FTC I

f cnt$^{\underline{s}}$ on $[a,b]$, Then

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Derivative of the integral is the integrand.

$$\int \text{integrand}\, dt$$

$$g(x) = \int_0^x \frac{t^2 \sin(t)}{t^4+1}\,dt \Rightarrow g'(x) = \frac{x^2 \sin(x)}{x^4+1}$$

Chain Rule Version of FTC I

$$g(u(x)) = \int_a^{u(x)} \frac{t^2 \sin(t)}{t^4+1}\,dt \Rightarrow$$

$$\frac{dg}{dx} = \frac{dg}{du}\cdot\frac{du}{dx} = \frac{(u(x))^2 \sin(u(x))}{(u(x))^4+1}\cdot u'(x)$$

$$\frac{d}{dx}\int_0^{x^2-x} \frac{t^2 \sin(t)}{t^4+1}\,dt = \left(\frac{(x^2-x)^2 \sin(x^2-x)}{(x^2-x)^4+1}\right)(2x-1)$$

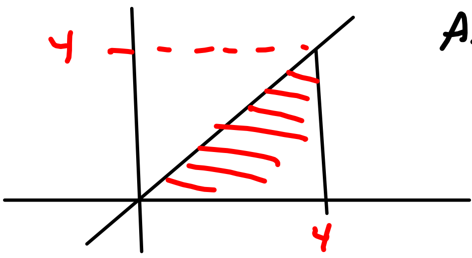
FTC II

f cnt^s on $[a,b]$, F any antiderivative of f. Then

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

(Always pick F, so that $C = 0$)

$$\int_0^4 x\,dx = \frac{1}{2}x^2\Big]_0^4 = \frac{1}{2}\left[4^2 - 0^2\right] = \frac{1}{2}[16] = 8$$

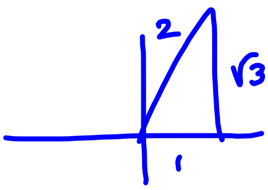


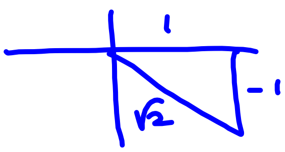
Area $= \frac{1}{2}bh = \frac{1}{2}(4)(4) = 8$ ✓

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \sec(t)\tan(t)\,dt = \sec(t)\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{3}} = \sec\left(\frac{\pi}{3}\right) - \sec\left(-\frac{\pi}{4}\right)$$

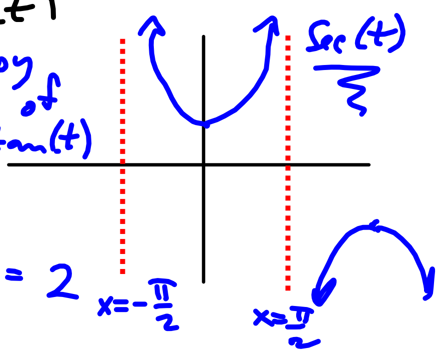
$$= \boxed{2 - \sqrt{2}}$$

Since $\frac{d}{dt}[\sec(t)] = \sec(t)\tan(t)$





Crappy graph of $\sec(t)\tan(t)$



$$\sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\sec\left(-\frac{\pi}{4}\right) = \sqrt{2}$$

6. **Question Details** SCalc8 4.2.022.

Use the form of the definition of the integral given in the theorem to evaluate the integral.

$$\int_1^5 (x^2 - 4x + 6)\, dx$$

Needful facts

$$\sum_{k=1}^{n} 1 = n, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{n^2 + \cdots}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + \cdots}{6}$$

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$$

$$x_k = a + k\Delta x = 1 + k\left(\frac{4}{n}\right)$$

$$= 1 + \frac{4k}{n}$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} (x_k^2 - 4x_k + 6)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \frac{4}{n}\sum_{k=1}^{n}\left(\left(1+\frac{4k}{n}\right)^2 - 4\left(1+\frac{4k}{n}\right) + 6\right)$$

$$2\left(\frac{4k}{n}\right)(1) = \frac{8k}{n}$$

$$= \frac{4}{n}\sum_{k=1}^{n}\left(1 + \frac{8k}{n} + \frac{16k^2}{n^2} - 4 - \frac{16k}{n} + 6\right)$$

$$= \frac{4}{n}\sum_{k=1}^{n}\left(3 - \frac{8k}{n} + \frac{16k^2}{n^2}\right)$$

$$= \frac{4}{n}\left[\sum 3 - \sum \frac{8k}{n} + \sum \frac{16k^2}{n^2}\right]$$

$$= \frac{4}{n}\left[3\sum 1 - \frac{8}{n}\sum k + \frac{16}{n^2}\sum k^2\right]$$

$$= \frac{4}{n}\left[3n - \frac{8}{n}\left(\frac{n^2+n}{2}\right) + \frac{16}{n^2}\left(\frac{2n^3+\cdots}{6}\right)\right]$$

$$= 12 - \left(\frac{\cancel{4}^{2}(8)}{\cancel{n^2}}\right)\left(\frac{\cancel{n^2}+\cdots}{\cancel{2}_{1}}\right) + \frac{\cancel{4}^{2}(16)}{\cancel{n^3}}\left(\frac{2\cancel{n^3}+\cdots}{\cancel{6}_{3}}\right)$$

$$\xrightarrow{n\to\infty} 12 - 16 + \frac{4(16)}{3}$$

$$= -4 + \frac{64}{3} =$$

Check with FTC II:

$$\int_1^5 (x^2-4x+6)\,dx = \left[\frac{x^3}{3} - \frac{4x^2}{2} + 6x\right]_1^5$$

$$= \left[\frac{1}{3}x^3 - 2x^2 + 6x\right]_1^5 = \frac{1}{3}(125) - 2(25) + 6(5)$$
$$- \left[\frac{1}{3} - 2 + 6\right]$$

$$= \frac{125}{3} - 50 + 30 - \frac{1}{3} + 2 - 6$$

$$= \frac{125}{3} - \frac{1}{3} - 20 - 4 = \frac{124}{3} - 60 = {}^{124}$$

$x^2 + \sqrt{1+2x}$ $\quad [4, 6]$

$\frac{b-a}{n} = \frac{6-4}{n} = \frac{2}{n}$

$x_k = a + \frac{2k}{n} = 4 + \frac{2k}{n}$

$\sum_{k=1}^{n} \left[\left(\left(4 + \frac{2k}{n}\right)^2 + \sqrt{1 + 2\left(4 + \frac{2k}{n}\right)} \right) \left(\frac{2}{n}\right) \right]$