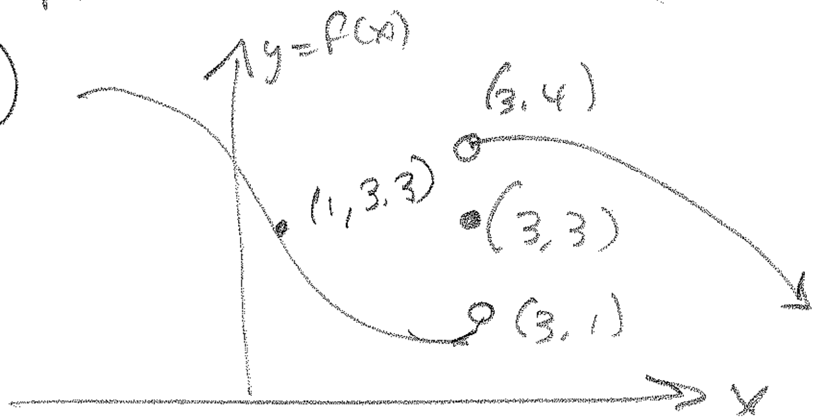
201 §1.5 #s 1, 3, 5, 6, 7, 11, 15, 19, 23–29, 35, 39

(1) $\lim_{x \to 2} f(x) = 5$ means... Answers vary.
It's <u>entirely</u> possible $f(2) = 3$. (Just not nearby!)

(3) Explain the meaning of...
$\lim_{x \to 1^-} f(x) = 3$ & $\lim_{x \to 1^+} f(x) = 7$

This says limit from the left as x approaches 1 is 3 and limit from the right as x approaches 1 is 7.

(5)



Find each of the following. If one doesn't exist, explain why.

(a) $\lim_{x \to 1} f(x) = 3.3$ (approx)

(b) $\lim_{x \to 3^-} f(x) = 1$

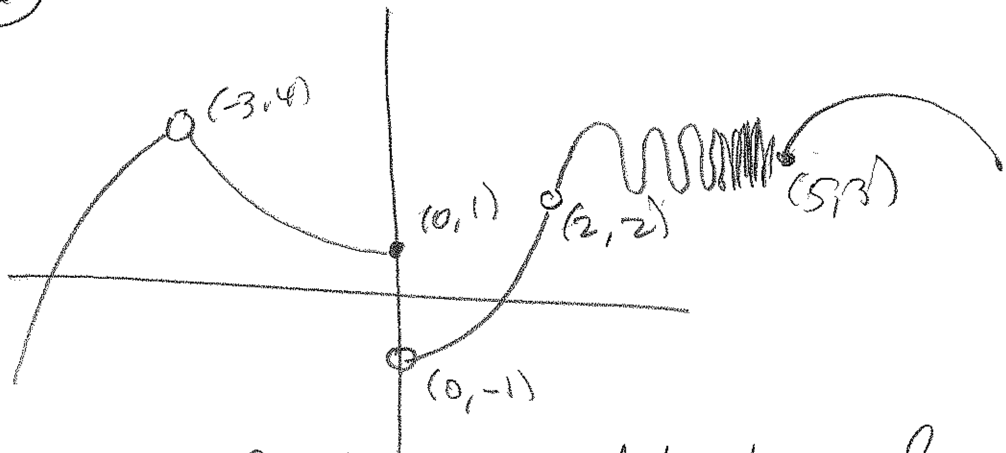
(c) $\lim_{x \to 3^+} f(x) = 4$

(d) $\lim_{x \to 3} f(x) \nexists$, b/c left- & right-hand limits don't agree.

(e) $f(3) = 3$

201 §1.5 #s 6, 7, 11, 15, 19, 23-29, 35, 39

(6)



For this function, h, state the value, if it exists. If it doesn't, state why.

(a) $\lim_{x \to -3^-} h(x) = 4$

(b) $\lim_{x \to -3^+} h(x) = 4$

(c) $\lim_{x \to 3} h(x) = 4$

(d) $h(-3)$ $\nexists$. see open dot @ (-3, 4) & no closed dot @ x = 3.

(e) $\lim_{x \to 0^-} h(x) = 1$

(f) $\lim_{x \to 0^+} h(x) = -1$

(g) $\lim_{x \to 0} h(x)$ $\nexists$, b/c $\lim_{x \to 0^-} h(x) = 1 \neq -1 = \lim_{x \to 0^+} h(x)$.

(h) $h(0) = 1$

(i) $\lim_{x \to 2} h(x) = 2$

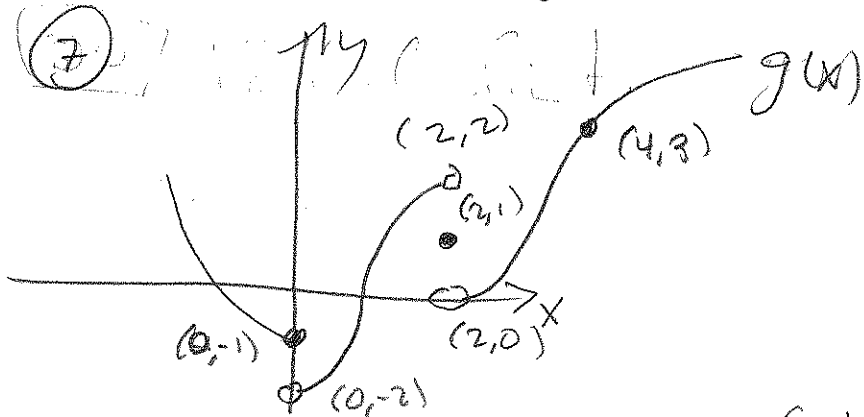
(j) $h(2)$ $\nexists$ No closed dot.

(k) $\lim_{x \to 5^+} h(x) = 3$

(l) $\lim_{x \to 5^-} h(x)$ $\nexists$.
oscillates from 2 to 4 infinitely often in the neighborhood of x = 5.

201 S' 1.5 #s 7, 11, 15, 19, 23-29, 35, 38

7



Same as #6

(a) $\lim_{x \to 0^-} g(x) = -1$

(b) $\lim_{x \to 0^+} g(x) = -2$

(c) $\lim_{x \to 0} g(x) \nexists$

left- & right-hand limits disagree.

(d) $\lim_{x \to 2^-} g(x) = 2$

(e) $\lim_{x \to 2^+} g(x) = 0$

(f) $\lim_{x \to 2} g(x) \nexists$

left- and right-hand limits disagree.

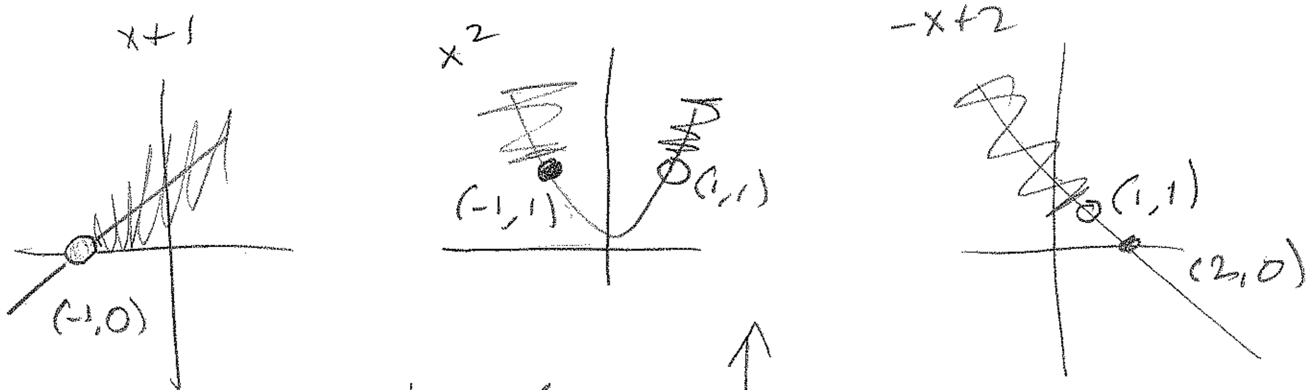
(g) $g(2) = 1$

(h) $\lim_{x \to 4} g(x) = 3$

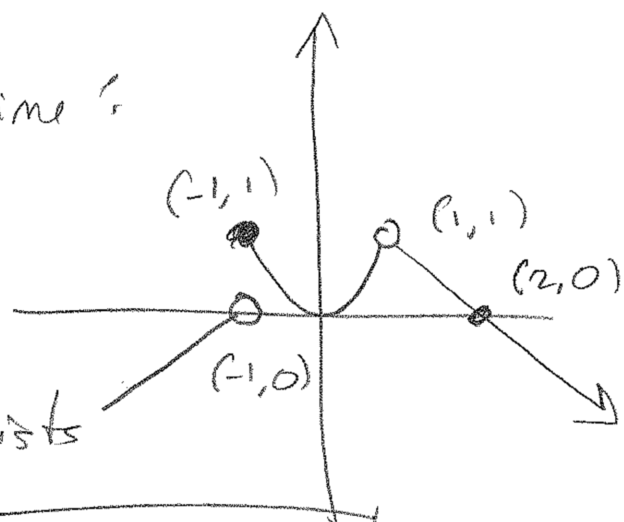
#s 11, 12 Sketch the graph and determine all $a \ni \lim_{x \to a} f(x)$ exists.

201 §1.5 #s 11, 15, 19, 23-29, 35, 38

(11) $f(x) = \begin{cases} 1+x & x < -1 \\ x^2 & -1 \le x < 1 \\ 2-x & x > 1 \end{cases}$



Combine:

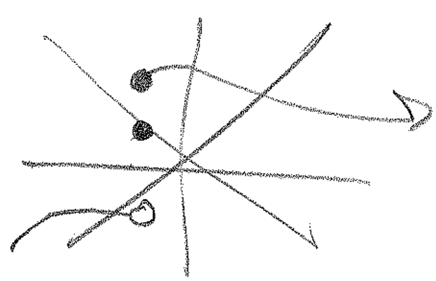


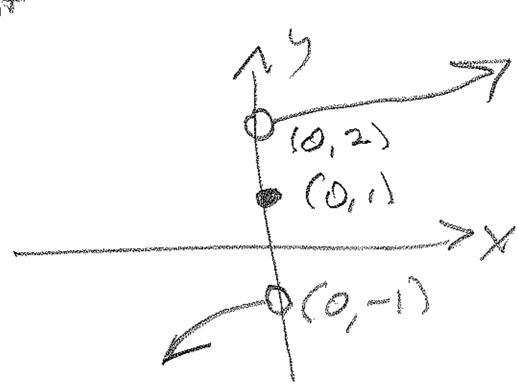
$\lim_{x \to a} f(x)$ exists

for $\boxed{a \in (-\infty, -1) \cup (-1, \infty)}$

#s 15-18 Sketch a function that satisfies the given properties

(15) $\lim_{x \to 0^-} f(x) = -1$ $\lim_{x \to 0^+} f(x) = 2$, $f(0) = 1$





201 §1.5 #s 19, 23–29, 35, 38

#s 19–22 Guess the limit by numerical method

19 $\lim_{x\to 2} \frac{x^2-2x}{x^2-x-2} = \lim_{x\to 2} f(x)$

$f(2.001) \approx .66678$ $\qquad f(1.9999) \approx .6666556$

$f(2.0001) \approx .666678$

$f(2.000001) \approx .66666678$

I'd guess $\lim_{x\to 2} f(x) = .\overline{6} = \frac{2}{3}$.

Analytic Check:

$$\frac{x^2-2x}{x^2-x-2} = \frac{x(x-2)}{(x+1)(x-2)} = \frac{x}{x+1} \xrightarrow{x\to 2} \frac{2}{2+1} = \frac{2}{3} \checkmark$$

$(x \neq 2)$

#s 23–26 Use numerical methods

23 $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0} f(x)$

My guess: $\boxed{\lim_{x\to 0} f(x) = .25}$

$f(.001) \approx .24998$

$f(-.001) \approx .25002$

Analytic Check:

$$\left(\frac{\sqrt{x+4}-2}{x}\right)\left(\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right) = \frac{x+4-4}{x(\sqrt{x+4}+2)} = \frac{1}{\sqrt{x+4}+2}$$

$$\xrightarrow{x\to 0} \frac{1}{\sqrt{0+4}+2} = \frac{1}{2+2} = \frac{1}{4} = .25 \checkmark$$

301 §1.5 #s 25–29, 35, 38

(25) $\lim_{x\to 1} \frac{x^6-1}{x^{10}-1} = \lim_{x\to 1} f(x)$

$f(.9999) \approx .60012$

$f(1.0001) \approx .59989$

My Guess: $\boxed{\lim_{x\to 1} f(x) = .6}$

Analytic Check:

$$\frac{x^6-1}{x^{10}-1} = \frac{(x-1)(x^5+x^4+x^3+x^2+x+1)}{(x-1)(x^9+x^8+\cdots+x+1)}$$

$$= \frac{x^5+x^4+x^3+x^2+x+1}{x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1} \xrightarrow{x\to 1} \frac{6}{10} = .6 \checkmark$$

(27) (a) Graph & zoom to estimate

$$\lim_{x\to 0} \frac{(\cos(2x) - \cos x)}{x^2} = \lim_{x\to 0} \frac{\cos^2 x - \sin^2 x - \cos x}{x^2}$$

$$= \lim_{x\to 0}\left(\frac{\cos x(\cos x - 1)}{x^2} - \frac{\sin^2 x}{x^2}\right)$$

$$= \lim_{x\to 0}\left(\frac{\cos x - 1}{x}\cdot\frac{\cos x}{x} - \left(\frac{\sin x}{x}\right)^2\right)$$

$\to 1 \qquad \to \infty \qquad \to 1$

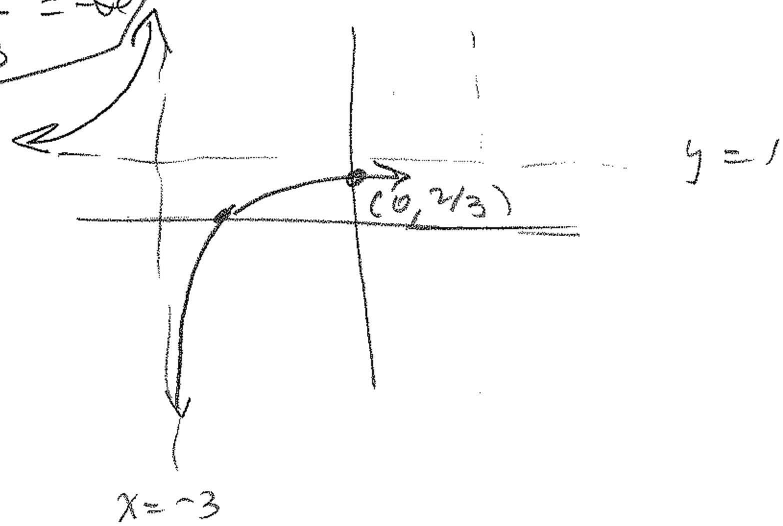
I don't think it exists

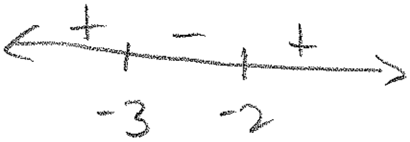
<u>But</u> my graph seems to think the $\lim_{x\to 0} f(x) \approx -.5$. My analysis isn't helping much. Numerically, $\boxed{y = L = -.5}$

201 §1.5 #s 29, 35, 39

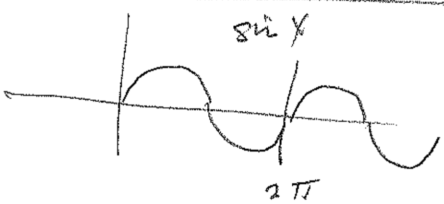
~~29~~ Determine the infinite limit #s 29–37

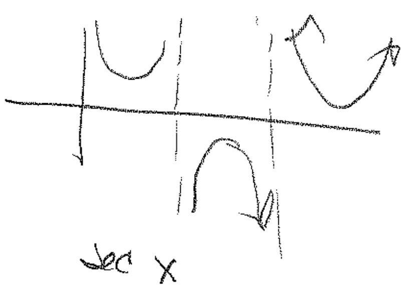
(29) $\boxed{\lim_{x\to -3^+} \frac{x+2}{x+3} = -\infty}$





(35) $\boxed{\lim_{x\to 2\pi^-} x \csc x = -\infty}$





x csc x won't be much different, qualitatively.

Just $x \approx 2\pi$ times graph of csc x.

201 $S^{?}$ 1.5 #38

38 (a) Find vertical asymptotes. (b) Confirm by graphing

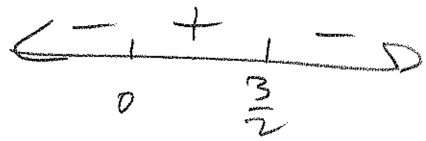
(a) $y = \dfrac{x^2+1}{3x-2x^2} = \dfrac{x^2+1}{x(3-2x)}$

$\boxed{x=0}$ $\quad 3-2x=0$

$2x = 3$

$\boxed{x=\frac{3}{2}}$ → vertical asymptotes

$\lim\limits_{x\to\infty} f(x) = \dfrac{1}{-2} = -\dfrac{1}{2} = y$



Rough sketch.