201 §2.4 ~~#s 1-31, 55, 70, 73~~ #s 3-25, 31-35, 39, 43, 52

#s 1-16 Differentiate

~~1~~ $f(x) = 3x^2 - 2\cos x \longrightarrow \boxed{f'(x) = 6x + 2\sin x}$

3 $f(x) = \sin x + \frac{1}{2}\cot x \longrightarrow \boxed{f'(x) = \cos x - \frac{1}{2}\csc^2 x}$

5 $y = \sec\theta\tan\theta \longrightarrow \boxed{y' = \sec\theta\tan\theta\tan\theta + \sec\theta\sec^2\theta}$

$= \sec\theta\tan^2\theta + \sec^3\theta$

7 $y = c\cos t + t^2\sin t \longrightarrow$

$\boxed{y' = -c\sin t + 2t\sin t + t^2\cos t}$

9 $y = \dfrac{x}{2-\tan x} \longrightarrow y' =$

$\boxed{y' = \dfrac{1(2-\tan x) - x(-\sec^2 x)}{(2-\tan x)^2}} = \dfrac{2-\tan x + x\sec^2 x}{(2-\tan x)^2}$

11 $f(\theta) = \dfrac{\sec\theta}{1+\sec\theta}$

$\longrightarrow \boxed{f'(\theta) = \dfrac{(\sec\theta\tan\theta)(1+\sec\theta) - \sec\theta(\sec\theta\tan\theta)}{(1+\sec\theta)^2}}$

$= \dfrac{\sec\theta\tan\theta + \sec^2\theta\tan\theta - \sec^2\theta\tan\theta}{(1+\sec\theta)^2}$

$= \dfrac{\sec\theta\tan\theta}{(1+\sec\theta)^2}$

201 §2.4 #5 ~~13-31~~ 55, 70, 73

(13) $y = \dfrac{t\sin t}{1+t} \Rightarrow \boxed{y' = \dfrac{(\sin t + t\cos t)(1+t) - (t\sin t)(1)}{(1+t)^2}}$

$= \dfrac{\sin t + t\sin t + t\cos t + t^2\cos t - t\sin t}{(1+t)^2}$

$= \dfrac{t^2\cos t + t\cos t + \sin t}{(t+1)^2}$

(15) $h(\theta) = \theta\csc\theta - \cot\theta \Rightarrow$

$\boxed{h'(\theta) = \csc\theta - \theta\csc\theta\cot\theta - \csc^2\theta}$

(17) $\dfrac{d}{dx}(\csc x) = -\csc x\cot x$

Pf $\dfrac{d}{dx}[\csc x] = \dfrac{d}{dx}\left[\dfrac{1}{\sin x}\right] = \dfrac{0\cdot\sin x - 1\cdot\cos x}{\sin^2 x}$

$= \dfrac{-\cos x}{\sin^2 x} = -\dfrac{1}{\sin x}\cdot\dfrac{\cos x}{\sin x} = \boxed{-\csc x\cot x}$ ∎

(19) $\dfrac{d}{dx}[\cot x] = -\csc^2 x$

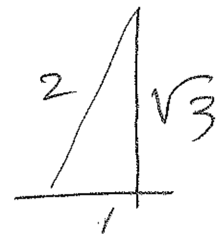
Pf $\dfrac{d}{dx}\left[\dfrac{\cos x}{\sin x}\right] = \dfrac{-\sin x\sin x - \cos x\cos x}{\sin^2 x}$

$= -\dfrac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\dfrac{1}{\sin^2 x} = -\csc^2 x$ ∎

201 §2.4 #s 21–31, 55, 70, 73

#s 21–24 Find tan. line @ given (x_1, y_1).

(21) $y = \sec x$, $\left(\frac{\pi}{3}, 2\right)$



$y' = \sec x \tan x \Rightarrow y'\left(\frac{\pi}{3}\right) = \sec\frac{\pi}{3}\tan\frac{\pi}{3} = 2\cdot\sqrt{3} = m$

$\longrightarrow \boxed{y = 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 2}$ does it

$= 2\sqrt{3}\,x - \frac{2\pi\sqrt{3}}{3} + 2$

$= 2\sqrt{3}\,x + \frac{6 - 2\pi\sqrt{3}}{3}$

(23) $y = \cos x - \sin x$, $(\pi, 1)$

$y' = -\sin x - \cos x \longrightarrow y'(\pi) = -\sin\pi - \cos\pi$

$= -0 - (-1)$

$= 1 = m$

$\boxed{y = 1(x - \pi) + 1}$

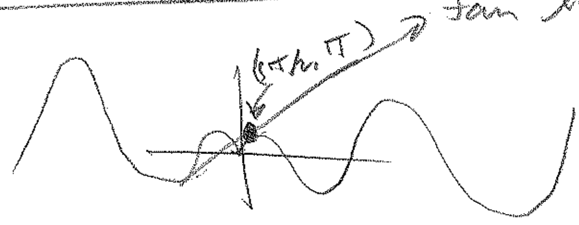
$= x - \pi + 1$

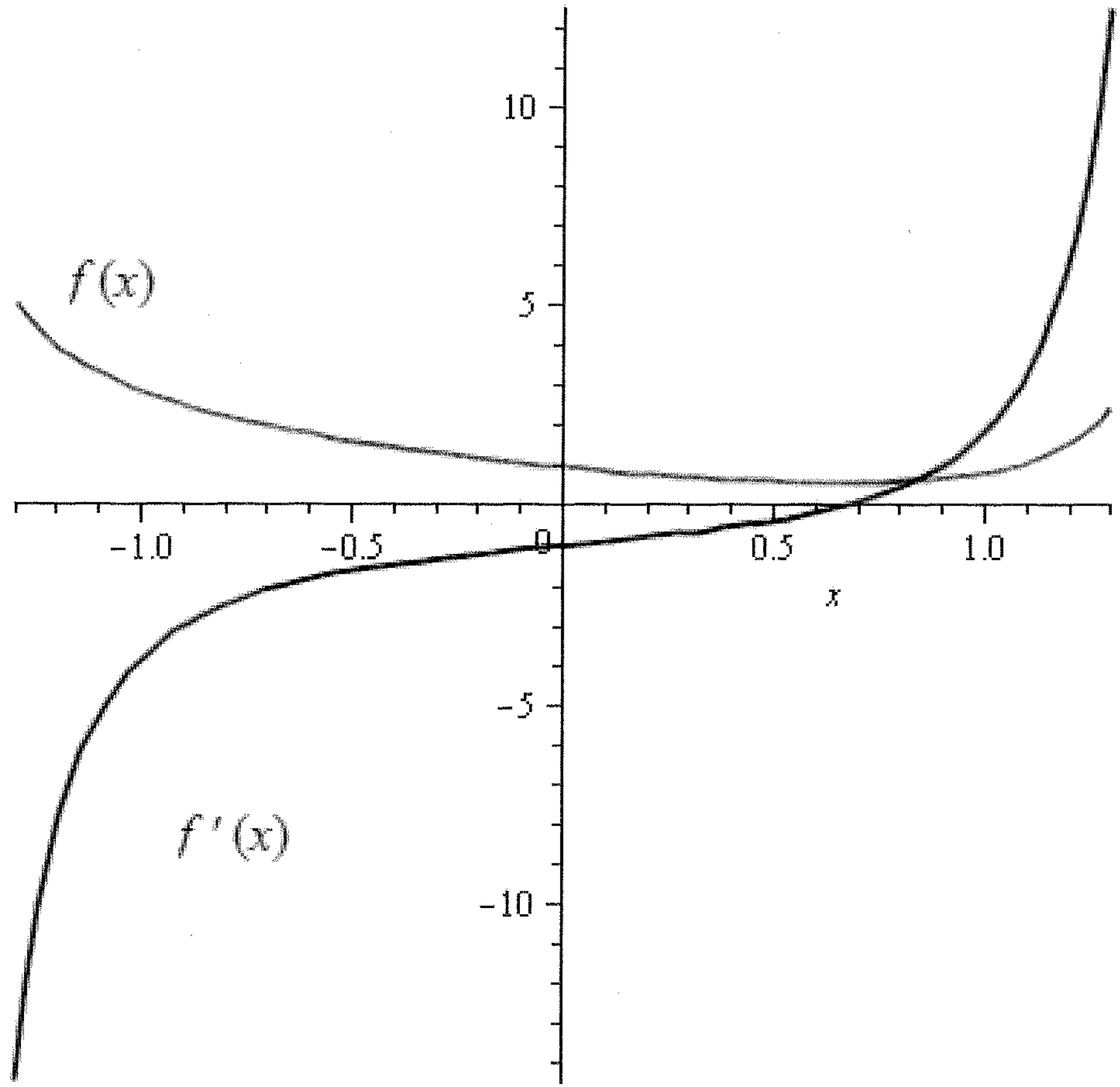
(25) (a) Find eq'n of tan line to $y = 2x\sin x$ @ $\left(\frac{\pi}{2}, \pi\right)$: $y' = 2\sin x + 2x\cos x$

$y'(\pi/2) = 2(1) + \frac{\pi}{2}(0) = 2$

$\boxed{y = 2\left(x - \frac{\pi}{2}\right) + \pi}$

(b)



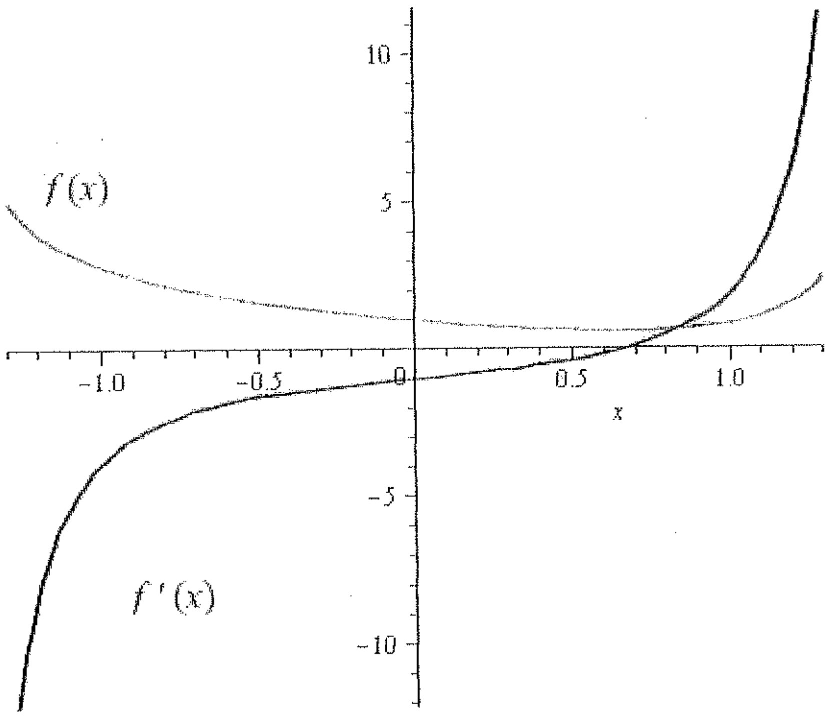


f(x)
f'(x)
10
5
−5
−10
−1.0
−0.5
0
0.5
1.0
x

201 §2.4 ~~#s 27-31, 35, 40, 43~~

27 (a) $f(x) = \sec x - x \Longrightarrow f'(x) = \sec x \tan x - 1$

(c) Check w/ graphs of f, f' on $\{x \mid |x| < \frac{\pi}{2}\}$



29 $H(\theta) = \theta \sin\theta \Longrightarrow$

$\boxed{H'(\theta) = \sin\theta + \theta\cos\theta} \longrightarrow$

$H''(\theta) = \cos\theta + \cos\theta - \theta\sin\theta$

$= \boxed{2\cos\theta - \theta\sin\theta}$

31 (a) Quotient Rule: $f(x) = \dfrac{\tan x - 1}{\sec x}$

$\longrightarrow f'(x) = \dfrac{(\sec^2 x)(\sec x) - (\tan x - 1)(\sec x \tan x)}{\sec^2 x}$

$= \dfrac{\sec^2 x - \tan^2 x + \tan x}{\sec x}$

201 §2.4 ~~#s 31, 55, 70, 73~~ #s 33-35, 39, 43, 52B

31 (b) $f(x) = \dfrac{\tan x - 1}{\sec x} = \dfrac{\frac{\sin x}{\cos x} - 1}{\frac{1}{\cos x}}$

$= \dfrac{\cos x}{1} \cdot \dfrac{\sin x - \cos x}{\cos x} = \sin x - \cos x$

$\rightarrow \boxed{f'(x) = \cos x + \sin x}$

(c) We show (a) & (b) are the same.

$\dfrac{\sec^2 x - \tan^2 x + \tan x}{\sec x} = \dfrac{1 + \tan x}{\sec x} = \dfrac{1}{\sec x} + \dfrac{\tan x}{\sec x}$

$= \cos x + \tan x \cos x = \cos x + \dfrac{\sin x}{\cos x} \cos x$

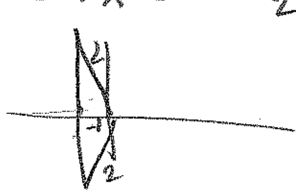
$= \cos x + \sin x$

33 For what values of x does $x + 2\sin x$ have a horizontal tangent?

$f'(x) = 1 + 2\cos x \overset{\text{SET}}{=} 0$

$\Rightarrow 2\cos x = -1$ ~~SOL'N: $\{x \mid x = ?$~~

$\Rightarrow \cos x = -\frac{1}{2}$



$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

so, $\boxed{\{x \mid x = \frac{2\pi}{3} + 2n\pi \text{ OR } x = \frac{4\pi}{3} + 2n\pi,\ n \in \mathbb{Z}\}}$

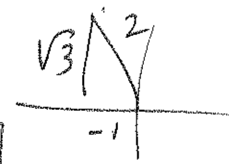
201 §2.4 # ~~5, 28~~, ~~33~~ 35, 39, 43, 52D

(35) Mass on a spring vibrates horizontally on smooth level surface. Its eq'n of motion is $x(t) = 8\sin t$, where t in secs & x in cm.

(a) Find Velocity $= \boxed{x'(t) = 8\cos t}$ and

Acceleration $= \boxed{x''(t) = -8\sin t}$

(b) Find x, x', x'' @ $t = \frac{2\pi}{3}$:



$$x\left(\frac{2\pi}{3}\right) = 8\sin\left(\frac{2\pi}{3}\right) = 8\cdot\frac{\sqrt{3}}{2} = \boxed{4\sqrt{3}\text{ cm}}$$

$$x'(2\pi/3) = 8\cos\left(\frac{2\pi}{3}\right) = (8)\left(-\frac{1}{2}\right) = \boxed{-4 \text{ cm/s}}$$

$$x''(2\pi/3) = -8\sin\frac{2\pi}{3} = (-8)\left(\frac{\sqrt{3}}{2}\right) = \boxed{-4\sqrt{3}\text{ cm/s}^2}$$

#s 39-48 Find the limit.

(39) $$\lim_{x\to 0}\frac{\sin(3x)}{x} = \lim_{x\to 0} 3\left(\frac{\sin(3x)}{3x}\right) = \boxed{3}$$

(43) $$\lim_{x\to 0}\frac{\sin(3x)}{5x^3-4x} = \lim_{x\to 0}\frac{\sin 3x}{x}\cdot\frac{1}{5x^2-4}$$

$$= \lim_{x\to 0}\frac{\sin(3x)}{3x}\cdot\frac{3}{5x^2-4} = 1\cdot\frac{3}{-4} = \boxed{-\frac{3}{4}}$$

201 §2.4 #52B

52 (a) $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$

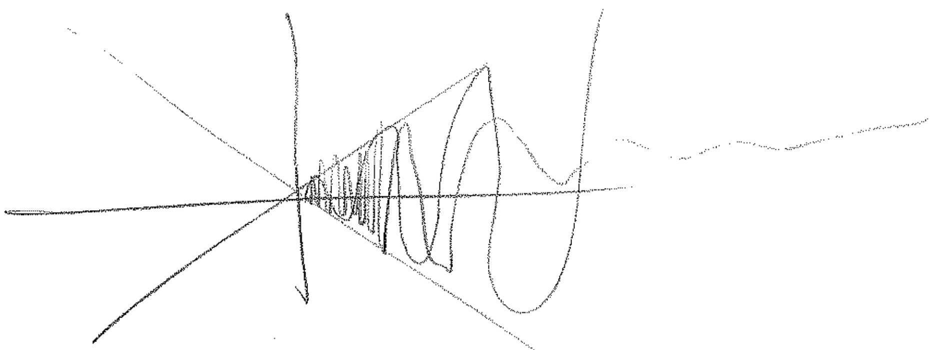
$= \lim_{u\to 0} \frac{\sin u}{u} = \boxed{1}$ Not sure I'm buying this.

Hmm $\sin\left(\frac{1}{x}\right) \xrightarrow[x\to\infty]{} 0$ } Maybe so.

(b) $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = 0$, by squeezing.

$-x \le x\sin\frac{1}{x} \le x$

(c) Graph 'em



Hmmm. I'll put the graphs in the notes.

Close to $x=0$, $\sin\frac{1}{x}$ is

Dampen it by $x\cdot\sin\frac{1}{x}$ &

Then look @ it as $x \to$ big

$\sin\left(\frac{1}{x}\right) \longrightarrow \sin(0) = 0$

$x \longrightarrow \infty$

It's an $\infty\cdot 0$ thing, which balances out (this time) @ $y=1$!