201 §3.2 #s 1–5, 9–11, 15, 17, 19, 25, 27

#s 1–4 Rolle's & Verify Hypotheses & find c(s).

1. $f(x) = 3x^2 - 12x + 5$ on $[1,3]$

f is polynomial $\Rightarrow$ cnt & difbl $\forall x \in \mathbb{R}$ ✓

$f(1) = 3 - 12 + 5 = -4$ ✓

$f(3) = 27 - 36 + 5 = -4$ ✓

$f'(x) = 6x - 12 \overset{\text{SET}}{=} 0 \Rightarrow$

$x = \boxed{2 = c}$

3. $f(x) = \sqrt{x} - \frac{1}{3}x$ on $[0,9]$

f is $\sqrt{x} - \frac{1}{3}x$ is cnts on $[0,\infty)$ ✓

and difbl on $(0,\infty)$ ✓

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{3} = \frac{1}{2\sqrt{x}} - \frac{1}{3} \overset{\text{SET}}{=} 0 \Rightarrow$

$\frac{3 - 2\sqrt{x}}{6\sqrt{x}} = 0 \Rightarrow 2\sqrt{x} = 3 \Rightarrow$

$\sqrt{x} = \frac{3}{2} \Rightarrow$

$\boxed{x = \frac{9}{4} = c}$

201 §3.2 #'s 5, 9–11, 15

(5) $f(x) = 1 - x^{2/3}$. Then $f(1) = f(-1) = 0$, BUT

$$f'(x) = -\frac{2}{3}x^{-\frac{5}{3}} = -\frac{2}{3x^{5/3}} \neq 0 \ \forall x.$$

This doesn't contradict Rolle's, b/c $f(x)$ isn't difbl @ $x = 0 \in (-1, 1)$

#s 9–12 Verify MVT Hypo. Find c(s).

(9) $f(x) = 2x^2 - 3x + 1$; $[0, 2]$

f is polynomial $\longrightarrow$ cnt's & difbl $\forall x \in \mathbb{R}$.

$$f'(x) = 4x - 3 \overset{SET}{=} \frac{f(2) - f(0)}{2 - 0}$$

$$4x - 3 = \frac{8 - 3(2) + 1 - [1]}{2}$$

$$4x - 3 = \frac{2}{2} = 1$$

$$4x = 4$$

$$\boxed{x = 1 = c}$$

201 § 3.2 #s 11, 15, 17, 19, 25, 27 ...

11 $f(x) = \sqrt[3]{x}$, $[0, 1]$

$= x^{\frac{1}{3}}$

cnts on $\mathbb{R}$ ✓

f co

difbl on $(-\infty, 0) \cup (0, \infty)$ ✓

$f'(x) = \frac{1}{3} x^{-\frac{2}{3}} \underset{=}{\text{SET}} \frac{f(1) - f(0)}{1 - 0}$

$\frac{1}{3x^{2/3}} = \frac{1-0}{1} = 1$

$1 = 3x^{2/3}$

$x^{2/3} = \frac{1}{3}$

$x = \left(\frac{1}{3}\right)^{3/2} = \frac{1}{3} \cdot \left(\frac{1}{3}\right)^{\frac{1}{2}} = \frac{1}{3} \cdot \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{9} = c}$

15 $f(x) = (x-3)^{-2}$. Show $\nexists$ $c \in (1, 4) \ni$

$f(4) - f(1) = f'(c)(4-1)$ (i.e, $\frac{f(4) - f(1)}{4-1} = f'(c)$)

$f'(x) = -2(x-3)^{-3} = -\frac{2}{(x-3)^3}$

$f(4) - f(1) = (4-3)^{-2} - (1-3)^{-2} = 1^{-2} - (-2)^{-2}$

$= 1 - \frac{1}{4} = \frac{3}{4} \underset{=}{\text{SET}} -\frac{2}{(x-3)^3}(4-1) = -\frac{2 \cdot 3}{(x-3)^3}$

$\frac{3}{4} = -\frac{6}{(x-3)^3}$

$(x-3)^3 = -8$

$x - 3 = \sqrt[3]{-8} = -2$

$x = 3 - 2 = \boxed{1 = x = c}$

But $c \notin (1, 4)$

No contradiction, b/c

f(x) not cnts @ $x = 3 \in [1, 4]$

201 § 3.2 #s 17, 19, 25, 27

(17) Show that $2x + \cos x = 0$ has exactly one root.

$2(10) + \cos(10) > 0$

$2(-10) + \cos(10) < 0$ $\Longrightarrow$ At <u>least</u> one. (IVT)

$f'(x) = 2 - \sin x > 0 \ \forall x \Longrightarrow$ At <u>Most</u> one

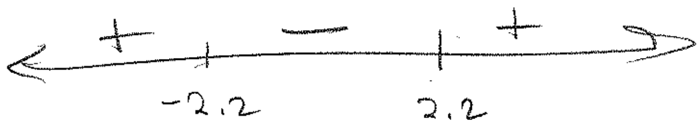
$f(x) = 2x + \cos x$ is strictly increasing $\forall x$.

(19) $x^3 - 15x + c = 0$ has at MOST one root.
in $[-2, 2]$

$f'(x) = 3x^2 - 15 \overset{\text{SET}}{=} 0$

$x^2 = 5$

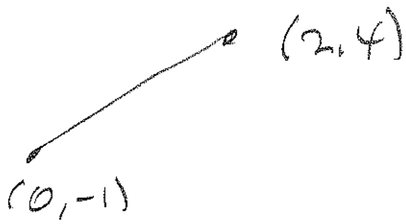
$x = \pm\sqrt{5} \approx \pm 2.236067977$



$f(x) = x^3 - 15x + c$ is <u>strictly</u> <u>decreasing</u> $\forall x \in [-2, 2]$

Can't cross x-axis twice between $\pm\sqrt{5}$.

(25) Does $\exists f \ni f(0) = -1$, $f(2) = 4$ $\&$
$f'(x) \leq 2 \ \forall x$?



$\dfrac{4-(-1)}{2-0} = \dfrac{5}{2} = 2.5$

No way f can have avg slope of $m = 2.5$ if its slope is everywhere less than 2!

Violates MVT.

201 §3.2 #27

27) Show that $\sqrt{x+1} < 1 + \frac{1}{2}x$ if $x > 0$

@ $x = 0$, we have $\sqrt{1} = 1$ on LHS
and 1 on RHS.

We show $f(x) = \sqrt{x+1} - \left(1 + \frac{1}{2}x\right) < 0$ if $x > 0$.

Note $f(0) = 0$.

$$f'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}} - \frac{1}{2}$$

$$= \frac{1}{2\sqrt{x+1}} + \frac{1}{2} < 0 \text{ if } x > 0$$

So it's decreasing, strictly.

Basically, $\sqrt{x+1}$ grows slower than $\frac{1}{2}x + 1$

CLOSE to $x=0$, it's a little harder to ~~be~~ sure. They're equal at $x=0$, but as soon as you move to the right, $\sqrt{x+1}$ is ducking underneath.