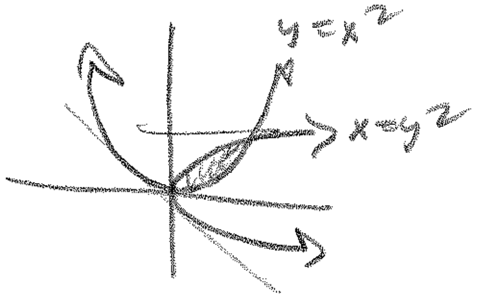
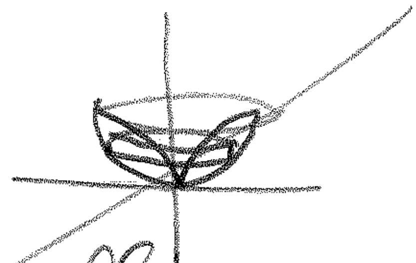
201 §5.2 #s 11, 13, 31

#s 1-18 Find volume obtained by rotating the region bdd by the given curves about the specified line. Sketch region, solid & typical disk or washer.

(11) $y=x^2$, $x=y^2$, about $y=1$





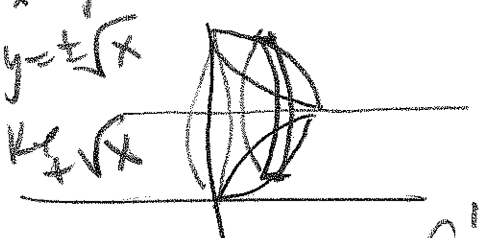
~~Washers stacked vertically $\int \ldots dy$~~

~~Upper - Lower = Right - Left (dy situation)~~

~~$y=x^2 \Rightarrow x=\pm\sqrt{y}$ Take $+\sqrt{y}$~~

~~So $\pi \int_0$~~

Nope rotate about $y=1$!



Washers on edge. $\int dx$ situation.

Outer^2 - Inner^2 =

$$\pi \int_0^1 \left((1-x^2)^2 - (1-\sqrt{x})^2\right) dx$$

$$= \pi \int_0^1 \left(x^4 - 2x^2 + 1 - (x - 2\sqrt{x} + 1)\right) dx$$

201 §5.2 #s 11, 13, 31

(11) cont'd

$$= \pi \int_0^1 (x^4 - 2x^2 + 1 - x + 2\sqrt{x} + 1)\,dx$$

$$= \pi \int_0^1 (x^4 - 2x^2 - x + 2x^{\frac{1}{2}} + 2)\,dx$$

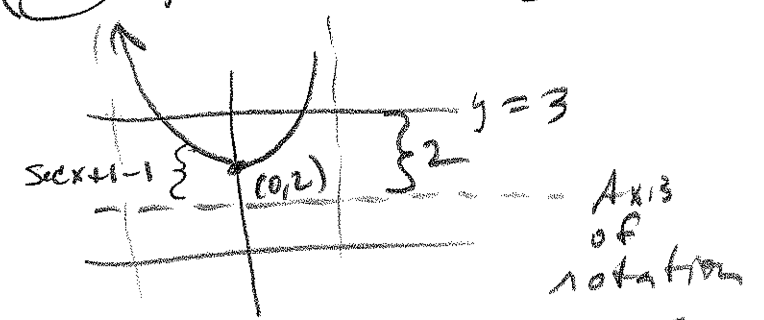
$$= \pi \left[\frac{1}{5}x^5 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2\left(\frac{3}{2}\right)x^{\frac{3}{2}} + 2x \right]_0^1$$

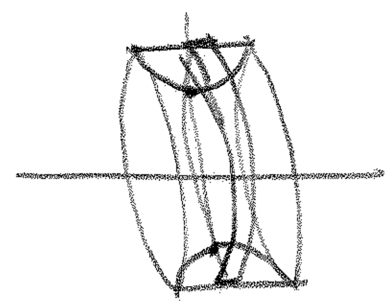
$$\begin{array}{r} 176 \\ -15 \\ \hline 161 \end{array}$$

$$= \pi\left[\frac{1}{5} - \frac{2}{3} - \frac{1}{2} + 3 + 2\right] = \pi\left[\frac{1}{5}\cdot\frac{6}{6} + \frac{2}{3}\cdot\frac{10}{10} - \frac{1}{2}\cdot\frac{15}{15} + \frac{5}{1}\cdot\frac{30}{30}\right]$$

$$= \pi\left[\frac{6 + 20 - 15 + 150}{30}\right] = \boxed{\frac{161\pi}{30}}$$

(13) $y = 1 + \sec x$, $y = 3$, about $y = 1$





outer² − inner² : $\pi \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(2^2 - (\sec x + 1 - 1)^2\right) dx$

$\sec x + 1 = 3$

$\sec x = 2$

$x = \frac{\pi}{3}$

$$= 2\pi \int_0^{\frac{\pi}{3}} (4 - \sec^2 x)\,dx = 2\pi \left[4x - \tan x\right]_0^{\frac{\pi}{3}}$$

2, $\sqrt{3}$

$$= 2\pi\left[\frac{4\pi}{3} - \tan\left(\frac{\pi}{3}\right) - (0 - 0)\right] = \boxed{\frac{8\pi^2}{3} - 2\pi\sqrt{3}} \approx 15.49614888$$

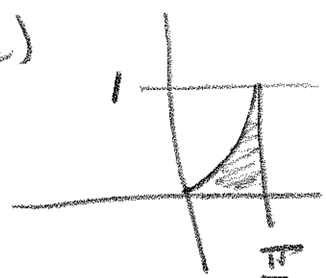
201 §5.2 #31

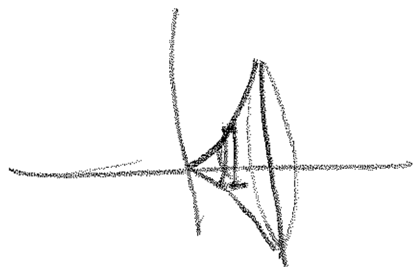
(31) Set up & then use graphing calc.

$y = \tan x,\ y = 0,\ x = \frac{\pi}{4}$

(a) about x-axis (b) about $y = -1$

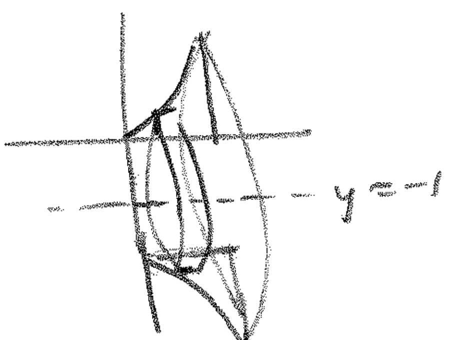
(a)





$$V = \pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx \approx .67419155 \approx \boxed{.67419}$$

(b)



$$V = \pi \int_0^{\frac{\pi}{4}} \left[(\tan x - (-1))^2 - (0 - (-1))^2 \right] dx$$

$$= \pi \int_0^{\frac{\pi}{4}} (\tan^2 x + 2 \tan x + 1 - 1) \, dx = \pi \int_0^{\frac{\pi}{4}} (\tan^2 x + 2 \tan x) \, dx$$

$$\approx 2.8517776 \approx \boxed{2.85178}$$