1. $\int_0^4 (x^2+3x)\,dx$

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{n} = \frac{4}{n}$$

$$x_k = a + k\Delta x = 0 + k\left(\frac{4}{n}\right) = \frac{4k}{n}$$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} f(x_k) = \frac{4}{n}\sum_{k=1}^{n}\left(x_k^2 + 3x_k\right)$$

$$= \frac{4}{n}\sum_{k=1}^{n}\left(\left(\frac{4k}{n}\right)^2 + 3\left(\frac{4k}{n}\right)\right) = \frac{4}{n}\sum_{k=1}^{n}\left(\frac{16k^2}{n^2} + \frac{12k}{n}\right)$$

$$= \frac{4}{n}\sum_{k=1}^{n}\frac{16}{n^2}k^2 + \frac{4}{n}\sum_{k=1}^{n}\frac{12}{n}k = \frac{64}{n^3}\sum_{k=1}^{n}k^2 + \frac{48}{n^2}\sum_{k=1}^{n}k$$

$$= \frac{64}{\cancel{n^3}}\cdot\frac{\cancel{n^3}+\cdots}{3} + \frac{48}{\cancel{n^2}}\cdot\frac{\cancel{n^2}+\cdots}{2}$$

$$\xrightarrow{n\to\infty} \frac{64}{3} + 24 = \frac{64+72}{3} = \frac{136}{3}$$

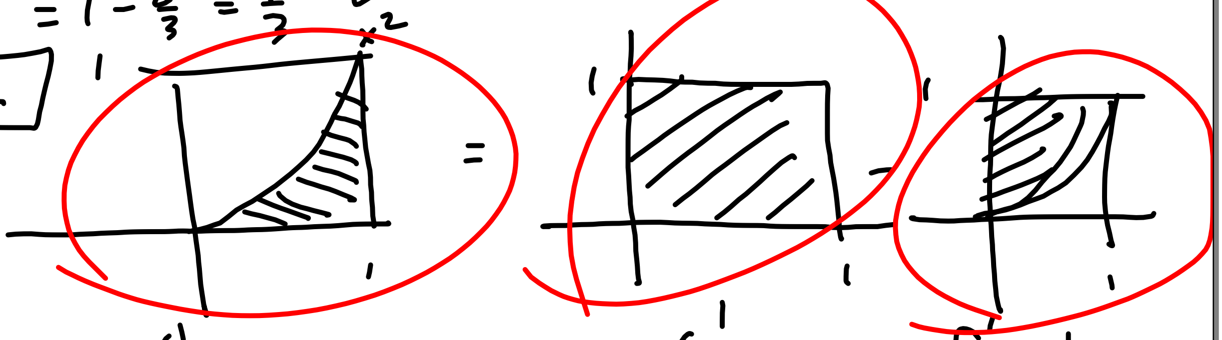
② $\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big]_0^1 = \frac{1}{3}$

$\dfrac{y^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}y^{\frac{3}{2}}$

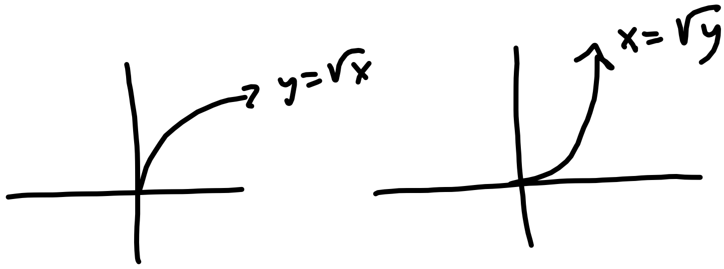
$\int_0^1 (1 - y^{\frac{1}{2}})\,dy = \left[y - \frac{2}{3}y^{\frac{3}{2}}\right]_0^1$

$= 1 - \frac{2}{3} = \frac{1}{3}$ ✓

B2



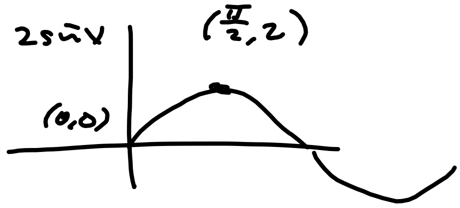
$\int_0^1 x^2\,dx = \int_0^1 1\,dy - \int_0^1 \sqrt{y}\,dy$



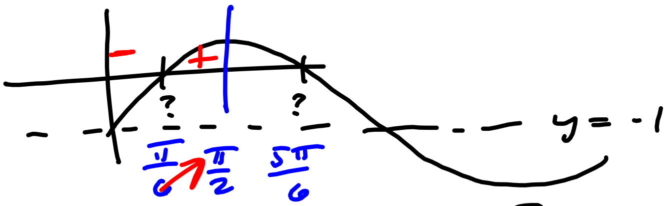
3

$$\int_0^{\frac{\pi}{2}} |2\sin x - 1|\, dx$$

$$|2\sin x - 1| = \begin{cases} 2\sin x - 1 & \text{if } 2\sin x - 1 \geq 0 \\ -(2\sin x - 1) & \text{if } 2\sin x - 1 < 0 \end{cases}$$



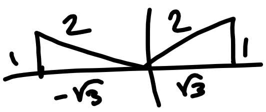
$2\sin x - 1$



$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

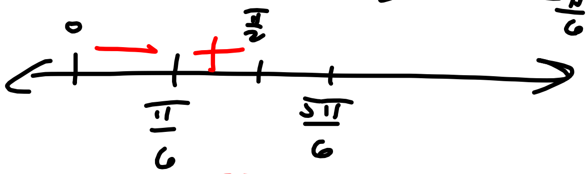


$\frac{\pi}{6}, \frac{5\pi}{6}$

$$-\int_0^{\frac{\pi}{6}} (2\sin x - 1)\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2\sin x - 1)\,dx$$

$$= -\left[-2\cos x - x\right]_0^{\frac{\pi}{6}} + \left[-2\cos x - x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$+ \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

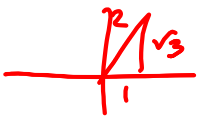


$$= \frac{2\sqrt{3}}{2} - \frac{\pi}{6} - (2 - 0)$$
$$- \left[\left(-2(0) - \frac{\pi}{2}\right) - \left(\sqrt{3} - \frac{\pi}{6}\right)\right]$$

Test: $2\sin\left(\frac{\pi}{12}\right) - 1 \approx -.48$

$2\sin\left(\frac{\pi}{3}\right) - 1 = 2\left(\frac{\sqrt{3}}{2}\right) - 1 = \sqrt{3} - 1$

$$-2 + \sqrt{3} - \frac{\pi}{6} - \left[-\frac{\pi}{2} - \sqrt{3} + \frac{\pi}{6}\right]$$



$$= 2\sqrt{3} + \frac{\pi}{6} - 2$$

$$-\frac{\pi}{3} + \frac{\pi}{2} = \frac{-2\pi + 3\pi}{6}$$
$$= \frac{\pi}{6}$$

4 a

$u = 3x+1$

$du = 3dx$

$\underline{dx} = \frac{du}{3}$

$$\int (3x+1)^4 dx = \int u^4 \cdot \frac{du}{3} = \frac{1}{3}\int u^4 du$$

$$= \frac{1}{3} \cdot \frac{u^5}{5} + C = \frac{1}{15}(3x+1)^5 + C$$

b $\int (3x+1)^4 x^2 dx$

$u = 3x+1$ $\quad du = 3dx$

$u-1 = 3x$ $\quad \frac{du}{3} = dx$

$\frac{u-1}{3} = x$

$$= \int (u^4)\left(\frac{u-1}{3}\right)^2\left(\frac{du}{3}\right)$$

$$\frac{1}{3}\int u^4\left(\frac{u^2-2u+1}{9}\right) du$$

$$= \frac{1}{27}\int (u^6 - 2u^5 + u^4)\, du = \frac{1}{27}\left[\frac{1}{7}u^7 - \frac{1}{3}u^6 + \frac{1}{5}u^5\right] + C$$

$$= \frac{1}{27}\left[\frac{1}{7}(3x+1)^7 - \frac{1}{3}(3x+1)^6 + \frac{1}{5}(3x+1)^5\right] + C$$

STOP

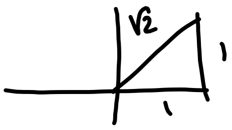
$$= \frac{1}{189}(\ )^7 - \frac{1}{81}(\ )^6 + \frac{1}{135}(\ )^5 + C$$

④ $\int \sec^4 x \tan x \, dx$

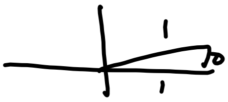
$= \int (\sec^3 x)(\sec x \tan x \, dx)$

$= \int u^3 \, du = \frac{1}{4}u^4 + C = \frac{1}{4}\sec^4 x + C$

⑤ $\int_0^{\frac{\pi}{4}} \sec^4 x \tan x \, dx = \left[\frac{1}{4} \sec^4 x \right]_0^{\frac{\pi}{4}}$



$= \frac{1}{4}(\sqrt{2})^4 - \frac{1}{4}(1)^4$



$= 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$

6a $\int_0^{3600} |r'(t)| dt$ = TOTAL Distance in ft, after 1 hr.

6b $\int_0^{3600} r'(t) dt$ = Net distance.

& Rob DID give the right answer.

7a $\frac{d}{dx} \int_0^x \frac{\sin(3t)}{t^2+4} dt$

$\frac{\sin(3x)}{x^2+4}$

b $\frac{d}{dx} \int_{x^2}^{\cos x} \frac{\sin(3t)}{t^2+4} dt = \frac{d}{dx}\left[\int_{x^2}^{0} \sim + \int_0^{\cos x} \right]$

$= \frac{d}{dx}\left[-\int_0^{x^2} \frac{\sin(3t)}{t^2+4} dt + \int_0^{\cos x} \frac{\sin(3t)}{t^2+4} dt \right]$

$= -\frac{\sin(3x^2)}{(x^2)^2+4}(2x) + \left(\frac{\sin(3\cos x)}{\cos^2 x+4}\right)(-\sin x)$

$\int_a^{f(x)} \sim$ a = constant

8i $\Delta x = \frac{b-a}{n} = \frac{4-2}{n} = \frac{2}{n}$

$\int_2^4$

x_k $a + k\Delta x$

$= 2 + \frac{2k}{n}$

B2

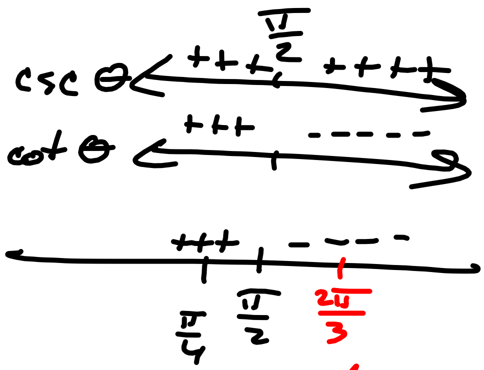
$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{3(x+h)}-\sqrt{3x}}{h}\cdot\frac{\sqrt{3(x+h)}+\sqrt{3y}}{\sqrt{3(x+h)}+\sqrt{3x}}$$

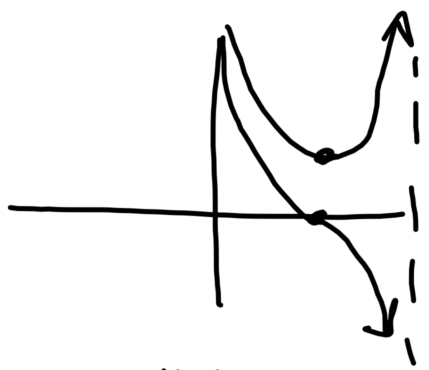
$$= \frac{3(x+h)-3x}{h(\sim\sim)} = \frac{3x+3h-3x}{h(\sim\sim)} = \frac{3h}{h\left(\sqrt{3(x+h)}+\sqrt{3x}\right)}$$

$$= \frac{3}{\sqrt{3(x+h)}+\sqrt{3x}} \xrightarrow{h\to 0} \frac{3}{\sqrt{3x}+\sqrt{3x}} = \frac{3}{2\sqrt{3x}}$$

B3

$$\int_{\frac{\pi}{4}}^{\cancel{\frac{\pi}{2}}\ \frac{2\pi}{3}} |\csc\theta\cot\theta|\,d\theta$$





$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{2}}^{\frac{2\pi}{3}}$$ w/ abs. value