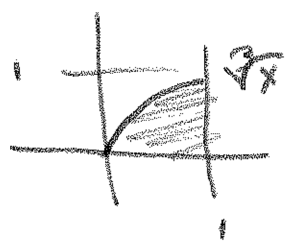
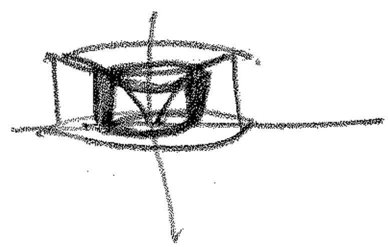
201 §5.3 #s 3, 6, 8, 9, 12, 15, 19, 20

#s 3-7 Use cylindrical shells to find the volume generated by rotating the region bdd by the given curves about the y-axis:

$$2\pi\int_a^b x\,f(x)\,dx$$

(3) $y=\sqrt[3]{x}$, $y=0$, $x=1$





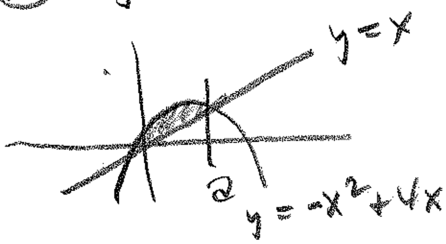
$$V=2\pi\int_0^1 x\cdot x^{\frac{1}{3}}\,dx = 2\pi\int_0^1 x^{\frac{4}{3}}\,dx = 2\pi\left[\frac{3}{7}x^{\frac{7}{3}}\right]_0^1$$

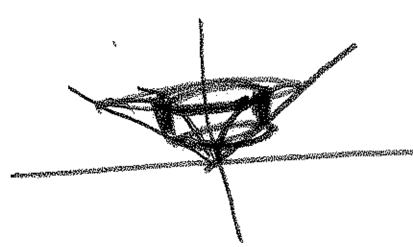
$$=2\pi\left(\frac{3}{7}\right)=\boxed{\frac{6\pi}{7}}$$

$$-x^2+4x=-x(x-4)$$

(6) $y=-x^2+4x$, $y=x$

$$-x^2+4x=x$$
$$-x^2+3x=0$$
$$x^2-3x=0$$
$$x=0,\ x=3$$
$$a=3$$





$$V=2\pi\int_0^3 x\left(-x^2+4x-x\right)dx = 2\pi\int_0^3\left(-x^3+3x^2\right)dx$$

$$=2\pi\left[-\frac{1}{4}x^4+3\cdot\frac{1}{3}x^3\right]_0^3=2\pi\left[-\frac{81}{4}+27\right]=2\pi\left[\frac{-81+108}{4}\right]$$

$$=2\pi\left[\frac{27}{4}\right]=\boxed{\frac{27\pi}{2}}\approx 42.41150082$$

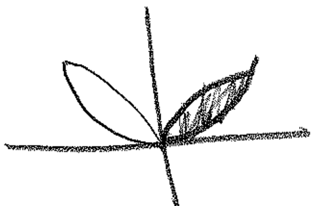
201 §8.3 #s 8, 9, 12, 15, 19, 20

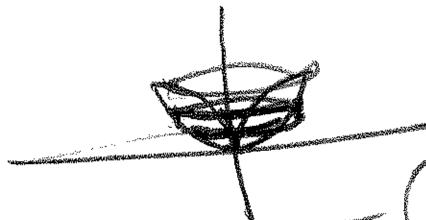
(8) V = vol. of solid obtained by rotating about the y-axis the region bdd by $y=\sqrt{x}$ & $y=x^2$. Find V by slicing & cylindrical shells. Draw pics.

(1) Slicing

washers: dy-sitch

outer^2 - inner^2





$$\pi\int_0^1\left((\sqrt{y})^2-(y^2)^2\right)dy$$

$y=\sqrt{x}$

$\Longrightarrow x=y^2$

$y=x^2=y$

$|x|=\sqrt{y}$

$x=\pm\sqrt{y}$ take $+\sqrt{y}$

$$=\pi\int_0^1(y-y^4)\,dy=\pi\left[\tfrac{1}{2}y^2-\tfrac{1}{5}y^5\right]_0^1$$

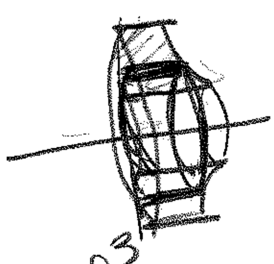
$$=\pi\left(\tfrac{1}{2}-\tfrac{1}{5}\right)=\frac{5-2}{10}\pi=\boxed{\frac{3}{10}\pi}$$

#s 9-14

Use shells to find vol of the solid obtained by rotating the region bdd by the given curves about the ~~specified ax~~ x-axis → For shells, this is dy-situation.

(9) $xy=1,\ x=0,\ y=1,\ y=3$

$x=\frac{1}{y}$



$$2\pi\int_a^b y\,f(y)\,dy$$

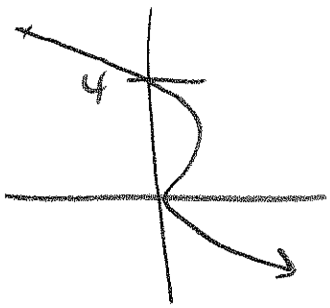
$$=2\pi\int_1^3 y\,\frac{1}{y}\,dy$$

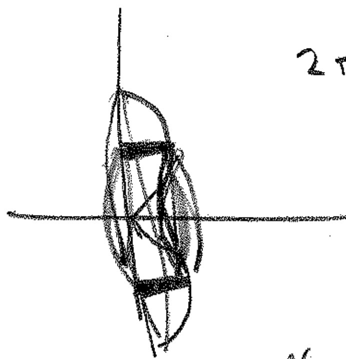
$$=2\pi\int_1^3 dy=2\pi[3-1]=\boxed{4\pi}$$

201 §5.3 #s 12, 15, 19, 20

(12) $x = 4y^2 - y^3$, $x = 0$

$x = y^2(4-y)$





$2\pi \int y\, g(y)\, dy$

$$V = 2\pi \int_0^4 y(4y^2 - y^3)\, dy = 2\pi \int_0^4 (4y^3 - y^4)\, dy$$

$$= \left[\frac{4}{4}y^4 - \frac{1}{5}y^5\right]_0^4 2\pi = \left(4^4 - \frac{1}{5}(4)^5\right)2\pi - (0-0)$$

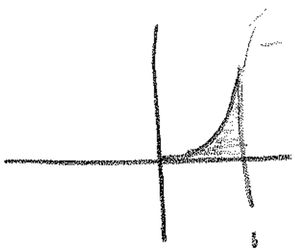
$$= 2\pi\left(256 - \frac{1024}{5}\right) = 2\pi\left(\frac{1280-1024}{5}\right) = \frac{256}{5} \cdot 2\pi = \boxed{\frac{512\pi}{5}}$$

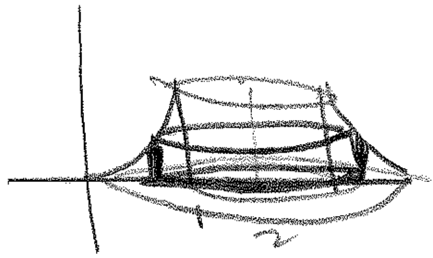
≈ 321.6990877

#s 15-20 use cylindrical shells to find vol. generated by rotating the region bdd by the given curves about the specified axis

201 §5.3 #s 15, 19, 20

(15) $y=x^4,\ y=0,\ x=1$ about $x=2$





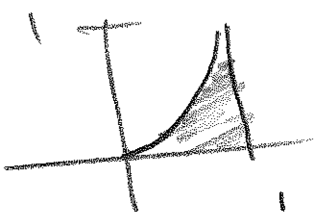
$$2\pi\int_0^1 x(1-x^4)\,dx$$

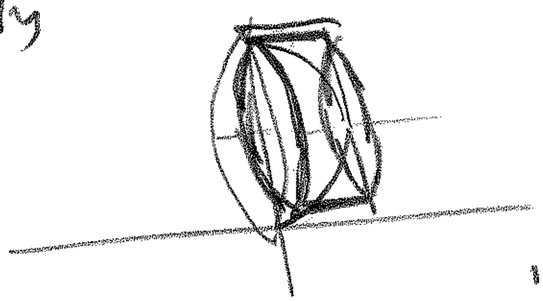
$$=2\pi\int_0^1 (x-x^5)\,dx = 2\pi\left[\tfrac{1}{2}x^2-\tfrac{1}{6}x^6\right]_0^1 = \left(\tfrac12-\tfrac16\right)2\pi = \tfrac13\cdot 2\pi$$

$$\boxed{=\frac{2\pi}{3}}$$

(19) $y=x^3,\ y=0,\ x=1$ about $y=1$

$\longrightarrow x=\sqrt[3]{y}$





$$2\pi\int_a^b y\,f(y)$$

$$V=2\pi\int_0^1 (1-y)(1-\sqrt[3]{y})\,dy = 2\pi\int_0^1 \left(1-\sqrt[3]{y}-y+y\sqrt[3]{y}\right)dy$$

$$=2\pi\int_0^1\left(1-y^{\frac13}-y+y^{\frac43}\right)dy = 2\pi\left[y-\tfrac34 y^{\frac43}-\tfrac12 y^2+\tfrac37 y^{\frac73}\right]_0^1$$

$$=2\pi\left[1-\tfrac34-\tfrac12+\tfrac32\right] = 2\pi\left[\tfrac54\right] = \boxed{\frac{5\pi}{2}}$$