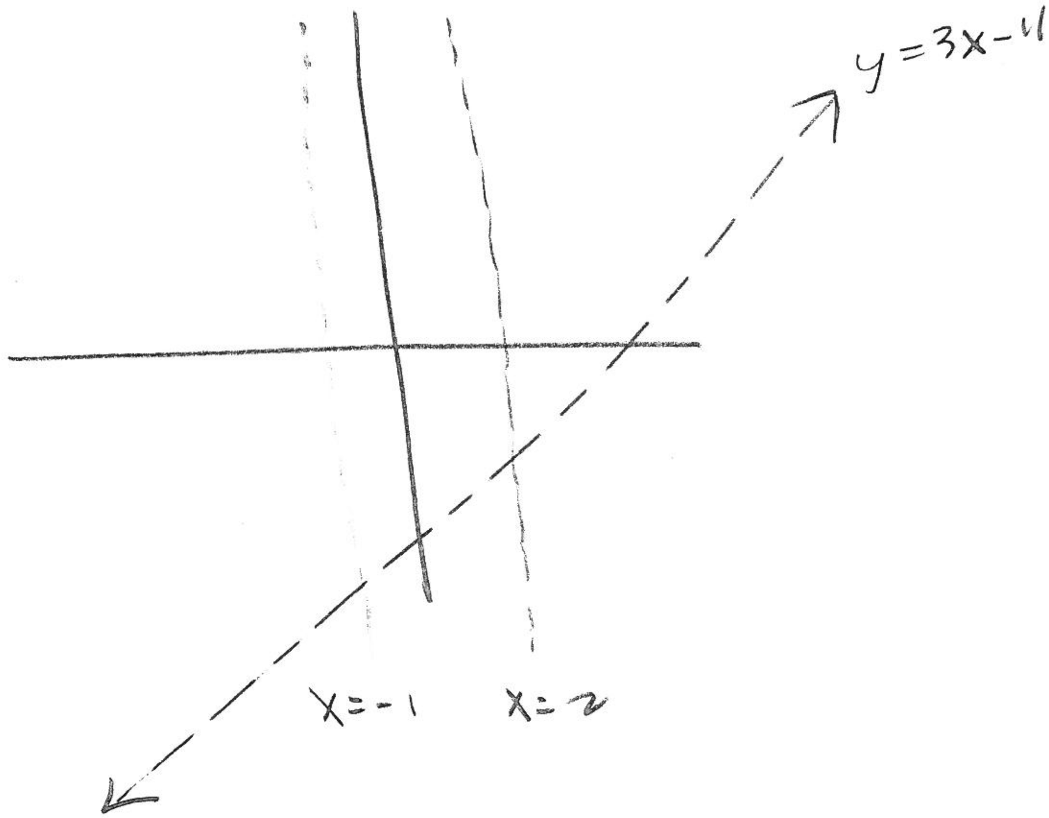
$$\frac{3x^3-14x^2+23x-10}{x^2-x-2}$$

$$= \frac{}{(x-2)(x+1)}$$

$\boxed{V.A.: x=-1, x=2 \quad O.A.: y=3x-11}$

O.A.:

$$\begin{array}{r|l} & 3x-11 \\ \hline x^2-x-2 & 3x^3-14x^2+23x-10 \\ & -(3x^3-3x^2-6x) \\ \hline & -11x^2 \end{array}$$



201 E3

7 10pts

$g(x) = x^2 - 2x - 8$, $h(x) = 2x^2 - 3x + 15$

$\Longrightarrow$ Vertical Distance .3

$f(x) = |g(x) - h(x)| = |x^2 - 2x - 8 - (2x^2 - 3x + 15)|$

$= |x^2 - 2x - 8 - 2x^2 + 3x - 15| = |-x^2 + x - 23|$

$= |x^2 - x + 23|$

$b^2 - 4ac = (-1)^2 - 4(1)(23) = 1 - 92 = -91$

No real roots $\Longrightarrow$

$f(x) = x^2 - x + 23$ $\longrightarrow$

$f'(x) = 2x - 1 \overset{\text{SET}}{=} 0 \longrightarrow$

$x = \frac{1}{2}$

$\frac{92-1}{4} = \frac{91}{4}$

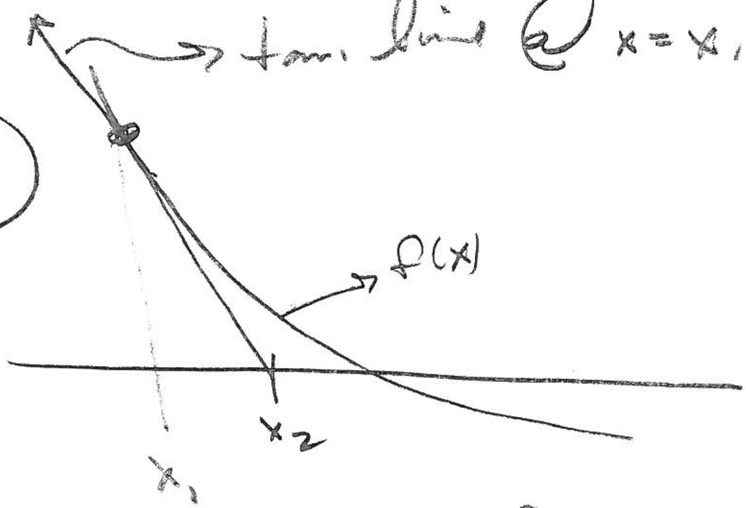
$$\begin{array}{r|rrr} \frac{1}{2} & 1 & -1 & 23 \\ & & \frac{1}{2} & -\frac{1}{4} \\ \hline & 1 & -\frac{1}{2} & \frac{91}{4} \end{array}$$

$\boxed{\frac{91}{4} = \text{Min Distance @ } x = \frac{1}{2}}$

201 E3

8

10 pts



Find intersection of tangent line with x-axis to find x_2. Repeat.

$$y = L(x) = f'(x_1)(x - x_1) + f(x_1) \overset{\text{SET}}{=} 0$$

$$\Rightarrow f'(x_1)(x - x_1) = -f(x_1)$$

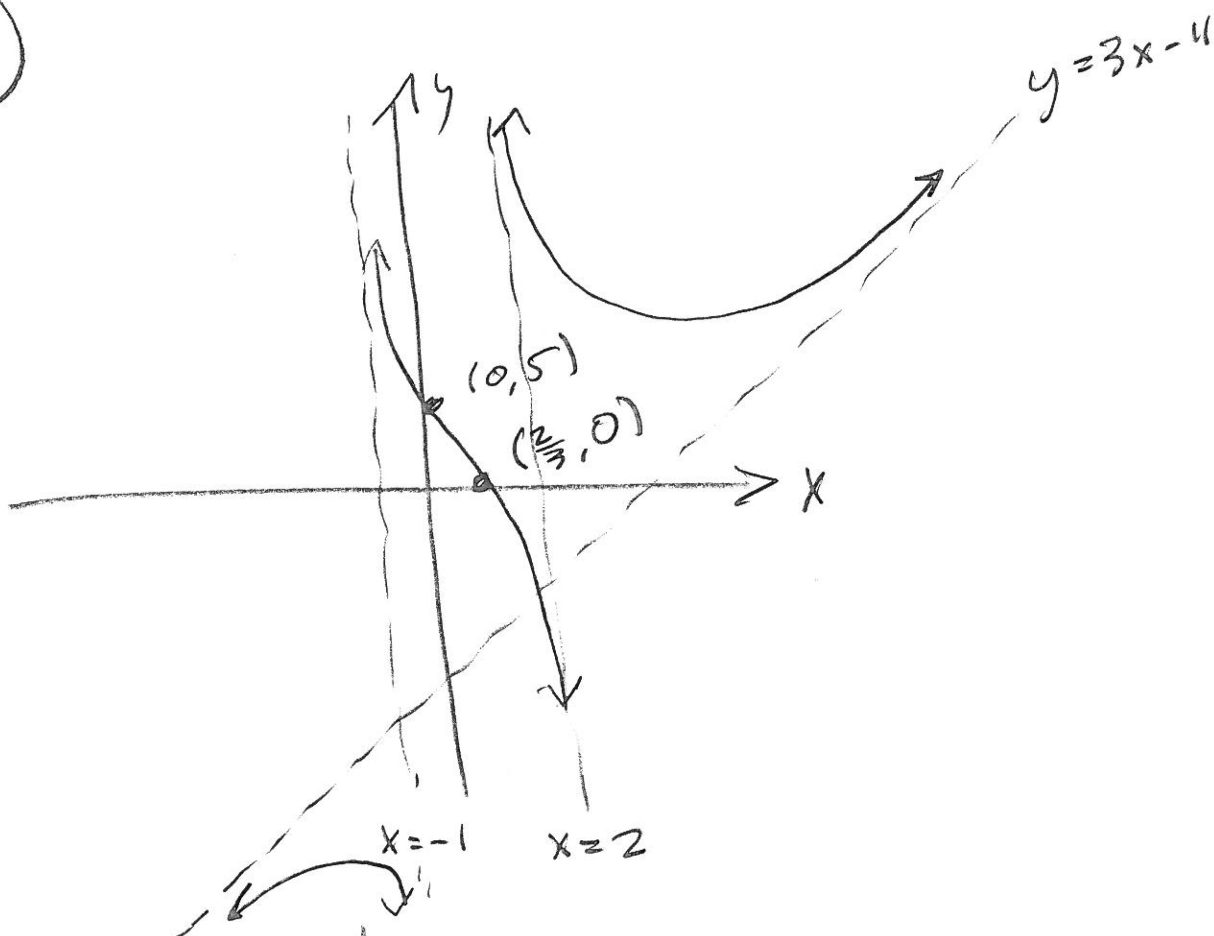
$$\Rightarrow (x - x_1) = -\frac{f(x_1)}{f'(x_1)}$$

$$\Rightarrow x = x_1 - \frac{f(x_1)}{f'(x_1)} = x_2$$

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ is the recursion

201 E3

B1



$x = \frac{2}{3}$ is root ∴

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -14 & 23 & -10 \\ & & 2 & -8 & 10 \\ \hline & 3 & -12 & 15 & 0 \end{array}$$

$(x - \frac{2}{3})(3x^2 - 12x + 15)$

$= 3(x - \frac{2}{3})(x^2 - 4x + 5)$

$= 3(x - \frac{2}{3})($

$x^2 - 4x + 5 = 0$

$3x^2 - 12x + 15 = 0$

$b^2 - 4ac = (-12)^2 - 4(3)(15)$

$= 144 - 180 = -36$ No real roots

201 E3

B2 10pts

$r = 3 \pm .01$

$A = \pi r^2$

$dA = 2\pi r\,dr = 2\pi(3)(.01) = .06\pi$

$\approx \boxed{.188495592}$

B3 10pts

$f(x) = \sqrt{x} = x^{\frac{1}{2}} \Longrightarrow f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$. $x_1 = 100$ $f'(x_1) = \frac{1}{20}$

$x = 97$

$L(x) = f'(x)(x - x_1) + f(x_1)$

$= \frac{1}{20}(97 - 100) + 10$

$= -\frac{3}{20} + \frac{200}{20} = \boxed{\frac{197}{20} \approx \sqrt{97}}$

$\boxed{9.85}$

201 E 3

B4 10pts $x^2 + 3xy + y^2 = 11$

$\longrightarrow 2x + 3y + 3xy' + 2yy' = 0$

$y'(3x+2y) = -2x-3y$

$$\boxed{y' = \frac{-2x-3y}{3x+2y}} \longrightarrow$$

$$y'\Big|_{\substack{x=2\\y=1}} = \frac{-2(2)-3(1)}{3(2)+2(1)} = \frac{-4-3}{6+2} = -\frac{7}{8}$$

$$\boxed{y = -\frac{7}{8}(x-2)+1}$$