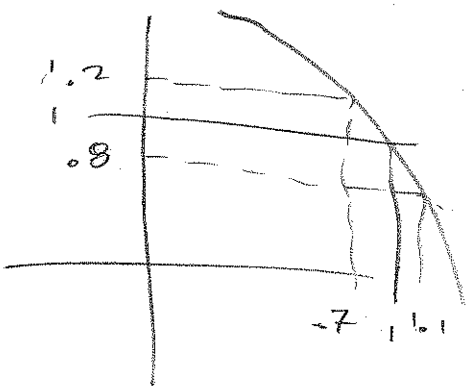
201 §1.7 #s 1, 11, 17, 18, 29, 32, 37

(1)



Use graph to find δ corresponding to $\varepsilon = 0.2$

Want $\delta \ni 0 < |x-1| < \delta$
$\Rightarrow |f(x) - 1| < 0.2$

Let $\boxed{\delta = 0.1}$ $\ni$ $0.9 < x < 1.1$ will keep $|f(x) - 1| < \varepsilon = .2$

(11) Want disk $\ni$ $A = \pi x^2 = 1000 \Rightarrow$

(a) radius is found by solving
$\pi x^2 = 1000$

$$x^2 = \frac{1000}{\pi}$$

$$x = \pm \frac{10\sqrt{10}}{\sqrt{\pi}} \quad \text{OR} \quad \frac{10\sqrt{10\pi}}{\pi}$$

Ditch the negative.

(b) If error is to be within $\pm 5\text{cm}^2$ of 1000cm^2

How close to $\frac{10\sqrt{10\pi}}{\sqrt{\pi}}$ must the radius be?

Want $|\pi x^2 - 1000| < 5 \longrightarrow$

$-5 < \pi x^2 - 1000 < 5 \Rightarrow$

$995 < \pi x^2 < 1005$

for §1.7 #s 4, 17, 18, 29, 32, 37

(11) cont'd

$$\Rightarrow \frac{995}{\pi} < x^2 < \frac{1005}{\pi}$$

$$\Rightarrow \sqrt{\frac{995}{\pi}} < x < \sqrt{\frac{1005}{\pi}}$$

$$\Rightarrow \sqrt{\frac{995}{\pi}} - \frac{10\sqrt{10\pi}}{\pi} < x - \frac{10\sqrt{10\pi}}{\pi} < \frac{\sqrt{1005}}{\pi} - \frac{10\sqrt{10\pi}}{\pi}$$

Pick the smalles in absolute value of

$$\left| \frac{\sqrt{1005\pi}}{\pi} - \frac{10\sqrt{10\pi}}{\pi} \right| \approx .044547488$$

and

$$\left| \frac{\sqrt{995\pi}}{\pi} - \frac{10\sqrt{10\pi}}{\pi} \right| \approx .044658996$$

$.044547488 = \delta$ is small enough tolerance to keep area within 5cm^2 of desired 1000 cm^2.

(17) $\lim_{x \to -3} (1-4x) = 13$

$\boxed{\text{Pf}}$ Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{4}$. Then, if $0 < |x+3| < \delta$, we have $|(1-4x) - 13|$

$= |-12-4x| = |4x+12| = 4|x+3| < 4\delta = 4\frac{\varepsilon}{4} = \varepsilon$ ■

201 §1.7 #s 18, 29, 32, 37

18 $\lim_{x \to -2} (3x+5) = -1$

Pf

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{3}$. Then, if $0 < |x+2| < \delta$, we have $|3x+5-(-1)|$

$= |3x+6| = 3|x+2| < 3\delta = 3\frac{\varepsilon}{3} = \varepsilon$ ∎

29 $\lim_{x \to 2} (x^2-4x+5) = 1$

SCRATCH:

$|x^2-4x+5-1| < \varepsilon$

$|x^2-4x+4| < \varepsilon$

$|x-2|^2 < \varepsilon$

$|x-2| < \sqrt{\varepsilon}$

$\delta = \sqrt{\varepsilon}$ works

Pf Let $\varepsilon > 0$ be given. Define $\delta = \sqrt{\varepsilon}$. Then, if $0 < |x-2| < \delta$, we have

$|x^2-4x+5-1| = |x^2-4x+4|$

$= |x-2|^2 < \delta^2 = (\sqrt{\varepsilon})^2 = \varepsilon$ ∎

201 §1.7 #s 32,37

(32) $\lim_{x \to 3} x^3 = 8$ is the claim.

<u>Scratch</u>

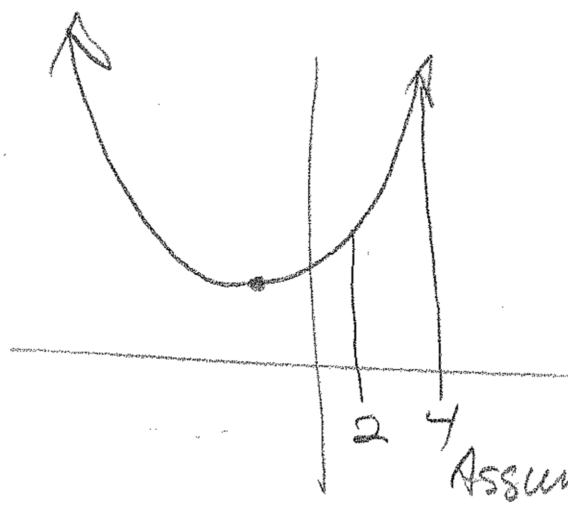
$x^3 - 8 = x^3 - 2^2 = (x-2)(x^2+2x+4)$

is gonna be $< \delta(x^2+2x+4)$, so we need a handle on x^2+2x+4.

Quick pic of $x^2+2x+4 = x^2+2x+1-1+4$

$= (x+1)^2+3$

$(h,k) = (-1,3)$



We're interested in the vicinity of $x=3$. Assuming $\delta \le 1$, we have

$2 < x < 4$ whenever

$|x-3| < 1$

Since x^2+2x+4 is increasing we see the biggest it can get is $4^2+2(4)+4$, when $x=4$, i.e.

$2 < x < 4 \Longrightarrow |x^2+2x+4| < \boxed{28}$. Now

Write proof!

201 §1.7 #s 32, 37

Pf (32)

<u>Proof</u> Let $\varepsilon > 0$ be given, Define $\delta = \min\{1, \frac{\varepsilon}{28}\}$. Then any time we have $0 < |x-3| < \delta$, we will also have $|x^3 - 8| = |x-2||x^2+2x+4|$

$$< |x^2+2x+4|\,\delta < 28\,\delta \leq 28 \cdot \frac{\varepsilon}{28} = \varepsilon \blacksquare$$

201 §1.7 #37

(37) $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ is the claim.

Scratch want $|\sqrt{x} - \sqrt{a}| < \varepsilon$

$$\Rightarrow \frac{|\sqrt{x} - \sqrt{a}|}{1}\left(\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}\right) = \frac{||x - a|}{\sqrt{x}+\sqrt{a}} < \frac{\delta}{\sqrt{x}+\sqrt{a}}$$

want that $< \varepsilon$.

want a string of "=" and "<"

ending with "$<\varepsilon$". Hmm mm.

$$\frac{\delta}{\sqrt{x}+\sqrt{a}} < \frac{\delta}{\sqrt{a}} \overset{\text{WANT}}{<} \varepsilon \Longrightarrow$$

$$\delta < \varepsilon\sqrt{a}$$

That's it!

Proof Let $\varepsilon > 0$ be given. Define $\delta = \varepsilon\sqrt{a}$

Then if $0 < |x - a| < \varepsilon$, we have

$$|\sqrt{x} - \sqrt{a}| = \frac{|x-a|}{\sqrt{x}+\sqrt{a}} < \frac{|x-a|}{\sqrt{a}} < \frac{\delta}{\sqrt{a}} = \frac{\varepsilon\sqrt{a}}{\sqrt{a}}$$

$= \varepsilon$ ∎