Test 1 Re-Take Solutions

$f(3)=21$

$(3,$

$=(x_1, y_1)$

(1) $m_{sec} = \dfrac{f(3.001)-f(3)}{3.001-3} = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{3(3.001)^2-2(3.001)-21}{.001} = 16.003$

$(x_2, y_2) = (3.001, f(3.001))$

$= (3.001, 21.016003)$

(2) $m = 16$ is my guess

(3) $y = 16(x-3)+21$

(4) (a)

$$\begin{array}{r|rrr} 5 & 5 & -13 & -10 \\ & & 25 & 60 \\ \hline & 5 & 12 & 50 \end{array}$$

$50 \neq 0$, so no factor of $x-5$ in the numerator

$\therefore \lim_{x\to 5^+} \dfrac{5x^2-13x-10}{x-5}$ $\nexists$

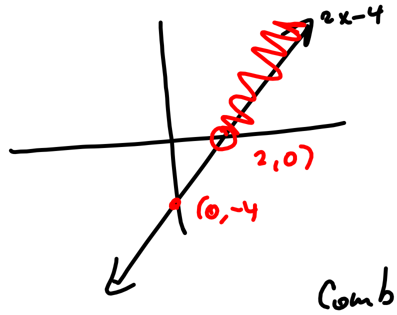
(b) $\lim_{x\to 5^-} \dfrac{5x^2-13x-10}{-(x-5)}$ $\nexists$

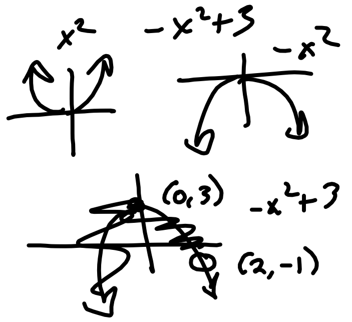
(c) $\lim_{x\to 5}$ —— $\nexists$

(5) $f(x) = \begin{cases} 2x-4 & \text{if } x \le 2 \\ -x^2+3 & \text{if } x > 2 \end{cases}$

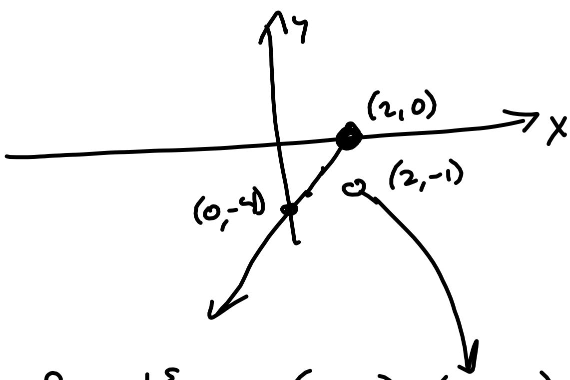
$2(2)-4=0$ ●

$-(2)^2+3=-1$ ○





Combine



Bonus f cnt^s on $(-\infty,2)\cup(2,\infty)$

$x=2$ is only prob.

(6) $7x^2-2x-3 = f(x) \Longrightarrow$

$$\frac{f(x+h)-f(x)}{h} = \frac{7(x+h)^2-2(x+h)-3-(7x^2-2x-3)}{h}$$

$$= \frac{7(x^2+2xh+h^2)-2x-2h-3-7x^2+2x+3}{h}$$

$$= \frac{7x^2+14xh+7h^2-2x-2h-3-7x^2+2x+3}{h}$$

$$= \frac{14xh+7h^2-2h}{h} = \frac{\cancel{h}(14x+7h-2)}{\cancel{h}}$$

$$= 14x+7h-2 \xrightarrow{h\to 0} \boxed{14x-2 = f'(x)}$$

(if $h \neq 0$)

(b) $f(x)=\sqrt{x} \Longrightarrow \dfrac{f(x+h)-f(x)}{h}$

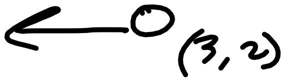
$$= \left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right) = \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \frac{x+h-x}{h(\sim\sim)} = \frac{h}{h(\sim\sim)} = \frac{1}{\sqrt{x+h}+\sqrt{x}} \xrightarrow{h\to 0} \frac{1}{\sqrt{x}+\sqrt{x}} =$$

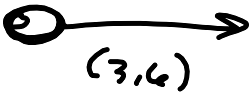
$$\sim\sim = \sqrt{x+h}+\sqrt{x}$$

$$= \boxed{\frac{1}{2\sqrt{x}} = f'(x)}$$

$\lim_{x\to -3^-} f(x) = 2$

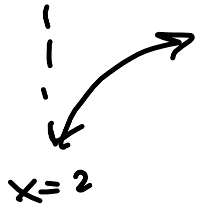


$\lim_{x\to -3^+} f(x) = 6$



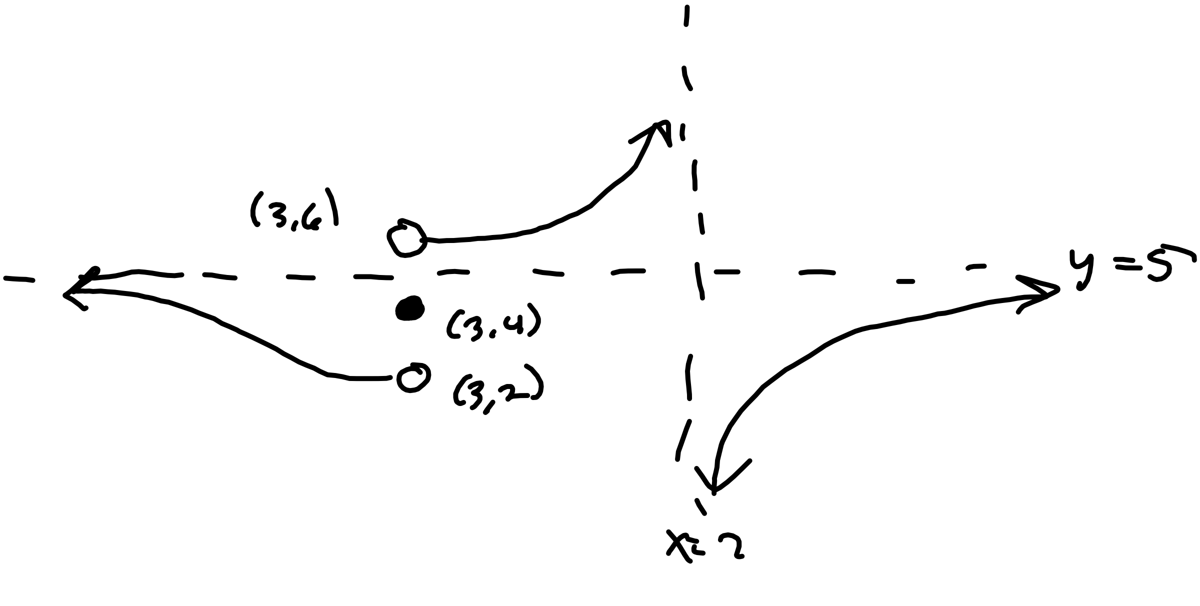
$\lim_{x\to 2^-} f(x) = \infty$

$\lim_{x\to 2^+} f(x) = -\infty$



$\lim_{|x|\to\infty} f(x) = 5$

$f(-3) = 4$



Claim $\lim_{x \to 2} (7x+2) = 16$

Proof Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{7}$.

Then $0 < |x-2| < \delta \implies |7x+2-16| = |7x-14|$

$= 7|x-2| < 7\delta = 7 \cdot \frac{\varepsilon}{7} = \varepsilon$ ∎

(9) $\cos\left(\frac{\pi}{6}x\right) = x - 1$ some where in $(0, 3)$

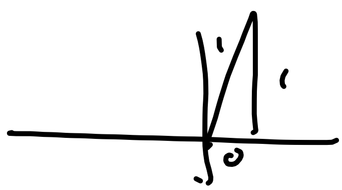
Proof Define $f(x) = \cos\left(\frac{\pi}{6}x\right) - x + 1$

Then $f(0) = \cos(0) - 0 + 1 = 2 > 0$

$f(3) = \cos\left(\frac{\pi}{6}\cdot 3\right) - 3 + 1 < 0$

$= \cos\left(\frac{\pi}{2}\right) - 2$

$= -2 < 0$



$\frac{\pi}{2}$

So f is cont &

$f(0) = 2 > 0$

$f(3) = -2 < 0$

$\Rightarrow \exists\, x \in (0, 3)$

$\ni f(x) = 0$.