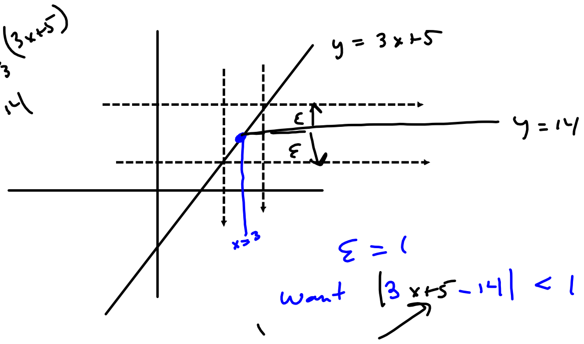
$\lim_{x\to 3}(3x+5) = 14$



$\varepsilon = 1$

want $|3x+5-14| < 1$

$\Rightarrow |3x-9| = 3|x-3| < 1$

$3|x-3| < 1$

$|x-3| < \frac{1}{3}$

Define $\delta = \frac{1}{3}$

Proof $\lim_{x\to 3}(3x+5) = 14$ $\quad \varepsilon =$ ANY, $\delta = \frac{\varepsilon}{3}$

Let $\varepsilon > 0$. Then define $\delta = \frac{\varepsilon}{3}$. Now, if $0 < |x-3| < \delta$, then

$|3x+5-14| = |3x-9| = 3|x-3| < 3\delta = 3\cdot\frac{\varepsilon}{3}$

$= \varepsilon$ ■

Next level of difficulty:

$\lim_{x\to 2}(x^2) = 4$

Scratch: want $|x^2-4| < \varepsilon$

$|x-2||x+2| < \varepsilon$

Need a handle on $|x+2|$

KEY: ASSUME $\delta < 1$

will be $< \delta$, ALWAYS

$x \to 2$, right?

So $\boxed{\delta \leq 1} \Rightarrow$

$1 < x < 3$

$1+2 < x+2 < 3+2$

$3 < x+2 < 5 \Rightarrow$

$|x+2| < 5$

Want

So, $|x-2||x+2| < |x-2|(5) = 5|x-2| < \varepsilon$

So $|x-2| < \frac{\varepsilon}{5}$

δ

DEFINE $\delta = \min\{1, \frac{\varepsilon}{5}\}$

etc.

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{5}\}$.

Then $0 < |x-2| < \delta \Rightarrow$

$|x^2-4| = |x-2||x+2| \leq 5|x-2| < 5\delta \leq 5\cdot\frac{\varepsilon}{5}$

$= \varepsilon$ ■ Q.E.D.

■