§5.4 #s 11, 13, 14, 16, 17, 21, 23

(11) A spring has natural length 20 cm. Compare the work $W_1$, done in stretching the spring from 20 to 30 cm with the work $W_2$ done in stretching it from 30 to 40 cm

$W = F \cdot D = Kx\,dx$

$F = Kx$

$$W_1 = \int_0^{10} Kx\,dx = K\left[\tfrac{1}{2}x^2\right]_0^{10} = 50K$$

$$W_2 = \int_{10}^{20} Kx\,dx = K\left[\tfrac{1}{2}x^2\right]_{10}^{20} = \tfrac{1}{2}\left[20^2 - 10^2\right]K$$

$$= \frac{300}{2}K = 150K = 3W_1$$

$$\boxed{W_2 = 3W_1}$$

Textbook converts to MKS. I left it in CGS.

~~#s~~ 13-20 Show how to approx w/ Riemann

(~~13~~) Sum. Write Integral. Evaluate

(13) Heavy rope, 50 ft long, weighs .5 lb/ft & hangs over edge of a building 120 ft high

(a) How much work is done, pulling rope to top of building?

Weight of rope $= (50)(.5) = 25$ lbs

As it raises, the weight drops .5 lbs/ft

201 §5.4 #s 13, 14, 16, 17, 21, 23

13 cntd $\quad F = 25 - x$ where $x$ = distance raised. Better get $\big(\underbrace{(50-x)}_{\text{Amt of rope left}}\,ft\big)\left(.5\,\frac{lb}{ft}\right)\Delta x$

$$W = \sum_{k=1}^{n} (25 - .5x_k)\,\Delta x$$

$$\xrightarrow{n\to\infty} \int_0^{50} \underbrace{(25 - .5x)}_{lbs}\,\underbrace{dx}_{ft} = \left[25x - \frac{1}{2}\cdot\frac{1}{2}x^2\right]_0^{50}$$

$$= 25(50) - \frac{1}{4}(2500) - (0-0)$$

$$= 1250 - 625$$

$$= 625 \text{ ft-lbs}$$

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I'm moving the whole rope an incremental distance. The textbook is moving an incremental piece of rope from its starting height to the top. Both are legit.

201 & 5.4 #s 14, 16, 17, 21, 23

(13 b) How much work is done in pulling half the rope to the top?

Using my same approach.

Amt of rope hanging over edge is $50 - x$, where $x$ is the amt you've pulled up. Its weight is $0.5(50-x)$

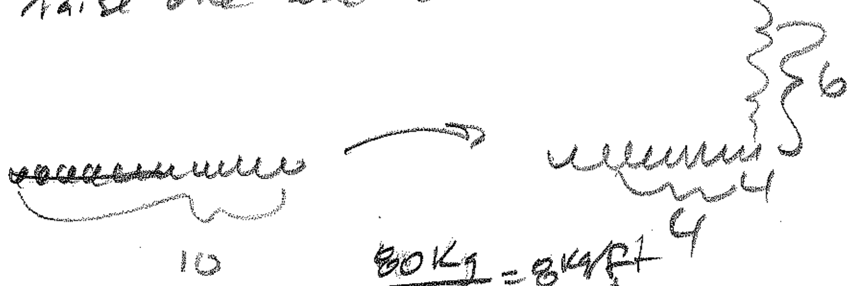
We pull up $\frac{1}{2}$ of it: $x = 0$ to $x = 25$.

$$\int_0^{25} (25 - .5x)\,dx = \left[25x - \frac{1}{4}x^2\right]_0^{25}$$

$$= 25(25) - \frac{1}{4}(25)(25) = 625 - \frac{625}{4} = \frac{3(625)}{4}$$

$$= \frac{1875}{4} \text{ ft-lbs.}$$

(14) A chain lying on the ground is 10 m long & its mass is 80 kg. How much work to raise one end of the chain to 6 m?




I'll try it book's way

$\frac{80\text{ Kg}}{10\text{ ft}} = 8\text{ Kg/ft}$

From $x = 0$ to $x = 6$, we lift $\Delta x$ - piece $6 - x$ m

x

$$\int_0^{6\text{ ft}} (6-x)\left(8\,\frac{\text{kg}}{\text{ft}}\right)\Delta x(\text{ft})$$

$$= 8\left[6x - \frac{1}{2}x^2\right]_0^6 = 8\,[36 - 18] = 164 \text{ Kg-m}$$

201 §5.4 #s 14, 16, 17, 21, 23

#14 corrected

Failed to convert mass to force. Need $9.8 m/s^2$

$$\text{Work} = 9.8\ m/s^2 \sum_{k=1}^{n} ((6-x_k)m)(8\ kg/m)(\Delta x\ m)$$

$$= \frac{kg \cdot m^2}{s^2} \xrightarrow{\cancel{mass}} (9.8)(8)\int_0^6 (6-x)\,dx$$

$$= (9.8)(8)\left[6x - \tfrac{1}{2}x^2\right]_0^6 = (9.8)(8)[36-18]$$

$$= 9.8(8)(18) = 1411.2\ \frac{kg \cdot m^2}{sec^2}$$

(16) Bucket weighs 4 lb, weightless rope.
80 ft well. 40 lbs $H_2O$ in bucket.
Raising @ rate of 2ft/s
Water leaks @ .2 lb/s
Find work done in pulling bucket up.

$44 - .2t$ = weight of bucket + water

2ft/sec ∴ $\frac{80ft}{2ft/sec}$ = 40 sec to top.

$$\text{work} = \int_0^{\cancel{44}\,40} \underbrace{(44-.2t)}_{\text{weight/force}}\,dt = \left[44t - \frac{.2}{2}t^2\right]_0^{40}$$

dt = dtime. Need d distance

201 §8.4 #s 16, 17, 21, 23

#16 cont'd

$x = 2t \longrightarrow$

$dx = 2dt \longrightarrow$

$dt = \frac{1}{2}dx$

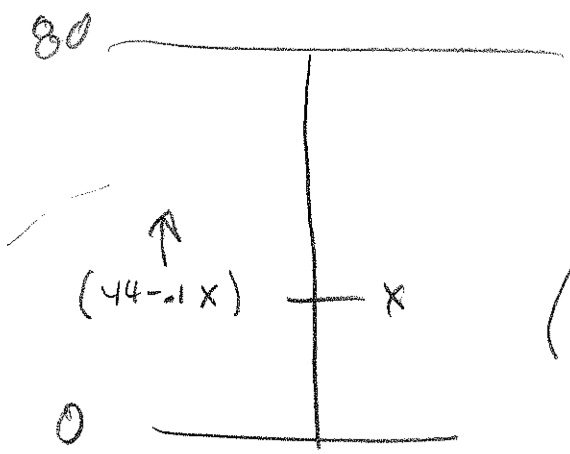
$$\int_0^{44} \underbrace{(44-.2t)}_{\substack{\text{weight}\\ \text{as } f(t).}} \underbrace{(2dt)}_{dx} = 4\int_0^{44}(22-.1t)\,dt$$

Need as f(x)!

$$= 4\left[22t - .\tfrac{1}{2}t^2\right]_0^{44} = 4\left[22(44) - .05(44)^2 - (0-0)\right]$$

$$= 3484.8 \text{ ft-lbs}$$

Something not Kosher, here.




Weight (x) = ?

Weight = 44 − (.2 lb/s)t

$$\left(.2t\,\frac{\text{lb}}{\text{s}}\right)\left(\frac{1\text{ s}}{2\text{ ft}}\right) = .1t\,\frac{\text{lb}}{\text{ft}}$$

$$x = 2t \Longrightarrow t = \frac{x}{2}$$

$$\left(.2\,\frac{\text{lb}}{\text{sec}}\right)(t\text{ sec}) = \left(.2\,\frac{\text{lb}}{\text{sec}}\right)\left(\frac{x}{2}\text{ sec}\right) = \underset{\text{lbs}}{.1x}$$

$$\int_0^{80}\overset{\text{lbs}}{(44 - .1x)}\overset{\text{ft}}{dx}$$

$$= \left[44x - .05x^2\right]_0^{80} = 44(80) - .05(80)^2 = \boxed{3200 \text{ ft-lbs}}$$

201 §5.4 #s 17, 21, 23

17 Leaky 10-kg bucket lifted to 12 m @ constant speed w/ rope weighing .8 kg/m

Starts w/ 36 kg $H_2O$, But leaks @ constant rate & finishes draining @ 12 m. How much work?

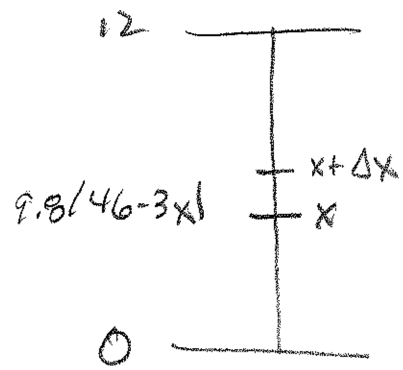
36+10=46 kg total

(0, 46) ; (12, 10)

Full @ x=0    Empty @ x=12

$m = \frac{10-46}{12-0} = -\frac{36}{12} = -3$

$-3 \frac{kg}{m}$




$F = (46-3x)(9.8\ m/s^2)$

$= (kg)(m/sec^2) = \text{Force}$

$\sum_{k=1}^{n} 9.8(46-3x_k)\Delta x$

$x_k = 0 + \frac{12}{n}k$, etc.

$9.8\int_0^{12} (46-3x)\,dx$

$= 9.8\left[46x - \frac{3}{2}x^2\right]_0^{12} = 9.8\left[46(12) - \frac{3}{2}(12)^2\right] = 3292.8$ lb-ft.

Oops! Forgot the rope!

It's $12(.8) - .8x = (12-x)(.8)$ in mass

So Force @ x is $((46-3x) + (12-x)(.8))\,9.8$

$= (46-3x+9.6-.8x)\,9.8 = (55.6-3.8x)(9.8)$

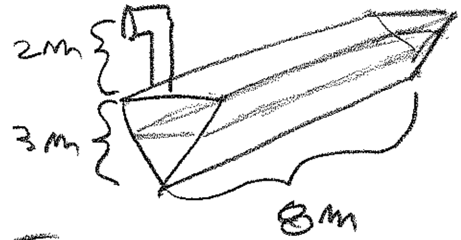
This gives $9.8\int_0^{12}(55.6-3.8x)\,dx = 9.8\left[55.6x - 1.9x^2\right]_0^{12}$

$= (55.6(12) - 1.9(12)^2)(9.8) = \boxed{3857.28}$ N-m

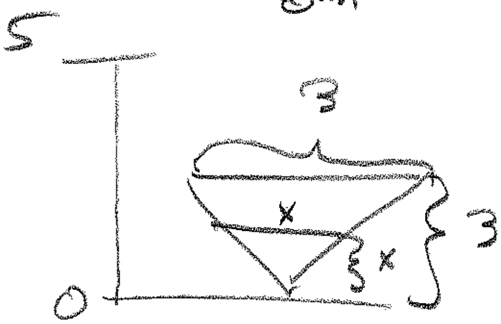
$m^2 = J$

201 § 5.4 #s 21, 23

(21) Tank is full of water. Find work required to pump it empty




we look at a strip/layer of thickness $dx = \Delta x$




width = height = x m
x-sectional /layer area is $x \cdot 8 = 8x$ = area

Layer is lifted $5-x$ m

$\therefore$ FORCE(x) = $(8x)(\Delta x)(9.8)(1000)$
$\quad m^2 \cdot m \quad m/s^2 \quad kg/m^3$

Distance (x) = $5-x$

$$W = 9800\int_0^3 8x(5-x)\,dx = 8(9800)\int_0^3 (5x - x^2)\,dx$$

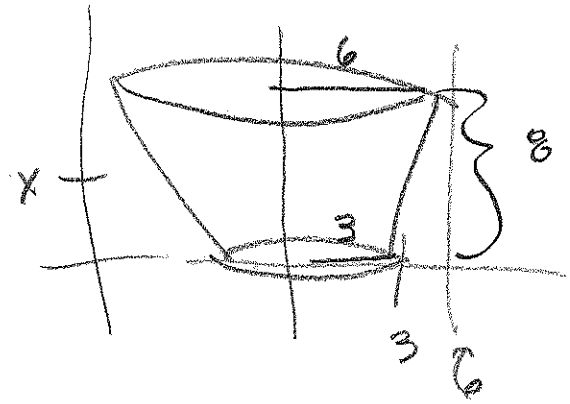
$$= 8(9800)\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^3 = 8(9800)\left[\frac{5}{2}(3)^2 - \frac{1}{3}(3)^3\right]$$

$$= 8(9800)\left[\frac{45}{2} - 9\right] = 8(9800)(22.5-9)$$

$$= 8(9800)(13.5) = 1{,}058{,}400$$

$$= \boxed{1.0584 \times 10^6 \text{ J}}$$

201 S 5.4 #23




Vol of a slice @ height $x$:

Radius: $(0,3), (8,6)$

$\rightarrow \frac{6-3}{8-0} = \frac{3}{8}$

No. $r = \frac{3}{8}(x-0) + 3$

$= \frac{3}{8}x + 3$

Area $\pi r^2 = \pi \left(\frac{3}{8}x\right)^2$

$= \frac{9\pi}{64}x^2$ @ $x$ $\rightarrow \pi\left(\frac{3}{8}x+3\right)^2$

Vol $= \left(\frac{9\pi}{64}x^2\right)\Delta x \longrightarrow F = 62.5\left(\frac{9\pi}{64}\right)x^2\Delta x$

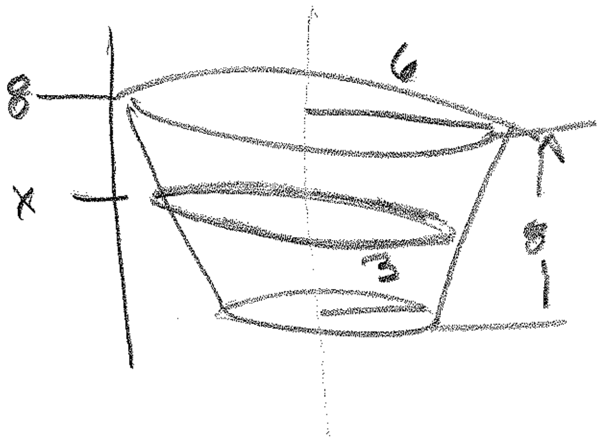
That slice is pumped $8-x$ ft

$W = 62.5\left(\frac{9\pi}{64}\right)\int_0^8 x^2(8-x)\,dx$

$62.5\left(\frac{9\pi}{64}\right)\int_0^8 \left(\frac{3}{8}x+3\right)^2(8-x)\,dx$

$\left(\frac{3}{8}x+3\right)^2 = \left(\frac{3}{8}\right)^2(x+8)^2 =$

201 §5.4 #23




Water slices

$$\underbrace{(\underbrace{\pi r^2}_{\text{Area}}\,\Delta x)}_{\text{Vol.}}\underbrace{(62.5)}_{\text{Density in force units}}\underbrace{(8-x)}_{\text{Distance}}$$

Force

$(0,r) = (0,3)$ $\quad m = \dfrac{6-3}{8-0} = \dfrac{3}{8}$

$(8,r) = (8,6)$

$$r = \frac{3}{8}(x-0)+3$$

$$= \frac{3}{8}x+3$$

$$\longrightarrow \text{Work} = 62.5\pi \int_0^8 \left(\frac{3}{8}x+3\right)^2(8-x)\,dx$$

Scratch

$$\left(\frac{3}{8}x+3\right)^2 = \left(\frac{3}{8}\right)^2(x+8)^2 = \frac{9}{64}(x^2+16x+64)$$

$$(x^2+16x+64)(8-x) = 8x^2+128x+512$$

$$-x^3-16x^2-64x$$

$$-x^3-8x^2+64x+512$$

$$= (62.5\pi)\left(\frac{9}{64}\right)\int_0^8(-x^3-8x^2+64x+512)\,dx$$

$$= \frac{(62.5)(9)\pi}{64}\left[-\frac{1}{4}x^4-\frac{8}{3}x^3+32x^2+512x\right]_0^8$$

201 8.5 #22 cont'd

$$= \frac{(62.5)(9)\pi}{64}\left[-\frac{1}{4}(8)^4-\frac{8}{3}(8)^3+22(8)^2+512(8)\right]$$

$\approx 86001.09889$ ft-lbs

Nowhere close. Book gets $\frac{3}{8}(16-x)=r$ where

I get $\frac{3}{8}(x+8)$

Book measures x from the top.

So I have 8-x for distance pumped. Book has x for that distance.

$\frac{3}{8}(0+8)=3$ ✓

$\frac{3}{8}(8+8)=6$ ✓

$\frac{\frac{3}{1}}{\frac{3}{8}}=\frac{3}{1}\cdot\frac{8}{3}=8$ ✓

$$\frac{3}{8}(x)+3 = \frac{3}{8}\left(x+\left(\frac{3}{\frac{3}{8}}\right)\right)=\frac{3}{8}(x+8)$$

$$\left(\frac{3}{8}(x+8)\right)^2=\frac{9}{64}(x^2+16x+64)$$

$$\begin{aligned}(8-x)(x^2+16x+64) &= 8x^2+128x+512\\ &\quad -x^3-16x^2-64x\\ \hline &\quad -x^3-8x^2+64x+512\end{aligned}$$

We'll have to work on this in class.