201 E1 FALL, 2007

(1) $(2, f(2)) = (2, 14) = (x_1, y_1)$

$f(x) = 2x^2 + 3x \Rightarrow f(2) = 2(2)^2 + 3(2)$

15 pts

$= 8 + 6 = 14$

$(x, f(x)) = (2.001, 14.011002) = (x_2, y_2)$

$f(2.001) = 2(4.004001) + 3(2.001)$

$= 8.008002 + 6.003$

$= 14.011002$

$\dfrac{f(x) - f(2)}{x - 2} = m_{AVG} = \dfrac{14.011002 - 14}{2.001 - 2}$

$= \dfrac{.011002}{.001} = 1000(.011002) = 11.002$

(2) 5 pts So, looks like $\boxed{m_{tan} = 11}$

Check: $4x + 3\Big|_{x=2} = 4(2) + 3 = 11$ ✓ Calculus!

(3) 5 pts $\boxed{y = 11(x - 2) + 14}$ STOP!

$= 11x - 22 + 14 = \boxed{11x - 8 = y}$ OK.

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4 (a) 5pts $\lim_{x \to -3^+} \frac{2x^2+13x+21}{|x+3|}$:

$\Rightarrow x > -3 \Rightarrow \frac{2x^2+13x+21}{x+3} = \frac{(2x+7)(x+3)}{x+3}$

$= 2x+7 \xrightarrow{x \to -3^+} 2(-3)+7 = -6+7 = \boxed{1}$

(b) 5pts $x \to -3^- \Rightarrow$

$x < -3 \Rightarrow \frac{2x^2+13x+21}{-(x+3)}$

$= -(2x+7) \xrightarrow{x \to -3^-} \boxed{-1}$

(c) 5pts $\lim_{x \to -3} \frac{2x^2+13x+21}{|x+3|}$ $\nexists$ b/c

left- and right-hand limits

disagree.

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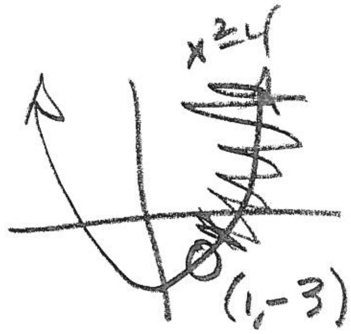
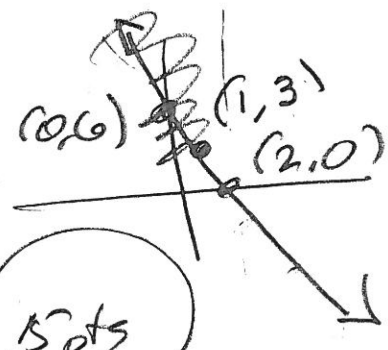
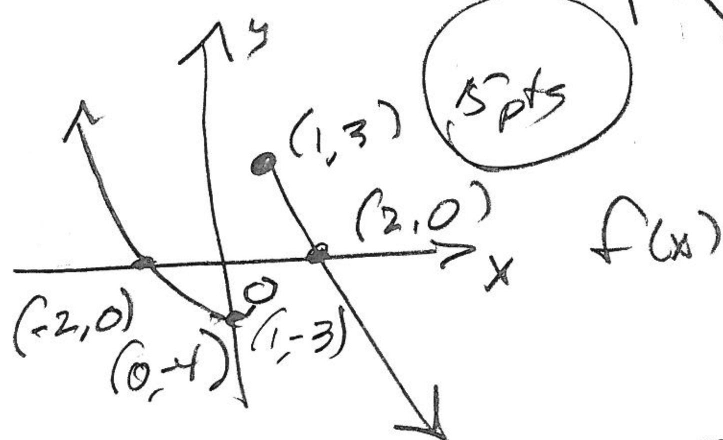
5 $f(x) = \begin{cases} x^2-4 & \text{if } x<1 \\ -3x+6 & \text{if } x \geq 1 \end{cases}$



$-3x+6 = -3(x-2)$



5 pts



$\boxed{\text{Bonus}}$ f cnts on $\boxed{(-\infty, 1) \cup (1, \infty)}$

5 pts

6a 10 pts $f(x) = 2x^2-3x+7$

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-3(x+h)+7-(2x^2-3x+7)}{h}$$

$$= \frac{2(x^2+2xh+h^2)-3x-3h-2x^2+3x}{h}$$

$$= \frac{2x^2+4xh+2h^2-3h-2x^2}{h} = \frac{4xh+2h^2-3h}{h} = 4x+2h-3 \xrightarrow{h\to 0} \boxed{4x-3}$$

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(6b) 5 pts $f(x) = \frac{1}{x} \Rightarrow \frac{f(x+h) - f(x)}{h}$

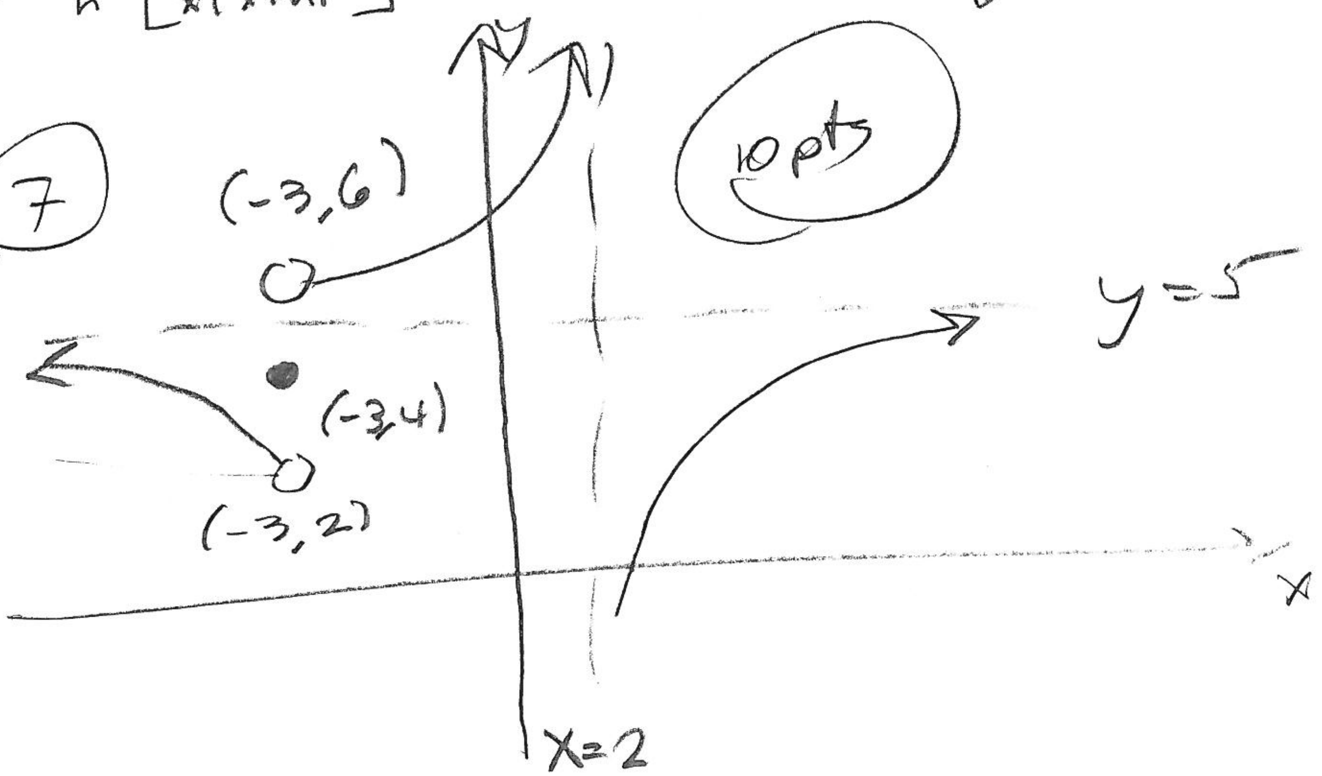
$$= \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{1}{h}\left[\frac{1}{x+h} - \frac{1}{x}\right]$$

$$= \frac{1}{h}\left[\frac{x}{x}\cdot\frac{1}{x+h} - \frac{x+h}{x+h}\cdot\frac{1}{x}\right]$$

$$= \frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right] = \frac{1}{h}\left[\frac{x-x-h}{x(x+h)}\right]$$

$$= \frac{1}{h}\cdot\left[\frac{-h}{x(x+h)}\right] = \frac{-1}{x(x+h)} \xrightarrow{h \to 0} \boxed{\frac{-1}{x^2} = f'(x)}$$

(7) 10 pts



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8 Claim $\lim_{x\to 2}(3x-7)=-1$

10pts

Proof

Let $\varepsilon>0$. Define $\delta=\frac{\varepsilon}{3}$. Then $0<|x-2|<\delta$

$\Rightarrow |(3x-7)-(-1)| = |3x-6| = 3|x-2|$

$< 3\delta = 3\cdot\frac{\varepsilon}{3} = \varepsilon$ ■

9 5pts $\sin(\frac{\pi}{3}x)-x+1=f(x)$.

$f(0)=\sin(0)-0+1=1$

$f(3)=\sin(\pi)-3+1=-2$

$f(0)=1>0>-2=f(3)$ &

f is a sum of sine & a polynomial,

hence cont's, so

$\exists\, c\in(0,3) \ni f(c)=0$, by IVT.

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B1 $\quad$ 5pts $\quad$ $\lim_{x\to 4} (3x^2 - 13x + 14) = 10$

<u>Scratch</u> $\quad 3x^2 - 13x + 14 - 10$

$= 3x^2 - 13x + 4$

$= \underbrace{(3x - 1)}_{\text{Need a bound}}\underbrace{(x - 4)}_{< \delta}$

$\delta \le 1 \Rightarrow \quad 3 < x < 5$

$9 < 3x < 15$

$8 < 3x - 1 < 14$ so $|3x - 1| < 14$

<u>Proof</u>
Let $\varepsilon > 0$. Define $\delta = \min\{1, \frac{\varepsilon}{14}\}$.

Then $0 < |x - 4| < \delta \Rightarrow |3x^2 - 13x + 14 - 10|$

$= |3x^2 - 13x + 4| = |3x - 1||x - 4| < 14\delta$

$\le 14 \cdot \frac{\varepsilon}{14} = \varepsilon$ ■

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B2 5pts

$$\frac{\sqrt{2x+2h}-\sqrt{2x}}{h} = \left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{h}\right)\left(\frac{\sqrt{2x+2h}+\sqrt{2x}}{\sqrt{2x+2h}+\sqrt{2x}}\right)$$

$$= \frac{2x+2h-2x}{h\left(\sqrt{2x+2h}+\sqrt{2x}\right)} = \frac{2h}{h\left(\sqrt{2x+2h}+\sqrt{2x}\right)}$$

$$= \frac{2}{\sqrt{2x+2h}+\sqrt{2x}} \xrightarrow{h\to 0} \frac{2}{\sqrt{2x}+\sqrt{2x}}$$

$$= \frac{2}{2\left(\sqrt{2x}\right)} = \boxed{\frac{1}{\sqrt{2x}}}$$

B3 5pts

$$-x^2 \le x^2\sin\left(\frac{\pi}{x}\right) \le x^2 \quad \forall\, x \neq 0$$

$$\begin{array}{ccccc} \downarrow{\scriptstyle x\to 0} & & \downarrow{\scriptstyle x\to 0} & & \downarrow{\scriptstyle x\to 0} \end{array}$$

Squeeze Theorem

$$0 \le \lim_{x\to 0} x^2\sin\left(\frac{\pi}{x}\right) \le 0$$

$$\Longrightarrow \lim_{x\to 0} x^2\sin\left(\frac{\pi}{x}\right) = 0$$

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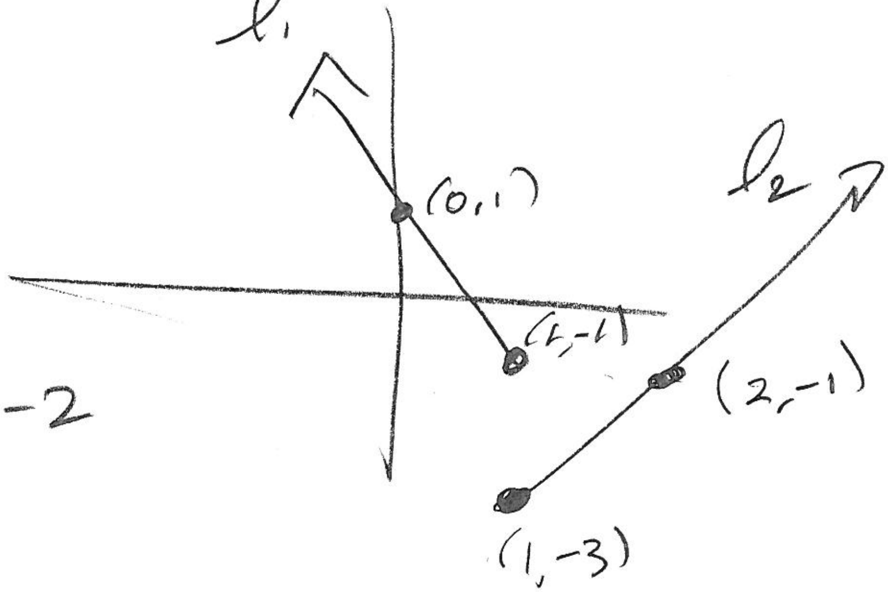
B4 5pts



l_1: $m = \frac{1+1}{0-1} = -2$

$y = -2x + 1$

l_2: $m = \frac{-1-(-3)}{2-1} = \frac{-1+3}{1} = 2$

$y = 2(x-1) - 3$

$= 2x - 2 - 3$

$= 2x - 5$

So $\boxed{f(x) = \begin{cases} -2x+1 & \text{if } x < 1 \\ 2x-5 & \text{if } x \geq 1 \end{cases}}$