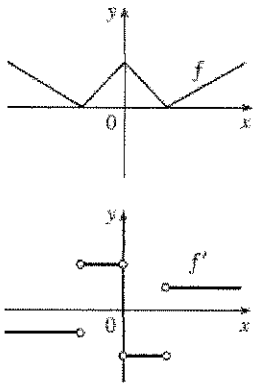
Hints for Exercises 4–11: First plot x-intercepts on the graph of f' for any horizontal tangents on the graph of f. Look for any corners on the graph of f—there will be a discontinuity on the graph of f'. On any interval where f has a tangent with positive (or negative) slope, the graph of f' will be positive (or negative). If the graph of the function is linear, the graph of f' will be a horizontal line.

9.



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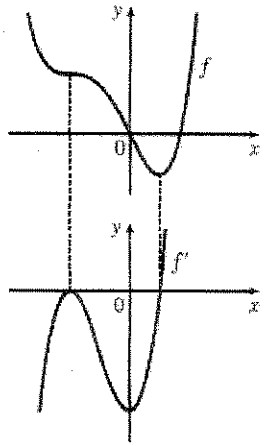
10.



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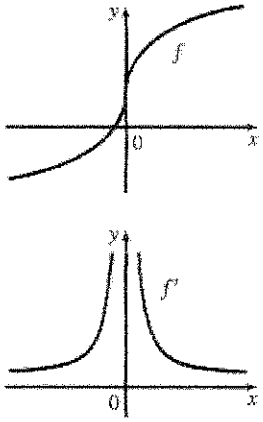
11.



201 §2.2 #s ~~5-11 odds~~, 19, 23-~~25~~ ALL, 41, 43

#s 19-29 Find derivative by defn of derivative. State $\mathcal{D}$ & R of f & f'.

(19) $f(x) = \frac{1}{2}x - \frac{1}{3} \longrightarrow$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[f(x+h) - f(x)\right]$$

$$= \frac{1}{h}\left[\frac{1}{2}(x+h) - \frac{1}{3} - \left(\frac{1}{2}x - \frac{1}{3}\right)\right] = \frac{1}{h}\left[\frac{1}{2}x + \frac{1}{2}h - \frac{1}{3} - \frac{1}{2}x + \frac{1}{3}\right]$$

$$= \frac{1}{h}\left[\frac{1}{2}h\right] = \boxed{\frac{1}{2} = f'(x)}$$

$\mathcal{D}(f) = \mathcal{D}(f') = \mathbb{R}$

$R(f) = \mathbb{R}$, $R(f') = \left\{\frac{1}{2}\right\}$

(23) $f(x) = -2x^3 + x^2 \Longrightarrow \dfrac{f(x+h)-f(x)}{h}$

$$= \frac{-2(x+h)^3 + (x+h)^2 - (-2x^3 + x^2)}{h}$$

$$= \frac{-2(x^3 + 3x^2h + 3xh^2 + h^3) + (x^2 + 2xh + h^2) + 2x^3 - x^2}{h}$$

$$= \frac{-2x^3 - 6x^2h - 6xh^2 - 2h^3 + x^2 + 2xh + h^2 + 2x^3 - x^2}{h}$$

$$= \frac{-6x^2h - 6xh^2 - 2h^3 + 2xh + h^2}{h} = -6x^2 - 6xh - 2h^2 + 2x + h$$

$\mathcal{D}(f) = R(f) = \mathbb{R}$

$h \to 0 \Rightarrow \boxed{-6x^2 + 2x \quad \mathcal{D}(f') = \mathbb{R},\ R(f') = (-\infty, \tfrac{1}{6}]}$

$-6x^2 + 2x$

$= -2(3x^2 - x)$

$= -2x(3x - 1)$

x	$-\infty$		0		$\frac{1}{3}$		$+\infty$
$-2x$		$+$	0	$-$		$-$	
$3x-1$		$-$		$-$	0	$+$	
$f'(x)$		$-$	0	$+$	0	$-$	
f	$+\infty$	$\searrow$	0	$\nearrow$	$-\frac{1}{9}$	$\searrow$	$-\infty$

201 §2.2 #s 24, 25, 41, 43

(24) $g(t) = \frac{t}{\sqrt{t}} \longrightarrow \frac{g(t+h)-g(t)}{h}$

$= \frac{1}{h}\left[\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}\right]$

$= \frac{1}{h}\left[\frac{1}{\sqrt{t+h}} \cdot \frac{\sqrt{t}}{\sqrt{t}} - \frac{1}{\sqrt{t}} \cdot \frac{\sqrt{t+h}}{\sqrt{t+h}}\right] = \frac{1}{h}\left[\frac{\sqrt{t}-\sqrt{t+h}}{\sqrt{t}\sqrt{t+h}}\right]$

still can't let $h \to 0$ ...

$= \frac{1}{h}\left[\frac{\sqrt{t}-\sqrt{t+h}}{\sqrt{t}\sqrt{t+h}}\right]\left[\frac{\sqrt{t}+\sqrt{t+h}}{\sqrt{t}+\sqrt{t+h}}\right]$

$= \frac{1}{h}\left[\frac{t-(t+h)}{\sqrt{t(t+h)}\left(\sqrt{t}+\sqrt{t+h}\right)}\right] = \frac{1}{h}\left[\frac{t-t-h}{\sqrt{t(t+1)}\left(\sqrt{t}+\sqrt{t+h}\right)}\right]$

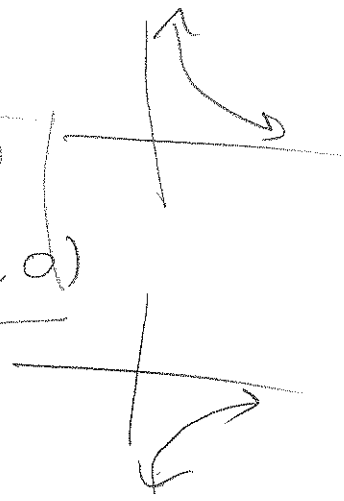
$= \frac{-1}{\sqrt{t(t+h)}\left(\sqrt{t}+\sqrt{t+h}\right)} \xrightarrow{h \to 0} \frac{-1}{\sqrt{t^2}\left(\sqrt{t}+\sqrt{t}\right)}$

$= \frac{-1}{|t|(2\sqrt{t})} = \boxed{-\frac{1}{2} \cdot \frac{1}{|t|\sqrt{t}}}$ OR $-\frac{1}{2}t^{-3/2}$ OR ...

$|t| = t$, since $t < 0 \notin \mathcal{D}(g)$

$\mathcal{D}(g) = (0,\infty)$, $\mathcal{R}(g) = (0,\infty)$

$\mathcal{D}(g') = (0,\infty)$, $\mathcal{R}(g') = (-\infty, 0)$



201 §2.2 #s 25, 41, 43

(25) $g(x) = \sqrt{9-x} \longrightarrow \dfrac{g(x+h)-g(x)}{h}$

$$= \frac{1}{h}\left[\sqrt{9-(x+h)} - \sqrt{9-x}\right]$$

$$= \left(\frac{\sqrt{9-(x+h)} - \sqrt{9-x}}{h}\right)\left(\frac{\sqrt{9-(x+h)} + \sqrt{9-x}}{\sqrt{9-(x+h)} + \sqrt{9-x}}\right)$$

$$= \frac{9-(x+h) - (9-x)}{h\left(\sqrt{9-(x+h)} + \sqrt{9-x}\right)}$$

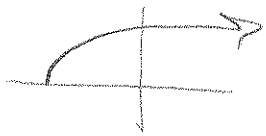
$$= \frac{9-x-h-9+x}{h\left(\sqrt{9-(x+h)} + \sqrt{9-x}\right)} = \frac{-1}{\sqrt{9-(x+h)} + \sqrt{9-x}}$$

$h \to 0 \longrightarrow \boxed{\dfrac{-1}{2\sqrt{9-x}}}$

$\sqrt{x}$

$\sqrt{9-x} = \sqrt{-x+9}$

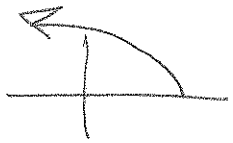
$\sqrt{x+9}$



$D(g) = (-\infty, 9]$, $R(g) = [0, \infty)$

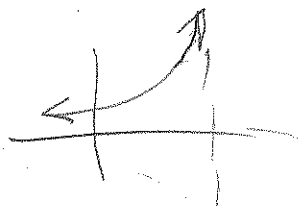
$D(g') = (-\infty, 9)$, $R(g') = (-\infty, 0)$

$\sqrt{-x+9}$

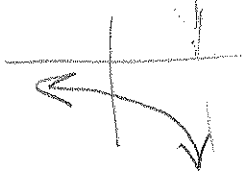


$-\frac{1}{2} \cdot \frac{1}{\sqrt{9-x}}$

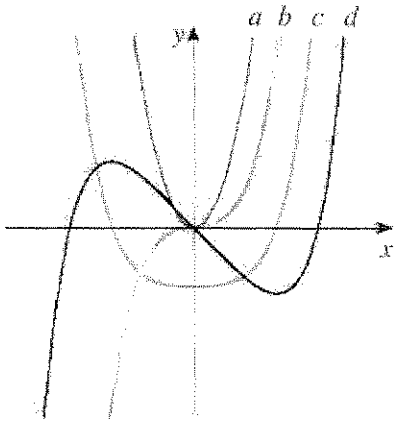
$\frac{1}{\sqrt{9-x}}$



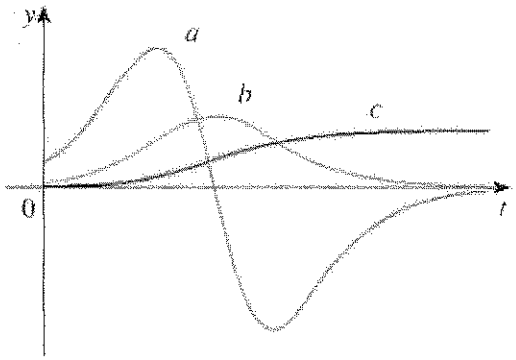
$-\frac{1}{2} \cdot \frac{1}{\sqrt{9-x}}$



42. The figure shows graphs of f, f', f'', and f'''. Identify each curve, and explain your choices.



43. The figure shows the graphs of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Identify each curve, and explain your choices.



41.

$a = f, b = f', c = f''$. We can see this because where a has a horizontal tangent, $b = 0$, and where b has a horizontal tangent, $c = 0$. We can immediately see that c can be neither f nor f', since at the points where c has a horizontal tangent, neither a nor b is equal to 0.

43.

We can immediately see that a is the graph of the acceleration function, since at the points where a has a horizontal tangent, neither c nor b is equal to 0. Next, we note that $a = 0$ at the point where b has a horizontal tangent, so b must be the graph of the velocity function, and hence, $b' = a$. We conclude that c is the graph of the position function.