

201 §2.6 #5 1-27, 45, 47

#5 1-4 (a) Find y' by implicit differentiation
(b) Solve for y explicitly & find y'
(c) Check your solution (compare.)

① (a) $9x^2 - y^2 = 1$

$$18x - 2yy' = 0$$

$$-2yy' = -18x$$

$$y' = \frac{9x}{y}$$

(b) $-y^2 = 1 - 9x^2$

$$y^2 = 9x^2 - 1$$

$$y = \pm \sqrt{9x^2 - 1} = \pm (9x^2 - 1)^{\frac{1}{2}}$$

$$\Rightarrow y' = \pm \frac{1}{2} (9x^2 - 1)^{-\frac{1}{2}} (18x) = \pm \frac{9x}{\sqrt{9x^2 - 1}}$$

$$\boxed{\text{(c)} = \pm \frac{9x}{y}}$$

(c) see above \rightarrow

③ (a) $\frac{1}{x} + \frac{1}{y} = 1$
 $x^{-1} + y^{-1} = 1$
 $-x^{-2} - y^{-2}y' = 0$
 $-y^{-2}y' = -x^{-2}$

$$\boxed{y' = \frac{y^2}{x^2}}$$

(b) $y + x = xy$
 $y - xy = -x$
 $y(1-x) = -x$
 $y = \frac{-x}{1-x} \rightarrow$

$$\boxed{y' = \frac{1(1-x) - x(-1)}{(1-x)^2} = \frac{1}{(1-x)^2} = \frac{x^2}{(1-x)^2} \cdot \frac{1}{x^2}}$$

(c) $-\frac{y^2}{x^2}$

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#55-20 Find (y') by implicit differentiation

$$\textcircled{5} \quad x^3 + y^3 = 1 \quad \rightarrow \quad \frac{dy}{dx}$$

$$3x^2 + 3y^2 y' = 0$$

$$3y^2 y' = -3x^2$$

$$y' = -\frac{x^2}{y^2}$$

$$\textcircled{7} \quad x^2 + xy - y^2 = 4$$

$$2x + y + xy' - 2yy' = 0$$

$$y'(x - 2y) = -2x - y$$

$$y' = \frac{-2x - y}{x - 2y}$$

$$\textcircled{9} \quad x^4(x+y) = y^2(3x-y)$$

$$4x^3(x+y) + x^4(1+y') = 2yy'(3x-y) + y^2(3-y')$$

$$4x^4 + 4x^3y + x^4 + x^4y' = 6xyy' - 2y^2y' + 3y^2 - y^2y'$$

$$x^4y' - 6xyy' + y^2y' = 6xyy' - 2y^2y' + 3y^2 - y^2y'$$

$$y'(x^4 - 6xy + y^2) = 3y^2 - 3x^4 - 4x^3y$$

$$y' = \frac{3y^2 - 3x^4 - 4x^3y}{x^4 - 6xy + y^2}$$

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$$(11) y \cos x = x^2 + y^2$$

$$y' \cos x - y \sin x = 2x + 2yy'$$

$$y' \cos x - 2yy' = 2x + y \sin x$$

$$y' = \frac{2x + y \sin x}{\cos x - 2y}$$

$$(13) 4 \cos x \sin y = 1$$

$$(-4 \sin x)(\sin y) + (4 \cos x)(\cos y) y' = 0$$

$$y' = \frac{-4 \sin x \sin y}{4 \cos x \cos y} = \boxed{-\tan x \tan y} \text{ to be checked.}$$

$$(15) \tan\left(\frac{x}{y}\right) = x + y$$

$$\left(\sec^2\left(\frac{x}{y}\right)\right) \left(\frac{y - xy'}{y^2}\right) = 1 + y'$$

$$\sec^2\left(\frac{x}{y}\right) (y - xy') = y^2 + y^2 y'$$

$$y \sec^2\left(\frac{x}{y}\right) - xy' \sec^2\left(\frac{x}{y}\right) = y^2 + y^2 y'$$

$$-xy' \sec^2\left(\frac{x}{y}\right) = y^2 - y \sec^2\left(\frac{x}{y}\right)$$

$$y' = \frac{y \sec^2\left(\frac{x}{y}\right) - y^2}{x \sec^2\left(\frac{x}{y}\right)}$$

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$$(17) \sqrt{xy} = 1 + x^2 y$$

$$(xy)^{\frac{1}{2}} = 1 + x^2 y$$

$$\frac{1}{2}(xy)^{-\frac{1}{2}}(y + xy') = 2xy + x^2 y'$$

$$\frac{1}{2}(xy)^{-\frac{1}{2}} y + \frac{1}{2}(xy)^{-\frac{1}{2}} xy' = 2xy + x^2 y'$$

$$\frac{1}{2}(xy)^{-\frac{1}{2}} xy' - x^2 y' = 2xy - \frac{1}{2}y(xy)^{-\frac{1}{2}}$$

$$y' = \frac{2xy - \frac{1}{2}y(xy)^{-\frac{1}{2}}}{\frac{1}{2}(xy)^{-\frac{1}{2}}x - x^2}$$

which could be given a workover, but to what purpose?

$$(19) y \cos x = 1 + \sin(xy)$$

$$y' \cos x - y \sin x = (\cos(xy))(y + xy')$$

$$y' \cos x - y \sin x = y \cos(xy) + xy' \cos(xy)$$

$$y' \cos x - xy' \cos(xy) = y \cos(xy) + y \sin x$$

$$y' = \frac{y \cos(xy) + y \sin x}{\cos x - x \cos(xy)}$$

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(21) If $f(x) + x^2(f(x))^3 = 10$ and $f(1) = 2$,
what is $f'(1)$?

$$f'(x) + 2x(f(x))^3 + (x^2)(3(f(x))^2)(f'(x)) = 0$$

$$\Rightarrow f'(x) + 3x^2 f'(x)(f(x))^2 = -2x(f(x))^3$$

$$\Rightarrow f'(x) = \frac{-2x(f(x))^3}{1 + 3x^2(f(x))^2}$$

$$\Rightarrow f'(1) = \frac{-2(1)(2)^3}{1 + 3(1)^2(2)^2} = \frac{-16}{1+12} = \boxed{-\frac{16}{13}}$$

(23) Treat y as independent variable and
 x as dependent. Then find dx/dy with
implicit differentiation. Phil

$$x^4 y^2 - x^3 y + 2x y^3 = 0$$

$$4x^3 x' y^2 + x^4 \cdot 2y - 3x^2 x' y - x^3 + 2x' y^3$$
$$+ 2x \cdot 3y^2 = 0$$

$$4x^3 y^2 x' - 3x^2 y x' + 2x' y^3 = -2x^4 y + x^3 - 6xy^2$$

$$x'(4x^3 y^2 - 3x^2 y + 2y^3) = -2x^4 y + x^3 - 6xy^2$$

$$x' = \frac{-2x^4 y + x^3 - 6xy^2}{4x^3 y^2 - 3x^2 y + 2y^3}$$

201 S' 206 #s 25, 27, 45, 47

#s 25-32 use implicit differentiation to find an eq'n of the tangent line to the curve at the given point.

(25) $y \sin(2x) = x \cos(2y)$ @ $(\frac{\pi}{2}, \frac{\pi}{4})$

$$y' \sin(2x) + y (\cos(2x))(2) = \cos(2y) - x (\sin(2y))(2y')$$

$$y' \sin(2x) + 2xy' \sin(2y) = \cos(2y) - 2y \cos(2x)$$

$$y' = \frac{\cos(2y) - 2y \cos(2x)}{\sin(2x) + 2x \sin(2y)} \rightarrow$$

$$y' \Big|_{(\frac{\pi}{2}, \frac{\pi}{4})} = \frac{\cos(2 \cdot \frac{\pi}{4}) - 2 \cdot \frac{\pi}{4} \cos(2 \cdot \frac{\pi}{2})}{\sin(2 \cdot \frac{\pi}{2}) + 2 \cdot \frac{\pi}{2} \sin(2 \cdot \frac{\pi}{4})}$$

$$= \frac{\cos(\frac{\pi}{2}) - \frac{\pi}{2} \cos(\pi)}{\sin(\pi) + \pi \sin(\frac{\pi}{2})} = \frac{0 - \frac{\pi}{2} \cdot (-1)}{0 + \pi} = \frac{1}{2}$$

$$\boxed{y = \frac{1}{2}(x - \frac{\pi}{2}) + \frac{\pi}{4}} = \frac{1}{2}x - \frac{\pi}{4} + \frac{\pi}{4} = \frac{1}{2}x$$

$y = m(x - x_1) + y_1$ $y = \frac{1}{2}x$!?

(27) $x^2 + xy + y^2 = 3$ @ $(1, 1)$

$$2x + y + xy' + 2yy' = 0$$

$$xy' + 2yy' = -2x - y$$

$$y' = \frac{-2x - y}{x + 2y} \Rightarrow y' \Big|_{(1,1)} = \frac{-2-1}{1+2} = \frac{-3}{3}$$

$$\boxed{y = -1(x-1) + 1}$$

STOP!

$$\boxed{y = -x + 2}$$

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(45) Find eq'n of tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (a) \quad (x_0, y_0)$$

$$\frac{1}{a^2} x^2 - \frac{1}{b^2} y^2 = 1$$

$$\frac{2}{a^2} x - \frac{2}{b^2} y y' = 0$$

$$-\frac{2}{b^2} y y' = -\frac{2}{a^2} x$$

$$y' = \frac{-\frac{2}{a^2} x}{-\frac{2}{b^2} y} = \frac{b^2}{a^2} \cdot \frac{x}{y} \longrightarrow$$

$$y' \big|_{(x_0, y_0)} = \frac{b^2}{a^2} \cdot \frac{x_0}{y_0} = \frac{b^2 x_0'}{a^2 y_0} \implies$$

$$\boxed{y = \frac{b^2 x_0}{a^2 y_0} (x - x_0) + y_0} \quad \text{IS FINE BY ME}$$

$$y = \frac{b^2 x_0}{a^2 y_0} x - \frac{b^2 x_0^2}{a^2 y_0} + y_0$$

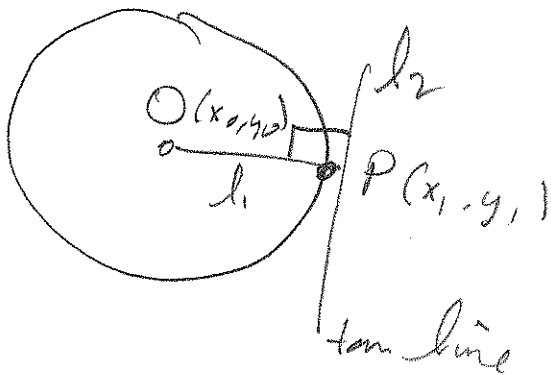
OR

$$y = \frac{b^2 x_0}{a^2 y_0} x + \frac{a^2 y_0^2 - b^2 x_0^2}{a^2 y_0}$$

Book Says $\frac{x_0 x}{a^2} - \frac{y_0 y}{b^2} = 1$ Are these the same?

201 § 2.6 # 47

(47) Show, using implicit differentiation, that any tangent to a point P is perpendicular to the radius OP



$$l_1 \perp l_2 \Rightarrow m_2 = \frac{y_1 - y_0}{x_1 - x_0}$$

We see that $m_2 = -\frac{1}{m_1}$, if we take the center to be $(x_0, y_0) = (0, 0)$.

$$x^2 + y^2 = r^2$$
$$2x + 2y y' = 0$$

$$y' = -\frac{x}{y} = -\frac{x_1}{y_1} = m_1$$

$$m_1 = -\frac{x_1}{y_1}$$

$$m_2 = \frac{y_1}{x_1}$$

So we could make this argument for a circle centered at the origin. Then argue that the geometry is the same, regardless of where the circle is moved to. Any body else have a good argument?