201 §5.1 #s 5, 9, 11, 13, 15, 17, 19, 23, 31, 35, 37

#s 5-12 Sketch region bdd by given curves
Draw approximating rectangle. Find area.

5. $y = x+1$, $y = 9-x^2$, $x=-1$, $x=2$

$9-x^2 = x+1$

$9-x^2-x-1=0$

$-x^2-x+8=0$

$x^2+x-8=0$

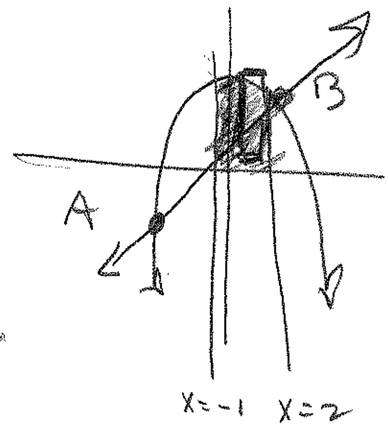
$x^2+x \qquad = 8$

$x^2+x+\left(\frac{1}{2}\right)^2 = 8+\frac{1}{4} = \frac{32+1}{4} = \frac{33}{4}$

$(x+1/2)^2 = \frac{33}{4}$

$x+\frac{1}{2} = \pm\frac{\sqrt{33}}{2}$

$x = \frac{-1\pm\sqrt{33}}{2}$ → -3.37228132 ; $+2.37228132$



$A \approx (-3.37, -2.37)$
$B \approx (2.37, 3.37)$

$$\text{Area} = \int_{-1}^{2}(9-x^2-(x+1))\,dx = \int_{-1}^{2}(-x^2-x+8)\,dx = \left[-\frac{1}{3}x^3-\frac{1}{2}x^2+8x\right]_{-1}^{2}$$

$= -\frac{1}{3}(2)^3-\frac{1}{2}(2)^2+8(2)-\left[-\frac{1}{3}(-1)^3-\frac{1}{2}(-1)^2+8(-1)\right]$

$= -\frac{8}{3}-\frac{4}{2}+16-\left[-\frac{1}{3}(-1)-\frac{1}{2}(1)-8\right]$

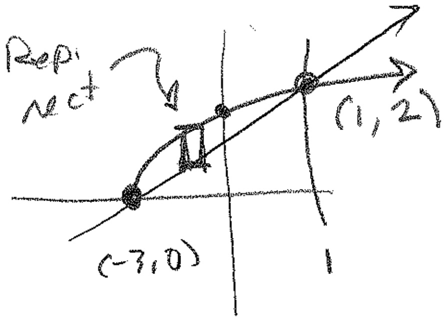
$= -\frac{8}{3}-2+16-\left[\frac{1}{3}-\frac{1}{2}-8\right]$

$= -\frac{8}{3}+14-\frac{1}{3}+\frac{1}{2}+8$

$= -\frac{9}{3}+22+\frac{1}{2} = -3+22+\frac{1}{2} = 19+\frac{1}{2} = \frac{38+1}{2} = \boxed{\frac{39}{2}}$

201 § 5.1 # 9, 11, 13, 15, 17, 19, 23, 31, 35, 37

(9) $y = \sqrt{x+3}$, $y = \frac{(x+3)}{2} = \frac{1}{2}x + \frac{3}{2}$



$\frac{x+3}{2} = \sqrt{x+3}$

$\frac{x^2+6x+9}{4} = x+3$

$x^2+6x+9 = 4x+12$

$x^2+2x-3 = 0$

$(x+3)(x-1) = 0$

$$\text{Area} = \int_{-3}^{1}\left(\sqrt{x+3} - \tfrac{1}{2}x - \tfrac{3}{2}\right)dx$$

$$= \int_{-3}^{1}\left((x+3)^{\frac{1}{2}} - \tfrac{1}{2}x - \tfrac{3}{2}\right)dx$$

$u = x+3$
$du = dx$

$$\int \sqrt{x+3}\, dx = \int u^{\frac{1}{2}} du = \tfrac{2}{3}u^{\frac{3}{2}} + C \Longrightarrow$$

$$\left[\tfrac{2}{3}(x+3)^{\frac{3}{2}} - \tfrac{1}{4}x^2 - \tfrac{3}{2}x\right]_{-3}^{1}$$

$$= \tfrac{2}{3}(1+3)^{\frac{3}{2}} - \tfrac{1}{4}(1)^2 - \tfrac{3}{2}(1) - \left[\tfrac{2}{3}(-3+3)^{\frac{3}{2}} - \tfrac{1}{4}(-3)^2 - \tfrac{3}{2}(-3)\right]$$

$$= \tfrac{2}{3}(4)^{\frac{3}{2}} - \tfrac{1}{4} - \tfrac{3}{2} - \left[\tfrac{2}{3}(0)^{3/2} - \tfrac{9}{4} + \tfrac{9}{2}\right]$$

$$= \tfrac{2}{3}(8) - \tfrac{1}{4} - \tfrac{6}{4} + \tfrac{9}{4} - \tfrac{18}{4} = \tfrac{16}{3} + \frac{-1-6+9-18}{4}$$

$$\begin{array}{r} -25 \\ 9 \\ \hline -16 \end{array}$$

$$= \tfrac{16}{3} - \tfrac{16}{4} = \tfrac{16}{3} - 4 = \tfrac{16}{3} - \tfrac{12}{3} = \boxed{\tfrac{4}{3}}$$

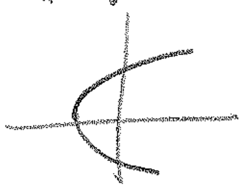
201 §5.1 #s 11, 13, 15, 17, 19, 23, 31, 35, 37

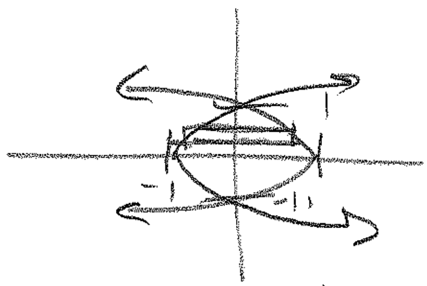
(11) ~~y =~~ $x = 1 - y^2$, $x = y^2 - 1$

$= -y^2 + 1$

$x = y^2$ $\quad$ $x = -y^2$ $\quad$ $x = -y^2 + 1$

$x = y^2 - 1$





$$A = \int_{-1}^{1} \left[(1 - y^2) - (y^2 - 1)\right] dy$$

$$= \int_{-1}^{1} (1 - y^2 - y^2 + 1)\, dy = \int_{-1}^{1} (2 - 2y^2)\, dy = 2\int_{-1}^{1} (1 - y^2)\, dy$$

$$= 2 \cdot 2 \int_{0}^{1} (1 - y^2)\, dy = 4\left[y - \tfrac{1}{3}y^3\right]_0^1 = 4\left[1 - \tfrac{1}{3} - (0 - 0)\right]$$

Symmetry

$$= 4\left[\tfrac{2}{3}\right] = \boxed{\tfrac{8}{3}}$$

201 §5.1 #s 13, 15, 17, 19, 23, 31, 35, 37

Same instructions

13

$y = 12 - x^2, \quad y = x^2 - 6$

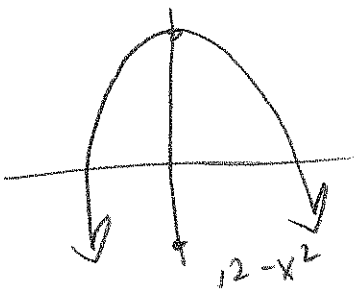
$= -x^2 + 12$

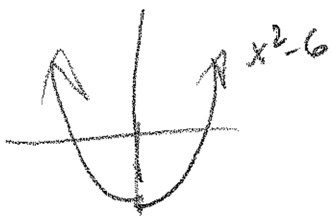
$-x^2 + 12 = x^2 - 6$

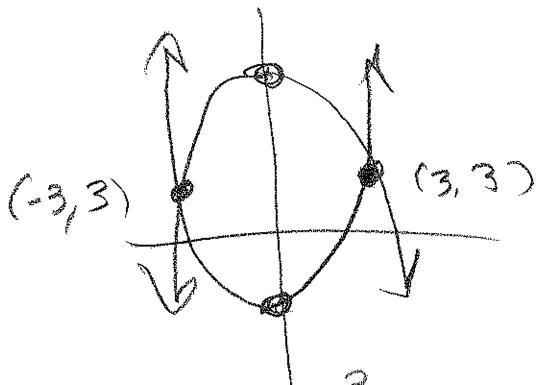
$-2x^2 + 18 = 0$

$x^2 - 9 = 0$

$x = \pm 3$







$$\text{Area} = \int_{-3}^{3} \left(-x^2 + 12 - (x^2 - 6)\right) dx = 2\int_0^3 (-2x^2 + 18)\, dx$$

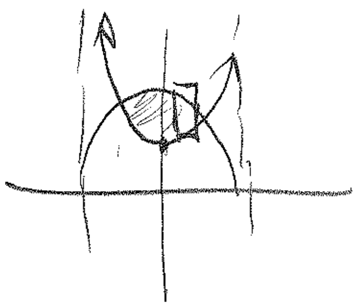
$$= 2\left[-\frac{2}{3}x^3 + 18x\right]_0^3$$

$$= 2\left[-\frac{2}{3}(3)^3 + 18(3) - (0 - 0)\right]$$

$$= 2\left[-\frac{2}{3}(27) + 54\right] = 2\left[-18 + 54\right] = 2[36] = \boxed{72}$$

201 §8.1 #s 15, 17, 19, 23, 31, 35, 37

15 $y = \sec^2 x$, $y = 8\cos x$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$



$$\text{Area} = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(8\cos x - \sec^2(x)\right) dx$$

$$= 2\int_{0}^{\frac{\pi}{3}} \left(8\cos x - \sec^2 x\right) dx$$

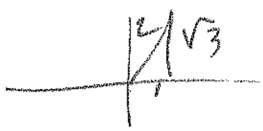
$$\sec^2 x = 8\cos x$$

$$\frac{1}{\cos^2 x} = 8\cos x$$

$$1 = 8\cos^3 x$$

$$\cos^3 x = \frac{1}{8}$$

$$\cos x = \frac{1}{2} \Longrightarrow x = \frac{\pi}{3}, -\frac{\pi}{3}$$



$$= 2\Big[8\sin x - \tan x\Big]_0^{\frac{\pi}{3}}$$

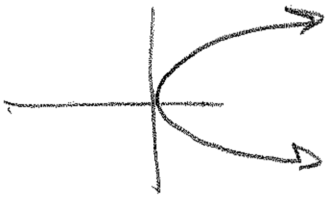
$$= 2\left[8\sin\left(\tfrac{\pi}{3}\right) - \tan\left(\tfrac{\pi}{3}\right) - (0-0)\right]$$

$$= 2\left[8\cdot\frac{\sqrt{3}}{2} - \sqrt{3}\right] = 2\left[4\sqrt{3} - \sqrt{3}\right]$$

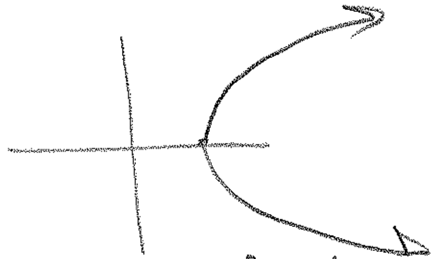
$$= 2\left[3\sqrt{3}\right] = \boxed{6\sqrt{3}} \approx 10.3923048~~5~~$$

201 §5.1 #s 17, 19, 23, 31, 35, 37

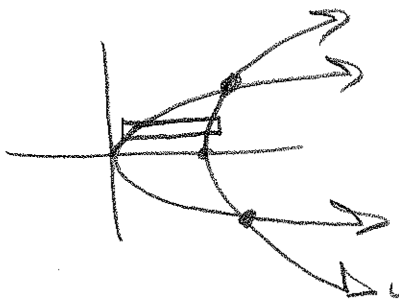
17) $x = 2y^2$, $x = 4 + y^2 = y^2 + 4$



$x = 2y^2$



$x = y^2 + 4$



"Right" is "up"

Rectangle height is

$y^2 + 4 - 2y^2 = -y^2 + 4 \overset{SET}{=} 0$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

$$\text{Area} = \int_{-2}^{2} (-y^2 + 4)\,dy$$

$$= 2\int_{0}^{2} (-y^2+4)\,dy = 2\left[-\tfrac{1}{3}y^3 + 4y\right]_0^2$$

Symmetry

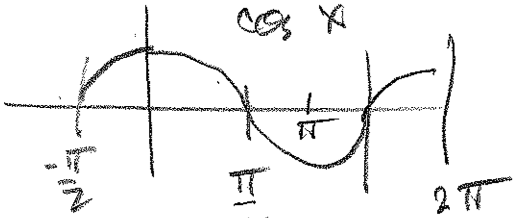
$$= 2\left[-\tfrac{1}{3}(2)^3 + 4(2) - (0+0)\right]$$

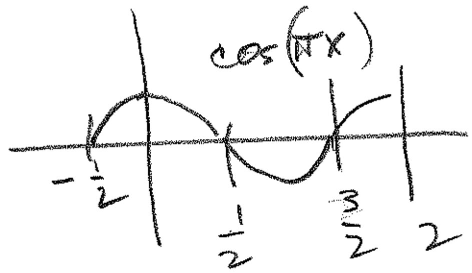
$$= 2\left[-\tfrac{8}{3} + 8\right] = 2\left[\frac{-8+24}{3}\right]$$

$$= 2\left[\frac{16}{3}\right] = \boxed{\frac{32}{3}}$$

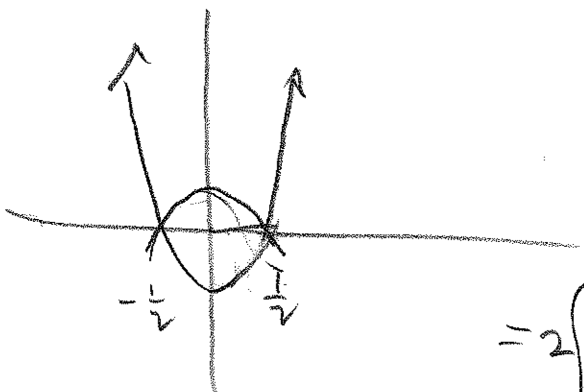
201 §5.1 #s 19, 23, 31, 35, 37

(19) $y = \cos(\pi x)$, $y = 4x^2 - 1$



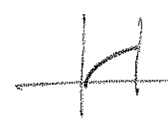


$$y = 4x^2 - 1 = 0$$
$$4x^2 = 1$$
$$x^2 = \frac{1}{4}$$
$$x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$



$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\cos(\pi x) - (4x^2 - 1)\right) dx$$

$$= 2\int_0^{\frac{1}{2}} \left(\cos(\pi x) - 4x^2 + 1\right) dx$$



$$= 2\left[\frac{1}{\pi}\sin(\pi x) - \frac{4}{3}x^3 + x\right]_0^{\frac{1}{2}}$$

$$= 2\left[\left(\frac{1}{\pi}\sin\left(\frac{\pi}{2}\right) - \frac{4}{3}\left(\frac{1}{2}\right)^3 + \frac{1}{2}\right) - \left(\frac{1}{\pi}\sin(0\pi) - \frac{4}{3}(0)^3 + 0\right)\right]$$

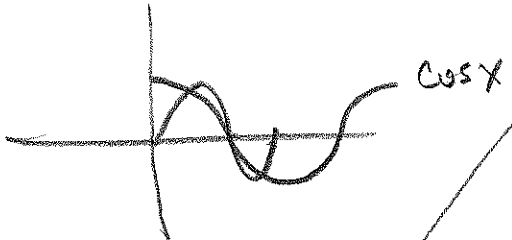
$$= 2\left[\frac{1}{\pi}(1) - \frac{4}{3}\cdot\frac{1}{8} + \frac{1}{2}\right] = 2\left[\frac{1}{\pi} - \frac{1}{6} + \frac{1}{2}\right]$$

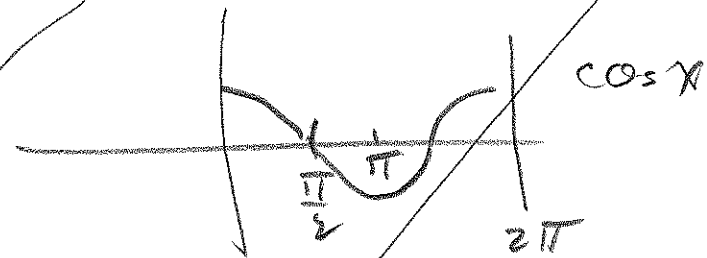
$$= \frac{2}{\pi} - \frac{1}{3} + 1 = \frac{2}{\pi}\cdot\frac{3}{3} - \frac{1}{3}\cdot\frac{\pi}{\pi} + \frac{1}{1}\cdot\frac{3\pi}{3\pi}$$

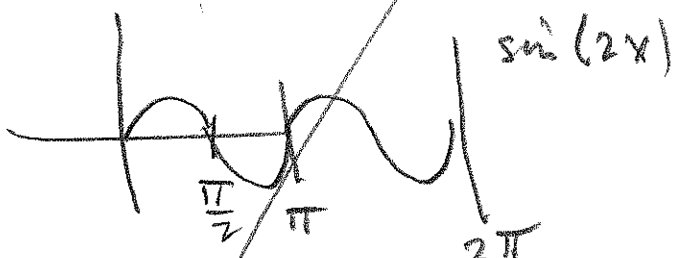
$$= \frac{6 - \pi + 3\pi}{3\pi} = \boxed{\frac{2\pi + 6}{3\pi}} \approx 1.303286439$$

201 § 5.1 #s 23, 31, 35, 37

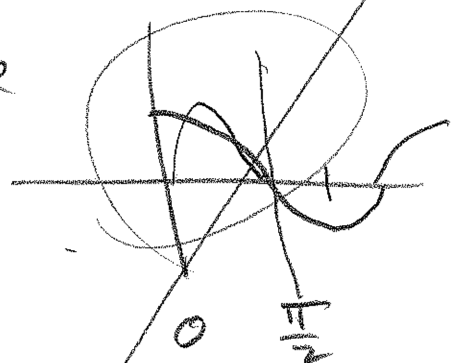
(23) $y = \cos x$, $y = \sin(2x)$, $x = 0$, $x = \frac{\pi}{2}$

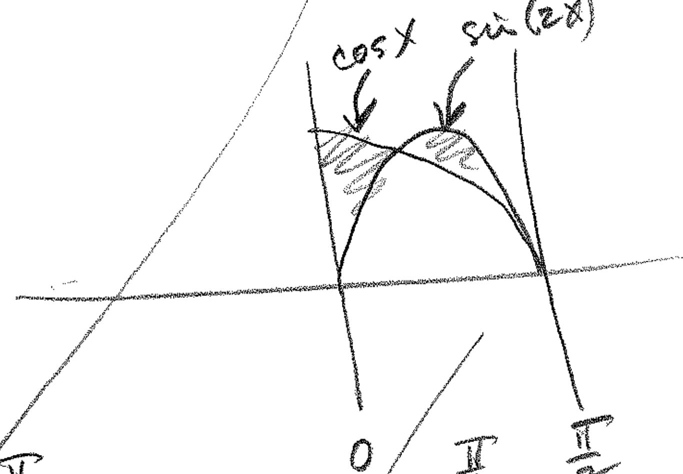






combine





Need to solve

$\sin(2x) = \cos x$

$\sin(2x) - \cos x = 0$

$2\sin x \cos x - \cos x = 0$

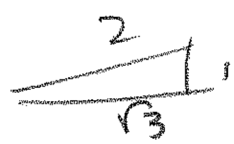
$\cos x\,(2\sin x - 1) = 0$

$\cos x = 0 \qquad \sin x = \frac{1}{2}$

$x = \frac{\pi}{2} \qquad x = \frac{\pi}{6}$

$$\int_0^{\frac{\pi}{6}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x - \cos x)\,dx$$

$$= \Big[\sin x + \cos x\Big]_0^{\frac{\pi}{6}} + \Big[-\cos x - \sin x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$



$\boxed{=1}$

$$= \left[\sin\frac{\pi}{6} + \cos\frac{\pi}{6} - (\sin 0 + \cos(0))\right]$$

$$+ \left[-\cos\frac{\pi}{2} - \sin\left(\frac{\pi}{2}\right) - \left(-\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\right)\right]$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} - (0+1) + \left[-0 - 1 - \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\right] = \frac{\sqrt{3}+1}{2} - 1 - 1 + \frac{\sqrt{3}+1}{2} = \sqrt{3} + 1 - 2 = \sqrt{3}$$

$= \sqrt{3} - 1$

201 $\int$ 5.1 #s 23, 31, 35, 37

#23's not working out right.

$$\int_0^{\frac{\pi}{6}} (\cos x - \sin(2x))\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin(2x) - \cos x)\,dx$$

$$= \left[\sin x + \tfrac{1}{2}\cos(2x)\right]_0^{\frac{\pi}{6}} + \left[-\tfrac{1}{2}\cos(2x) - \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

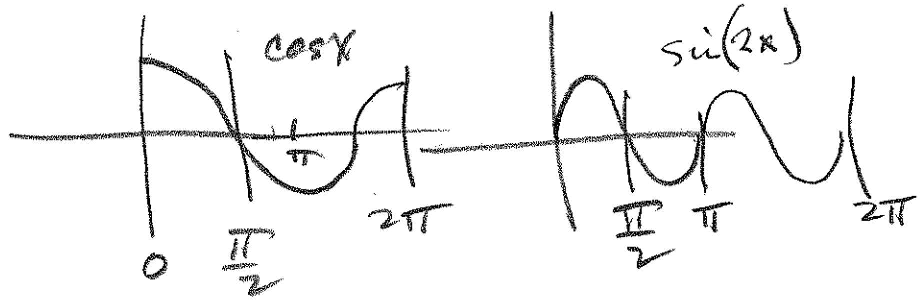
$$= \left[\sin\tfrac{\pi}{6} + \tfrac{1}{2}\cos\left(\tfrac{\pi}{3}\right) - \left(\sin(0) + \tfrac{1}{2}\cos(0)\right)\right]$$
$$+ \left[-\tfrac{1}{2}\cos(\pi) - \sin\tfrac{\pi}{2} - \left(-\tfrac{1}{2}\cos\tfrac{\pi}{3} - \sin\left(\tfrac{\pi}{6}\right)\right)\right]$$

$$= \tfrac{1}{2} + \tfrac{1}{2}\left(\tfrac{1}{2}\right) - \left(0 + \tfrac{1}{2}\right)$$
$$+ -\tfrac{1}{2}(-1) - 1 - \left(-\tfrac{1}{2}\left(\tfrac{1}{2}\right) - \tfrac{1}{2}\right)$$
$$= \tfrac{1}{2} + \tfrac{1}{4} - \tfrac{1}{2} + \tfrac{1}{2} - 1 + \tfrac{1}{4} + \tfrac{1}{2} = \boxed{\tfrac{1}{2}}$$ Yep.

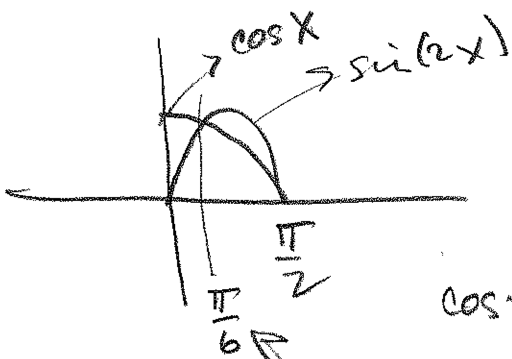
=

201 §5.1 #s 23, 31, 35, 37

23 $y = \cos x,\ y = \sin(2x),\ x = 0,\ x = \frac{\pi}{2}$



Combine:



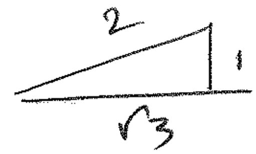
$$\cos x = \sin(2x)$$
$$\sin(2x) - \cos x = 0$$
$$2\sin x \cos x - \cos x = 0$$
$$\cos x\,(2\sin x - 1) = 0$$

$\cos x = 0$ $\quad$ $2\sin x - 1 = 0$

$x = \frac{\pi}{2}$ $\quad$ $2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$



$$\text{Area} = \int_0^{\frac{\pi}{2}} \left|\cos x - \sin(2x)\right| dx$$

$$= \int_0^{\frac{\pi}{6}} (\cos x - \sin(2x))\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin(2x) - \cos x)\,dx$$

$$= \left[\sin x + \tfrac{1}{2}\cos(2x)\right]_0^{\frac{\pi}{6}} + \left[-\tfrac{1}{2}\cos(2x) - \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left[\sin\tfrac{\pi}{6} - \tfrac{1}{2}\cos\left(\tfrac{\pi}{3}\right) - \left(\sin 0 + \tfrac{1}{2}\cos(0)\right)\right]$$

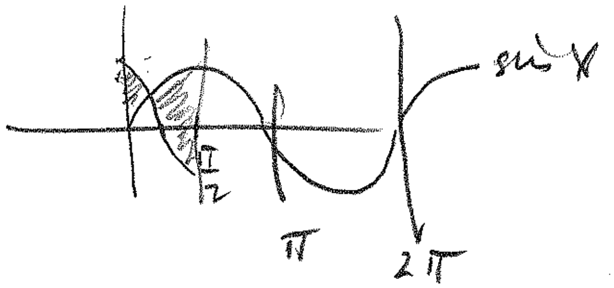
$$+ \left[-\tfrac{1}{2}\cos\pi - \sin\tfrac{\pi}{2} - \left(-\tfrac{1}{2}\cos\tfrac{\pi}{3} - \sin\tfrac{\pi}{6}\right)\right] = \left[\tfrac{1}{2} - \tfrac{1}{2}\cdot\tfrac{1}{2} - 0 - \tfrac{1}{2}\right]$$

$$+ \left[-\tfrac{1}{2}\cdot 0 - 1 - \left(-\tfrac{1}{2}\cdot\tfrac{1}{2} - \tfrac{1}{2}\right)\right] = \tfrac{1}{2} - \tfrac{1}{4} - \tfrac{1}{2} - 1 + \tfrac{1}{4} + \tfrac{1}{2}$$

201 §5.1 #s 31, 35, 37

(31) Evaluate the integral & interpret as the area of a region. Sketch the region

$$\int_0^{\frac{\pi}{2}} |\sin x - \cos(2x)|\, dx$$ Similar to the last one!



It is the area between sin x & cos(2x) over $[0, \frac{\pi}{2}]$

$\sin x = \cos(2x)$

$\sin x - \cos(2x) = 0$

$\sin x - (1 - 2\sin^2 x) = 0$

$2\sin^2 x + \sin x - 1 = 0$ THE missed sign.

$2u^2 + u - 1 = 0$

$$\int_0^{\frac{\pi}{6}} (\cos(2x) - \sin x)\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x - \cos(2x))\,dx$$

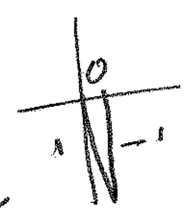
.52359878

$(2u - 1)(u + 1)$ → Here's where I got signs wrong

$u = +\frac{1}{2}$ $u = -1$

$\sin x = -\frac{1}{2}$ $\sin x = -1$

? $x = \frac{3\pi}{2} \notin D$



~~Sign is wrong~~ fixed in class

I expect $x = \frac{\pi}{6}$, not $\frac{\pi}{2}$.

I get $\frac{\pi}{2}$ & $-\frac{\pi}{6}$ or $\frac{7\pi}{6}$. No.

'201 S5.1 #s 35,37

#s 33-36. Use graph to find APPROXIMATE x-coords of points of intersections then find the approximate area of region bdd by the curves

35. $y = 3x^2 - 2x$, $y = x^3 - 3x + 4$

I think there's an algebra trick, available.

Let me see...

$x^3 - 3x + 4$ doesn't have a "clean" intercept

Let's look @ $y_1 - y_2 = x^3 - 3x + 4 - (3x^2 - 2x)$

$= x^3 - 3x^2 - x + 4$

$\pm 1, \pm 2, \pm 4$ are rational guesses

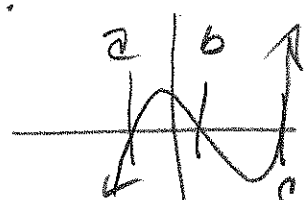
Grapher suggests $x = -1$ is a good guess

$$\begin{array}{r|rrrr} -1 & 1 & -3 & -1 & 4 \\ & & -1 & 4 & -3 \\ \hline & 1 & -4 & 3 & \end{array}$$

Nope

ok. Resorting to technology:

Find x-ints of $|y_1 - y_2|$:



$$\text{Area} = \int_a^b |x^3 - 3x^2 - x + 4|\,dx - \int_b^c |x^3 - 3x^2 - x + 4|\,dx$$

$a \approx -1.114908$

$b \approx 1.2541017$

$c \approx 2.8608059$

I'm going to integrate

$\int_a^c |x^3 - 3x^2 - x + 4|\,dx$ and

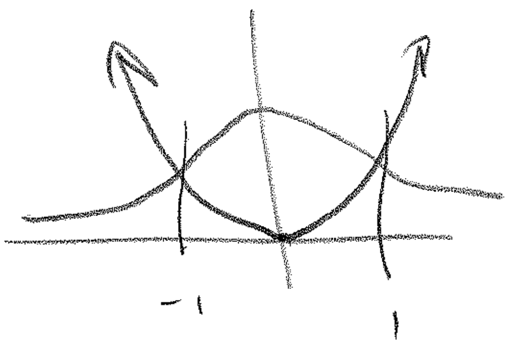
let Calculator tell me:

Area ≈ 8.3781589

201 §5.1 #57

(37) Graph the region between the curves.
use calculator to compute area accurate to 5 places

$$y = \frac{2}{x^4+1}, \quad y = x^2$$



$$\frac{2}{x^4+1} = x^2$$

$$x^2(x^4+1) = x^6 + x^2 = 2$$

$$x^6 + x^2 - 2 = 0$$

$$u^3 + u - 2 = 0$$

$$\begin{array}{r|rrrrrrr}
+1 & 1 & 0 & 0 & 0 & 1 & 0 & -2 \\
 & & 1 & 1 & 1 & 1 & 2 & 2 \\
\hline
-1 & 1 & 1 & 1 & 1 & 2 & 2 & 0 \\
 & & -1 & 0 & -1 & 0 & -2 & \\
\hline
 & 1 & 0 & 1 & 0 & 2 & 0 &
\end{array}$$

$$(x-1)(x+1)(x^4+x^2+2)$$

$$u^2 + u + 2 = 0$$

$$u^2 + u = -2$$

$$u^2 + u + \left(\tfrac{1}{2}\right)^2 = -2 + \tfrac{1}{4}$$

No real roots

Symmetry:

$$\int_{-1}^{1} = 2\int_{0}^{1}$$

$$\text{Area} = 2\int_0^1 \left(\frac{1}{x^4+1} - x^2\right) dx$$

$$\approx 1.0672793$$

$$\approx \boxed{1.06728}$$