

Do all your work and put all your answers WITH your work, CIRCLED, on the white paper provided. All I want on this sheet is your NAME! Spend no more than 2 minutes on any single problem on your first pass through the test. If you don't finish a problem in 2 or 3 minutes, start a fresh sheet of paper for the next problem, and so on.

- (15 pts) The point  $P(2,14)$  lies on the graph of  $f(x) = 2x^2 + 3x$ . Estimate the slope of this curve at  $P$ , by evaluating the slope between the point  $P$  and the point  $Q(2.001, f(2.001))$ .
- (5 pts) Any guess as to what the actual slope is at  $x = 2$ ?
- (5 pts) Based on your answer to #2, write the equation of the tangent line to  $f(x)$  at  $x = 2$ .
- (5 pts each) Evaluate the following limits, if they exist. If one does not exist, explain why.

a.  $\lim_{x \rightarrow -3^+} \frac{2x^2 + 13x + 21}{|x + 3|}$

b.  $\lim_{x \rightarrow -3^-} \frac{2x^2 + 13x + 21}{|x + 3|}$

c.  $\lim_{x \rightarrow -3} \frac{2x^2 + 13x + 21}{|x + 3|}$

- (15 pts) Sketch the graph of the piecewise-defined function  $f(x) = \begin{cases} x^2 - 4 & \text{if } x < 1 \\ -3x + 6 & \text{if } x \leq 1 \end{cases}$ . Label intercepts with ordered pairs.

**Bonus** (5 pts) On what intervals is  $f$  in #5 continuous?

- Simplify  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  for the following functions:

a. (10 pts)  $f(x) = 2x^2 - 3x + 7$

b. (5 pts)  $f(x) = \frac{1}{x}$

- (10 pts) Sketch a plausible graph of a function  $f$  that has the following properties:

a.  $\lim_{x \rightarrow -3^-} f(x) = 2$

c.  $\lim_{x \rightarrow 2^-} f(x) = \infty$

e.  $\lim_{|x| \rightarrow \infty} f(x) = 5$

b.  $\lim_{x \rightarrow -3^+} f(x) = 6$

d.  $\lim_{x \rightarrow 2^+} f(x) = -\infty$

f.  $f(-3) = 4$

- (10 pts) Prove that  $\lim_{x \rightarrow 2} (3x - 7) = -1$ , using the  $\epsilon - \delta$  definition of limit.

- (5 pts) Prove that the equation  $\sin\left(\frac{\pi}{3}x\right) = x - 1$  has a root in the interval  $(0, 3)$ , but *do not solve!*

**BONUS SECTION:** Work any 3 bonus questions for up to 15 bonus points.

- (5 pts) Prove that  $\lim_{x \rightarrow 4} (3x^2 - 13x + 14) = 10$

- (5 pts) Evaluate  $\lim_{h \rightarrow 0} \frac{\sqrt{2x+2h} - \sqrt{2x}}{h}$ , if it exists. If it does not, state why.

- (5 pts) See if you can squeeze out a convincing argument to find  $\lim_{x \rightarrow 0} \left( x^2 \sin\left(\frac{\pi}{x}\right) \right)$



4. (5 pts) Write the definition of the piecewise-defined function from its graph:

