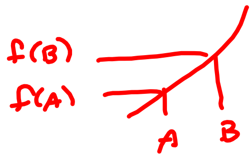
Devon asks §4.3 #71

Prove $1 \le \sqrt{1+x^3} \le 1+x^3$ if $x \ge 0$,

& use it to show $1 \le \int_0^1 \sqrt{1+x^3}\,dx \le 1.25$

First of all, if a function is increasing and $A \le B$, then $f(A) \le f(B)$



$\int_a^x g(t)\,dt$ is an increasing function of x, if $g(x) \ge 0$. (Think areas of funcs above the x-axis. ~~Also,~~

Also, btw, $\sqrt{x}$ is increasing.

Now, let's write a nice proof.

Assume $x \ge 0$, i.e.,

$0 \le x$. Then

$0^3 \le x^3$, i.e.,

$0 \le x^3$

$\Rightarrow$ $1 \le 1+x^3$

$\Rightarrow$ $\sqrt{1} \le \sqrt{1+x^3}$, i.e.,

$1 \le \sqrt{1+x^3}$, and, $\sqrt{1+x^3} \le 1+x^3$ $\forall\ x \ge 0$,

since $\sqrt{*} \le *$ whenever $* \ge 1$, and $1+x^3 \ge 1$, mos' def!

So, $1 \le \sqrt{1+x^3} \le 1+x^3$. $\blacksquare$

Now, $\int_a^x \sim$ is increasing, so it preserves the sense of the inequalities! As long as what's inside is ≥ 0, which $1+x^3$, 1, $\sqrt{1+x^3}$ all are.

$$\int_0^1 1\,dx \le \int_0^1 \sqrt{1+x^3}\,dx \le \int_0^1 (1+x^3)\,dx$$

$$1 \le \int_0^1 \sqrt{1+x^3}\,dx \le 1.25 \;! \quad \blacksquare$$