1.

(a) $\lim_{x\to 2}[f(x)+5g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2}[5g(x)]$ [Limit Law 1]

$= \lim_{x\to 2} f(x) + 5\lim_{x\to 2} g(x)$ [Limit Law 3]

$= 4 + 5(-2) = -6$

(b) $\lim_{x\to 2}[g(x)]^3 = \left[\lim_{x\to 2} g(x)\right]^3$ [Limit Law 6]

$= (-2)^3 = -8$

(c) $\lim_{x\to 2}\sqrt{f(x)} = \sqrt{\lim_{x\to 2} f(x)}$ [Limit Law 11]

$= \sqrt{4} = 2$

(d) $\lim_{x\to 2}\dfrac{3f(x)}{g(x)} = \dfrac{\lim_{x\to 2}[3f(x)]}{\lim_{x\to 2} g(x)}$ [Limit Law 5]

$= \dfrac{3\lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)}$ [Limit Law 3]

$= \dfrac{3(4)}{-2} = -6$

(e) Because the limit of the denominator is 0, we can't use Limit Law 5. The given limit, $\lim_{x\to 2}\dfrac{g(x)}{h(x)}$, does not exist because the denominator approaches 0 while the numerator approaches a nonzero number.

(f) $\lim_{x\to 2}\dfrac{g(x)\,h(x)}{f(x)} = \dfrac{\lim_{x\to 2}[g(x)\,h(x)]}{\lim_{x\to 2} f(x)}$ [Limit Law 5]

$= \dfrac{\lim_{x\to 2} g(x)\cdot \lim_{x\to 2} h(x)}{\lim_{x\to 2} f(x)}$ [Limit Law 4]

$= \dfrac{-2\cdot 0}{4} = 0$

2. (a) $\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2 + 0 = 2$

(b) $\lim_{x\to 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) $\lim_{x\to 0}[f(x)g(x)] = \lim_{x\to 0} f(x)\cdot\lim_{x\to 0} g(x) = 0\cdot 1.3 = 0$

(d) Since $\lim_{x\to -1} g(x) = 0$ and g is in the denominator, but $\lim_{x\to -1} f(x) = -1 \neq 0$, the given limit does not exist.

(e) $\lim_{x\to 2} x^3 f(x) = \left[\lim_{x\to 2} x^3\right]\left[\lim_{x\to 2} f(x)\right] = 2^3\cdot 2 = 16$

(f) $\lim_{x\to 1}\sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$

5. $$\lim_{t\to -2} \frac{t^4-2}{2t^2-3t+2} = \frac{\lim_{t\to -2}(t^4-2)}{\lim_{t\to -2}(2t^2-3t+2)} \qquad \text{[Limit Law 5]}$$

$$= \frac{\lim_{t\to -2} t^4 - \lim_{t\to -2} 2}{2\lim_{t\to -2} t^2 - 3\lim_{t\to -2} t + \lim_{t\to -2} 2} \qquad \text{[1, 2, and 3]}$$

$$= \frac{16-2}{2(4)-3(-2)+2} \qquad \text{[9, 7, and 8]}$$

$$= \frac{14}{16} = \frac{7}{8}$$

9. $$\lim_{x\to 2} \sqrt{\frac{2x^2+1}{3x-2}} = \sqrt{\lim_{x\to 2} \frac{2x^2+1}{3x-2}} \qquad \text{[Limit Law 11]}$$

$$= \sqrt{\frac{\lim_{x\to 2}(2x^2+1)}{\lim_{x\to 2}(3x-2)}} \qquad \text{[5]}$$

$$= \sqrt{\frac{2\lim_{x\to 2} x^2 + \lim_{x\to 2} 1}{3\lim_{x\to 2} x - \lim_{x\to 2} 2}} \qquad \text{[1, 2, and 3]}$$

$$= \sqrt{\frac{2(2)^2+1}{3(2)-2}} = \sqrt{\frac{9}{4}} = \frac{3}{2} \qquad \text{[9, 8, and 7]}$$

10.

(a) The left-hand side of the equation is not defined for $x = 2$, but the right-hand side is.

(b) Since the equation holds for all $x \neq 2$, it follows that both sides of the equation approach the same limit as $x \to 2$, just as in Example 3. Remember that in finding $\lim_{x\to a} f(x)$, we never consider $x = a$.

11. $$\lim_{x\to 5} \frac{x^2-6x+5}{x-5} = \lim_{x\to 5} \frac{(x-5)(x-1)}{x-5} = \lim_{x\to 5}(x-1) = 5-1 = 4$$

13. $\lim_{x\to 5} \frac{x^2-5x+6}{x-5}$ does not exist since $x-5 \to 0$, but $x^2-5x+6 \to 6$ as $x \to 5$.

23. $$\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x\to -4} \frac{\frac{x+4}{4x}}{4+x} = \lim_{x\to -4} \frac{x+4}{4x(4+x)} = \lim_{x\to -4} \frac{1}{4x} = \frac{1}{4(-4)} = -\frac{1}{16}$$

25. $\displaystyle\lim_{t\to 0}\frac{\sqrt{1+t}-\sqrt{1-t}}{t}=\lim_{t\to 0}\frac{\sqrt{1+t}-\sqrt{1-t}}{t}\cdot\frac{\sqrt{1+t}+\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}=\lim_{t\to 0}\frac{\left(\sqrt{1+t}\right)^2-\left(\sqrt{1-t}\right)^2}{t\left(\sqrt{1+t}+\sqrt{1-t}\right)}$

$$=\lim_{t\to 0}\frac{(1+t)-(1-t)}{t\left(\sqrt{1+t}+\sqrt{1-t}\right)}=\lim_{t\to 0}\frac{2t}{t\left(\sqrt{1+t}+\sqrt{1-t}\right)}=\lim_{t\to 0}\frac{2}{\sqrt{1+t}+\sqrt{1-t}}$$

$$=\frac{2}{\sqrt{1}+\sqrt{1}}=\frac{2}{2}=1$$

31. $\displaystyle\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=\lim_{h\to 0}\frac{(x^3+3x^2h+3xh^2+h^3)-x^3}{h}=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}$

$$=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}=\lim_{h\to 0}(3x^2+3xh+h^2)=3x^2$$

39. $-1\le\cos(2/x)\le 1 \quad\Rightarrow\quad -x^4\le x^4\cos(2/x)\le x^4$. Since $\lim\limits_{x\to 0}\left(-x^4\right)=0$ and $\lim\limits_{x\to 0}x^4=0$, we have $\lim\limits_{x\to 0}\left[x^4\cos(2/x)\right]=0$ by the Squeeze Theorem.

43. $\left|2x^3-x^2\right|=\left|x^2(2x-1)\right|=\left|x^2\right|\cdot|2x-1|=x^2\,|2x-1|$

$$|2x-1|=\begin{cases}2x-1 & \text{if } 2x-1\ge 0\\ -(2x-1) & \text{if } 2x-1<0\end{cases}=\begin{cases}2x-1 & \text{if } x\ge 0.5\\ -(2x-1) & \text{if } x<0.5\end{cases}$$

So $\left|2x^3-x^2\right|=x^2[-(2x-1)]$ for $x<0.5$.

Thus, $\displaystyle\lim_{x\to 0.5^-}\frac{2x-1}{|2x^3-x^2|}=\lim_{x\to 0.5^-}\frac{2x-1}{x^2[-(2x-1)]}=\lim_{x\to 0.5^-}\frac{-1}{x^2}=\frac{-1}{(0.5)^2}=\frac{-1}{0.25}=-4.$

45. Since $|x|=-x$ for $x<0$, we have $\displaystyle\lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{|x|}\right)=\lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{-x}\right)=\lim_{x\to 0^-}\frac{2}{x}$, which does not exist since the denominator approaches 0 and the numerator does not.

46. Since $|x|=x$ for $x>0$, we have $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{|x|}\right)=\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x}\right)=\lim_{x\to 0^+}0=0.$

49. (a) (i) $\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} \frac{x^2+x-6}{|x-2|} = \lim_{x\to 2^+} \frac{(x+3)(x-2)}{|x-2|}$

$= \lim_{x\to 2^+} \frac{(x+3)(x-2)}{x-2}$ [since $x-2>0$ if $x\to 2^+$]

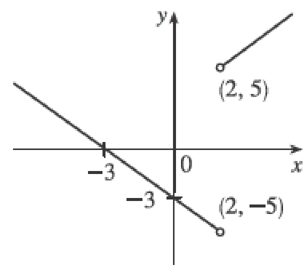
$= \lim_{x\to 2^+} (x+3) = 5$

(ii) The solution is similar to the solution in part (i), but now $|x-2| = 2-x$ since $x-2<0$ if $x\to 2^-$.

Thus, $\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} -(x+3) = -5$.

(b) Since the right-hand and left-hand limits of g at $x=2$ are not equal, $\lim_{x\to 2} g(x)$ does not exist.

(c)



54. $\lim_{v\to c^-}\left(L_0\sqrt{1-\frac{v^2}{c^2}}\right) = L_0\sqrt{1-1} = 0$. As the velocity approaches the speed of light, the length approaches 0.

A left-hand limit is necessary since L is not defined for $v>c$.

58. (a) $\lim_{x\to 0} f(x) = \lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x^2\right] = \lim_{x\to 0}\frac{f(x)}{x^2}\cdot\lim_{x\to 0} x^2 = 5\cdot 0 = 0$

(b) $\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x\right] = \lim_{x\to 0}\frac{f(x)}{x^2}\cdot\lim_{x\to 0} x = 5\cdot 0 = 0$

63. Since the denominator approaches 0 as $x\to -2$, the limit will exist only if the numerator also approaches 0 as $x\to -2$. In order for this to happen, we need $\lim_{x\to -2}\left(3x^2+ax+a+3\right) = 0 \quad\Leftrightarrow$

$3(-2)^2 + a(-2) + a + 3 = 0 \quad\Leftrightarrow\quad 12-2a+a+3=0 \quad\Leftrightarrow\quad a=15$. With $a=15$, the limit becomes

$\lim_{x\to -2}\frac{3x^2+15x+18}{x^2+x-2} = \lim_{x\to -2}\frac{3(x+2)(x+3)}{(x-1)(x+2)} = \lim_{x\to -2}\frac{3(x+3)}{x-1} = \frac{3(-2+3)}{-2-1} = \frac{3}{-3} = -1.$