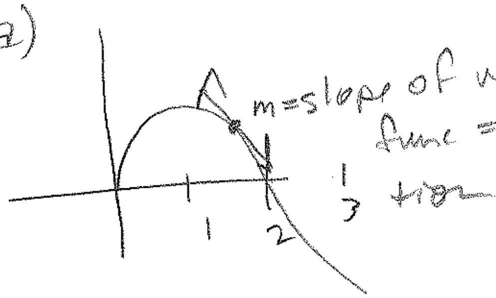
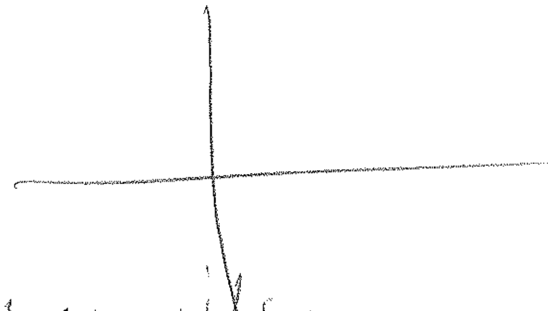
201 § 2.7 #'s 5, 6, 8, 11, 15

(5) Graphs of 2 velocity funcs.
when is each slowing / speedin

(a)



(b)



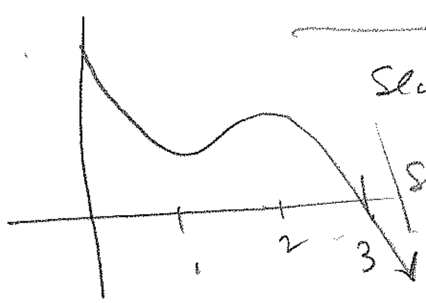
Slowing down: Acceleration, is negative

Acceleration $= a = \frac{dv}{dt}$

on $\boxed{(1, 3) \text{ is slowing}}$

speeding up: (0, 1) tangent line has positive slope.

(b)

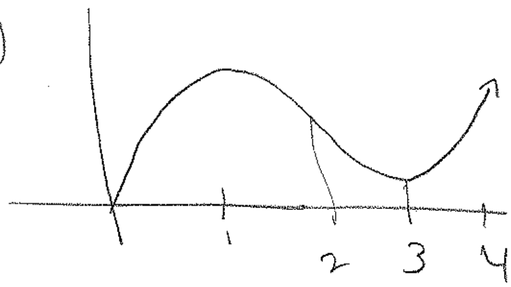


$\boxed{\begin{array}{l} \text{Slowing: } (0,1) \cup (2,3) \\ \text{Speeding-up: } (1,2) \end{array}}$

201 §2.7 #s 6, 8, 11, 15

(6) Graphs o POSITION funcs

(a)

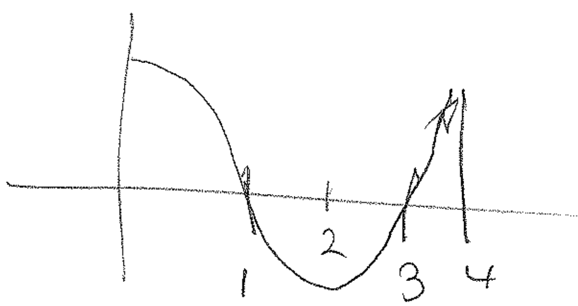


1st Look @ concavity

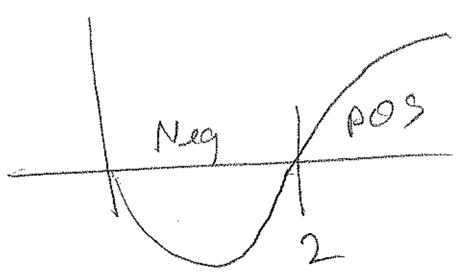
Speeding-up : (2,4) concave up

slowing-down : (0,2) concave down.

Look @ velocity:



Look @ accelerations:



slowing accelerating

201 § 2.7 #s 8, 11, 15

(8) $v_0 = +80 \text{ ft/s}$

$s_0 = 0$

$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

$s(t) = \text{height}$

$= -\frac{32}{2}t^2 + 80t + 0$

$v(t) = s'(t) = \text{velocity}$

$= -16t^2 + 80t$

$a(t) = s''(t) = v'(t)$

$= \text{acceleration}$

(a) Max height: $s'(t) = 0$

$-32t + 80 = 0$

$t = \frac{80}{32} = \frac{5}{2} \text{ s}$

$s(\frac{5}{2})$:

$$\begin{array}{c|ccc} \frac{5}{2} & -16 & 80 & 0 \\ & & -40 & 100 \\ \hline & -16 & 40 & \boxed{100} \end{array}$$

$\boxed{100 = s(\frac{5}{2}) \quad 100 \text{ ft is max ht.}}$

(b) Find velocity when $s(t) = 96$ on its way up/down.

$s(t) = 96$

$-16t^2 + 80t = 96$

$-16t^2 + 80t - 96 = 0$

$t^2 - 5t + 6 = 0$

$t = 2, 3$

$s'(2) = v(2) = -32(2) + 80$

$= -64 + 80$

$= \boxed{16 \text{ ft/s on way up.}}$

$s'(3) = -32(3) + 80$

$= \boxed{-16 \text{ ft/s on way down}}$

§ 2.7 #s 11, 15

(11) Computer chips in square wafers. Wants to keep side length close to 15 mm and wants to know how the area $A(x)$ changes when side length x changes

(a) Find $A'(15)$ & explain its meaning.

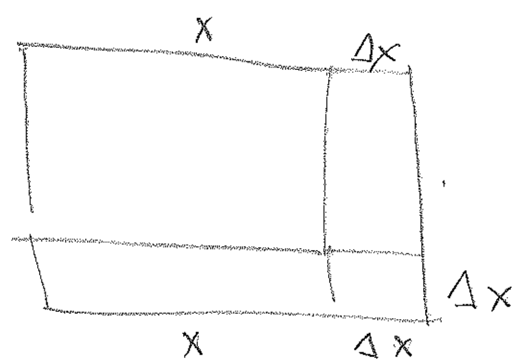
$A(x) = x^2$ $A(x)$ in cm^2, x in cm

$A'(x) = 2x \Longrightarrow A'(15) = 30 \ cm^2/cm$

This means rate of Area increase wrt length of a side is $30 \ cm^2/cm$.

(b) The rate of change of area wrt length of a side is $\frac{1}{2}$ its perimeter, $4x$, since

$A'(x) = 2x = \frac{1}{2}[4x] = \frac{1}{2}[\text{perimeter}]$



Geometrically, you can see the area increasing by an amount

$x\Delta x + x\Delta x + (\Delta x)^2$
$= 2x\Delta x + (\Delta x)^2$

If $\Delta x = 1$, then we get $2x+1$, close to $2x$

If Δx = small, we get $2x\Delta x + (\Delta x)^2$, which, neglecting $(\Delta x)^2$ as small is a lot like what we'll do in § 2.9!

201 §2.7 #15

(15) (a) Spherical balloon $V = \frac{4}{3}\pi r^3$, where radius ~~of~~ r is in μm. Find AVG RATE OF CHANGE in V when r changes from

(i) 5 μm to 8 μm

(ii) 5 to 6 μm

(iii) 5 to 5.1 μm

(i) $\dfrac{V(8)-V(5)}{8-5} = \dfrac{\frac{4}{3}\pi\left[8^3-5^3\right]}{3} = \dfrac{\frac{4}{3}\pi\left[(8-5)(8^2+40+25)\right]}{3}$

$= \frac{4}{9}\pi\left[3(64+65)\right] = \left(\frac{4}{9}\right)(3)\pi\left[129\right] = \frac{4}{3}\pi\left[129\right]$

$= 4\pi\left[43\right] = 172\pi \ \mu m^3/\mu m \approx 540.354 \ \mu m^3/\mu m$

(ii) $\dfrac{V(6)-V(5)}{6-5} = \dfrac{\frac{4}{3}\pi\left[6^3-5^3\right]}{6-5} = \frac{4}{3}\pi\left[(6-5)(6^2+30+5^2)\right]$

$= \frac{4}{3}\pi\left[36+30+25\right] = \frac{4}{3}\pi\left[66+25\right] = \frac{4}{3}\pi\left[91\right] = \dfrac{364\pi}{3} \ \mu m^3/\mu m$

$\approx 381.1799 \ \mu m^3/\mu m$

(iii) $\dfrac{V(5.1)-V(5)}{.1} = \dfrac{\frac{4}{3}\pi\left[5.1^3-5^3\right]}{.1} = 10\left(\frac{4}{3}\pi\right)(7.651)$

$= \dfrac{306.04\pi}{3} \ \mu m^3/\mu m \approx 320.484 \ \mu m^3/\mu m$

201 §2.7 #15

(15) A spherical balloon's ~~volume is $V = \frac{4}{3}\pi r^3$ ft^3~~,
~~where r is radius in feet~~ #16 stuff.

Surface area is $4\pi r^2 = S$ in ft^2

Find rate of increase in S.A. when r =

(a) 1 ft : $\left.\frac{dS}{dr}\right|_{r=1} = \left. 8\pi r \right|_{r=1} = 8\pi$

(b) 2 ft : $\left.\frac{dS}{dr}\right|_{r=2} = 8\pi(2) = 16\pi$

(c) 3 ft : $\left.\frac{dS}{dr}\right|_{r=3} = 8\pi(3) = 24\pi$

Rate of increase is, apparently a <u>linear</u> function! $(1, 8\pi)$, $(2, 16\pi)$, $(3, 24\pi)$

$m = 8\pi$!