MAT 201
100 Points

Test 2, Fall, 2017
Covers Chapter 2

Name KEY

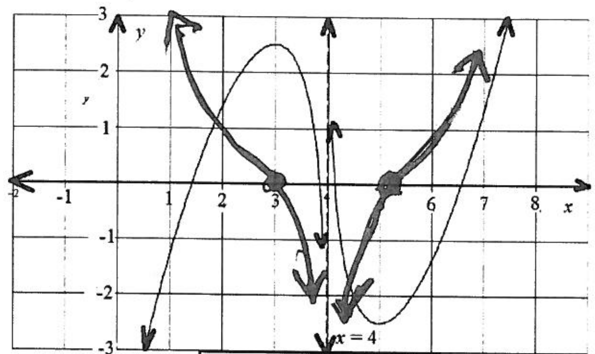
All I want on this cover sheet is your NAME.

Do all work and put all answers on the white paper provided. Do not write on the backs of the white pages. Leave a margin at the top left corner on every page. "201-G11" works really well for the top left corner of every page.

Leave room between problems. Do not squeeze work in to fit a page. Start a fresh page. When in doubt on how long a problem will turn out to be, start a fresh page.

1. Let $f(x)=2x^2-5x$
   a. (5 pts) Find an equation of the tangent line to f at $(2,-2)$.
   b. (5 pts) Sketch a graph of $f(x)$ and the tangent line to f at $(2,-2)$.

2. (10 pts) The graph of a function f is given on the right. On the same set of axes, sketch a graph of f'.



Graph of f for

3. Differentiate the following with respect to the indicated independent variable.
   a. $f(x)=x^{\frac{5}{2}}-3x^2+11x+5-2x^{-\frac{2}{3}}$
   b. $g(t)=\sin(5t)\cos(3t)$
   c. $h(\rho)=\dfrac{\tan(\rho)}{(2\rho-5)}$
   d. $r(w)=(7w^2+5w)^6$
   e. $Q(x)=\cos(6w)-6\cos(w)$ (It's a triiiiiick!)

4. Consider the relation $x^2-xy-y^2=1$.
   a. (10 pts) Use implicit differentiation to find $y'=\dfrac{dy}{dx}$
   b. (5 pts) Find an equation of the tangent line to the curve at the point $(2,1)$.

5. (10 pts) A lighthouse is exactly $\sqrt{3}$ miles from the nearest point P on a straight shoreline, and its light makes 6 revolutions per minute. How fast is the beam of light moving along the shoreline, when it's 1 mile from P?

6. The height of a triangle with a base *exactly* 8 cm is measured. The maximum error in measuring the height is ± 0.1 cm.
   a. (5 pts) Use a differential to estimate the error in the calculation of the area of the triangle.
   b. (5 pts) What is the relative error?
   c. (5 pts) What is the percentage error?

Be sure to see the back for Bonus!

BONUS SECTION: Work up to 15 points' worth.

1. (10 pts) Prove that $\lim_{x \to 4}(x^2 - 3x + 2) = 6$

2. Use the figure at the right:
   a. (5 pts) Show $dx = \Delta x, dy,$ and Δy .
   b. (5 pts) Is the tangent line an over- or under-estimate? Why?

3. (5 pts) Sketch the graph of $h(x) = \dfrac{(x-2)(x+2)}{x-3}$. Show all intercepts and asymptotes.

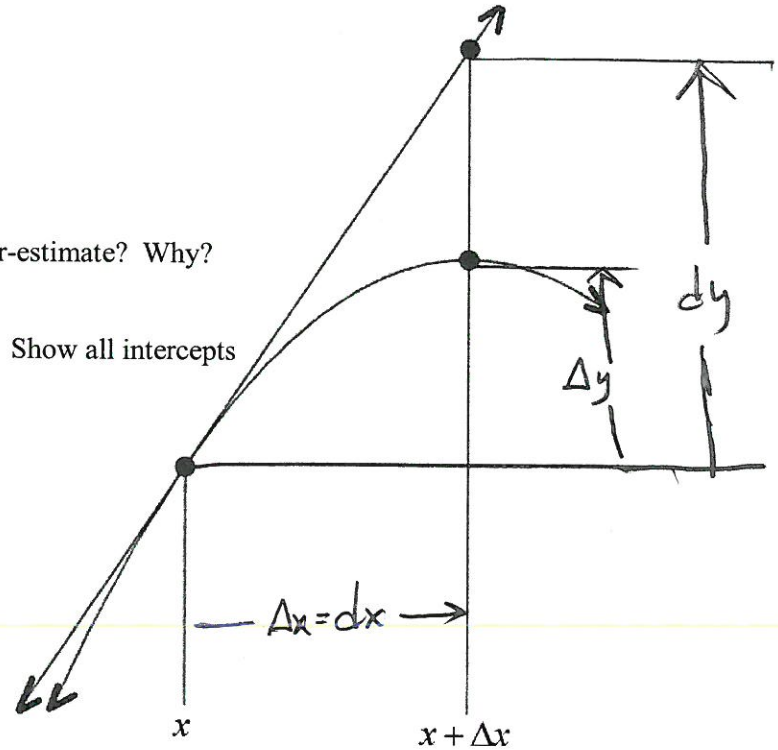


Figure for Bonus #2

B2 b

Overestimate, b/c slope is decreasing, i.e., the tangent line lies above the curve. ($y'' < 0$!)

Function is CONCAVE DOWN

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1) $f(x) = 2x^2 - 5x$ $f(2) = 8 - 10 = -2$

a) $f'(x) = 4x - 5 \longrightarrow$

5 pts

$f'(2) = 4(2) - 5 = 3 = m_{tan} \longrightarrow$

$$\boxed{y = 3(x-2) - 2}$$

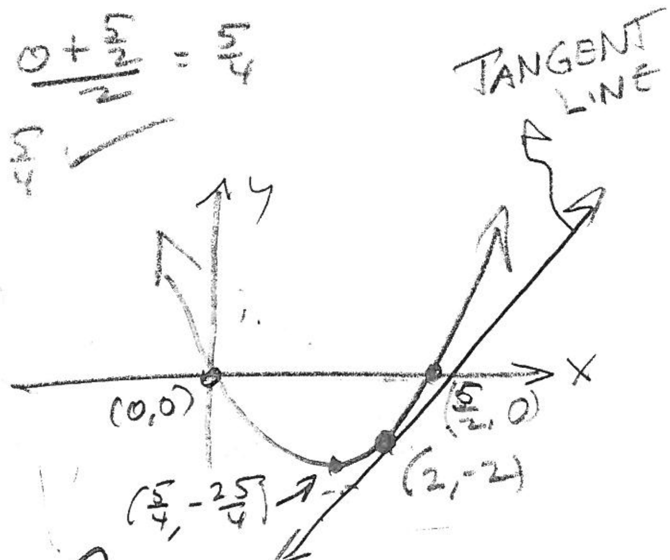
b) $2x\left(x - \frac{5}{2}\right) = f(x) \overset{SET}{=} 0 \Longrightarrow x \in \left\{0, \frac{5}{2}\right\}$

So, vertex is @ $x = \dfrac{0 + \frac{5}{2}}{2} = \frac{5}{4}$

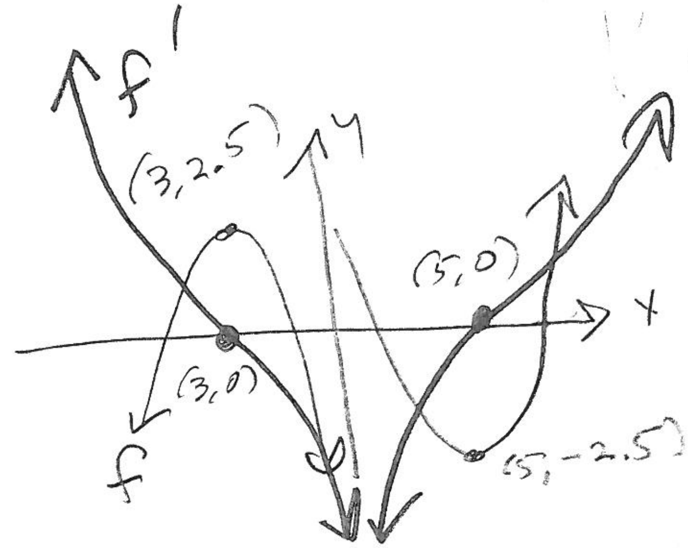
$f'(x) = 4x - 5 = 0 \longrightarrow x = \frac{5}{4}$

$f\left(\frac{5}{4}\right) = 2\left(\frac{5}{4}\right)\left(\frac{5}{4} - \frac{5}{2}\right)$

$= \left(\frac{5}{2}\right)\left(-\frac{5}{4}\right) = -\frac{25}{4}$



2)



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5 pts ea

(3) (a) $f(x) = x^{\frac{5}{2}} - 3x^2 + 11x + 5 - 2x^{-\frac{2}{3}}$

$\Rightarrow f'(x) = \frac{5}{2}x^{\frac{3}{2}} - 6x + 11 + \frac{4}{3}x^{-\frac{5}{3}}$

(b) $g(t) = \sin(5t)\cos(3t)$

$\Rightarrow g'(t) = 5\cos(5t)\cos(3t) + \sin(5t)(-3\sin(3t))$

(c) $h(p) = \frac{\tan(p)}{2p-5} \longrightarrow$

$h'(p) = \frac{(\sec^2(p))(2p-5) - (\tan(p))(2)}{(2p-5)^2}$

(d) $r(w) = (7w^2 + 5w)^6 \longrightarrow$

$r'(w) = 6(7w^2 + 5w)^5(14w + 5)$

(e) $Q(x) = \cos(6w) - 6\cos(w) \longrightarrow$

$Q'(x) = 0$!

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4 a 10 pts $x^2 - xy - y^2 = 1 \longrightarrow$

$2x - y - xy' - 2yy' = 0 \longrightarrow$

$-xy' - 2yy' = -2x + y \longrightarrow$

$$y' = \frac{-2x+y}{-x-2y} = \boxed{\frac{2x-y}{x+2y} = y'}$$

b 5 pts $$y'\Big|_{(x,y)=(2,1)} = \frac{2(2)-1}{2+2(1)} = \frac{3}{4}$$

$$\longrightarrow \boxed{y = \frac{3}{4}(x-2) + 1}$$

5 Fresh Page. Biggie.

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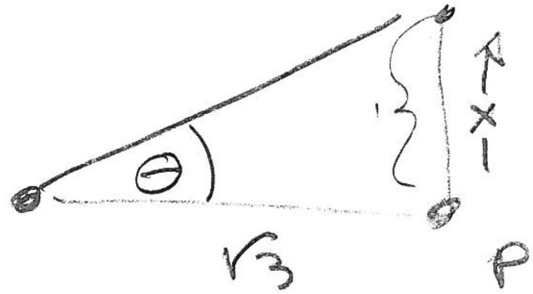
5

10 pts



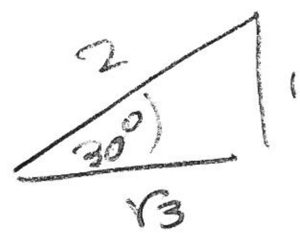
Want $\left.\frac{dx}{dt}\right|_{x=1}$, given the light revolves

@ 6 rpm. $= 6 \frac{\text{revs}}{\text{min}}$

$$\left(6 \frac{\text{revs}}{\text{min}}\right)\left(\frac{2\pi \text{ radians}}{1 \text{ rev}}\right) = \frac{12\pi \text{ radians}}{\text{min}}$$

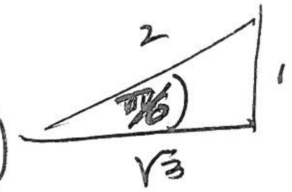
Now

$$\frac{\text{miles}}{\text{miles}} \quad \frac{x}{\sqrt{3}} = \tan\theta$$



$$x = \sqrt{3}\tan\theta$$

$$\frac{dx}{dt} = \left(\sqrt{3}\sec^2\theta\right)\left(\frac{d\theta}{dt}\right)$$



$$= \left(\sqrt{3}\sec^2\frac{\pi}{6}\right)\left(12\pi \frac{\text{radians}}{\text{min}}\right)$$

$$= \left(\sqrt{3}\right)\left(\frac{2}{\sqrt{3}}\right)^2(12\pi) = \sqrt{3}\left(\frac{4}{3}\right)(12\pi)$$

87.06236948 mi/min.

$$= \frac{48\sqrt{3}\pi}{3} = \left(16\pi\sqrt{3}\ \frac{\text{mi}}{\text{min}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)$$

$$= 16\left(60\pi\sqrt{3}\right)) = 960\sqrt{3}\pi \approx \boxed{5{,}223.74269}$$

$\frac{\text{mi}}{\text{hr}}$!

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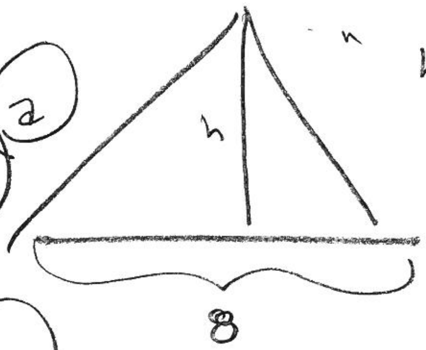
6



$h = 4 \pm .1$ ''

(a) 5pts

$$\text{Area} = \tfrac{1}{2} b h = \tfrac{1}{2}(8) h = 4h$$

$$dA = 4\,dh$$

$$= 4(\pm .1) = \boxed{\pm .4 \text{ cm}}$$

$\approx \Delta A$

$=$ Error (max)

(b) 5pts

$$\text{Area} = \tfrac{1}{2}(8)(4) = 16$$

So $\dfrac{dA}{A} = \dfrac{\pm .4}{16} = \boxed{\pm .025}$

$\approx$ relative error

(c) 5pts

$\boxed{\% \text{ error} \approx 2.5\%}$

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(1) 10pts

~~$x^2 - 3x + 2 \xrightarrow{x \to 4} 6$~~

~~Proof~~ ~~Scratch: Assume $\delta \le 1$. Then~~

~~$3 < x$~~

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(B1) $\lim_{x \to 4} (x^2 - 3x + 2) = 6$

Scratch: $x^2 - 3x + 2 = 6$

$x^2 - 3x - 4 = 0$ (want)

$\underbrace{(x-4)}_{< \delta}\underbrace{(x+1)}_{\text{Need a bound.}} = 0$

Assume $\delta \le 1$. Then

$3 < x < 5 \longrightarrow$

$4 < x+1 < 6 \longrightarrow$

$|x+1| < 6$, so

Proof Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{6}\}$.

Then $0 < |x-4| < \delta \longrightarrow$

$|x^2 - 3x + 2 - 6| = |x^2 - 3x - 4| = |x+1||x-4|$

$< 6|x-4| < 6\delta \le 6 \cdot \frac{\varepsilon}{6} = \varepsilon$ ∎

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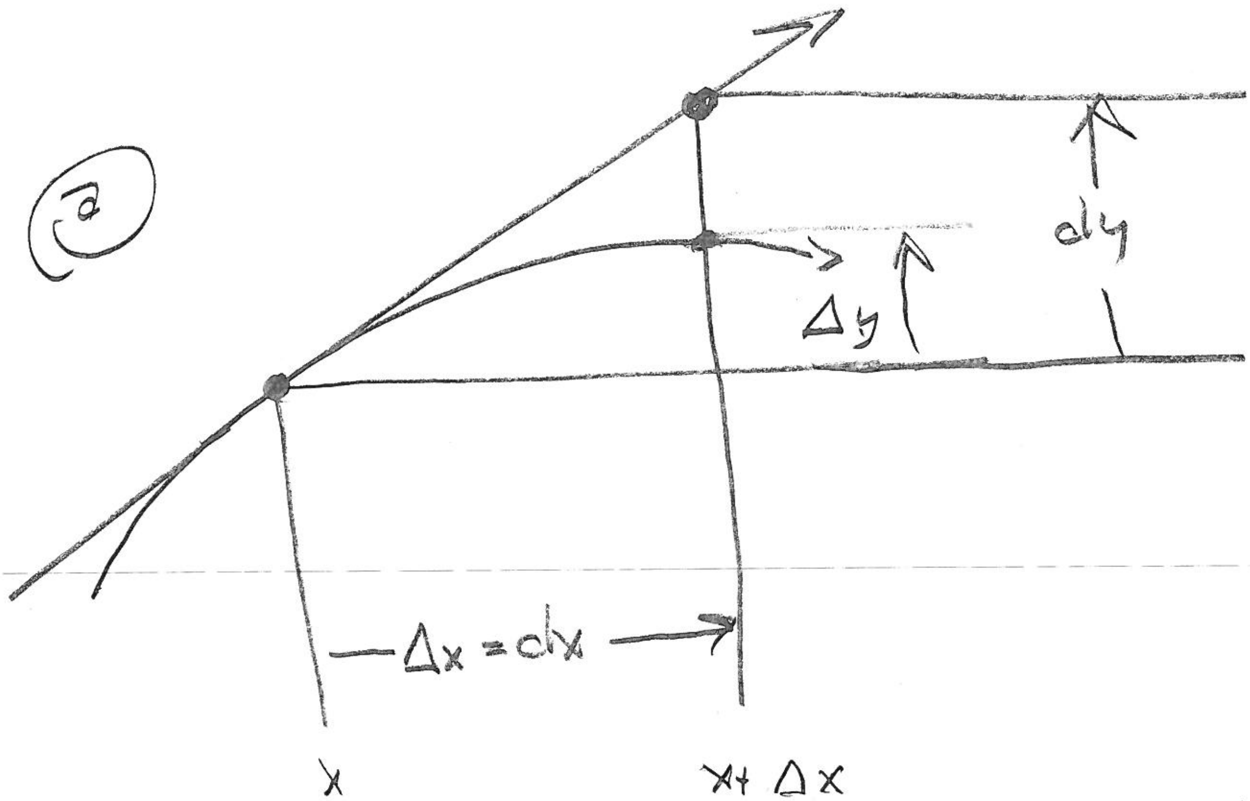
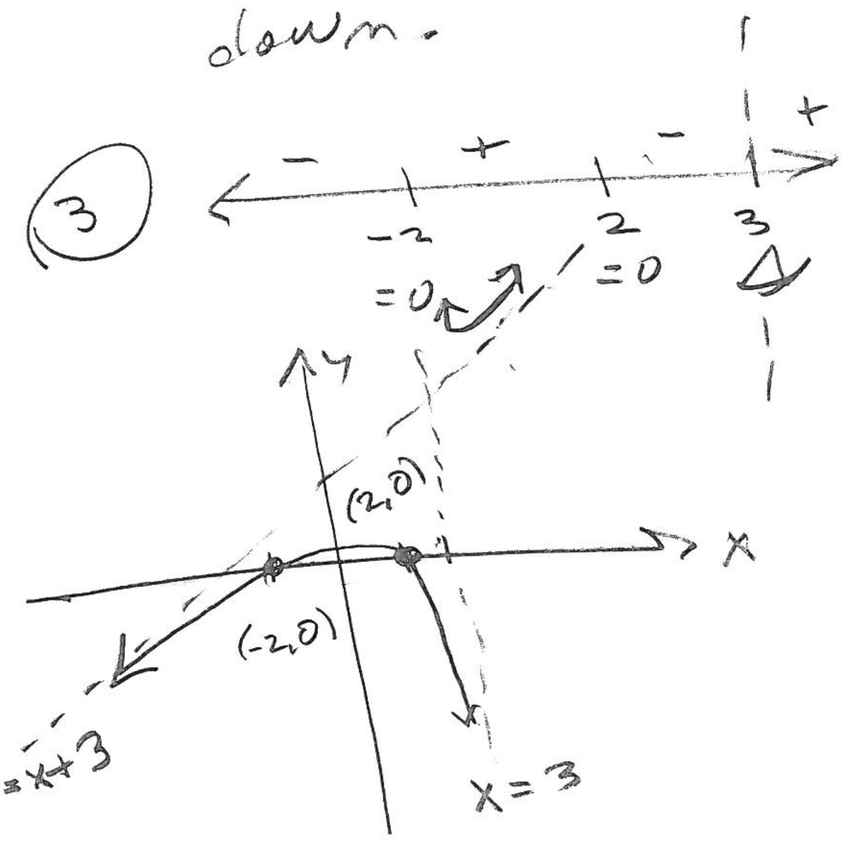
(2)

(a)



(b) tangent line is an over-estimate. f(x) is concave down.

(3)



$$\begin{array}{r|rrr} 3 & 1 & 0 & -4 \\ & & 3 & 9 \\ \hline & 1 & 3 & 5 \end{array}$$

$y = x + 3$ is O.A.