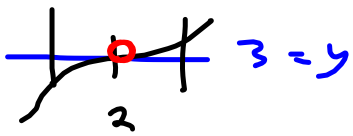
SCalc8 1.5.001. (3354117) (Remove) -- view 2m

Explain what is meant by the equation

$$\lim_{x \to 2} f(x) = 3.$$

Is it possible for this statement to be true and yet $f(2) = 4$? Explain.



This says I can make f(x) as close to y = 3 as I desire, by taking x sufficiently close to 2.

$| f(x) - 3 | <$ small by making $| x - 2 | <$ some other sufficiently small.

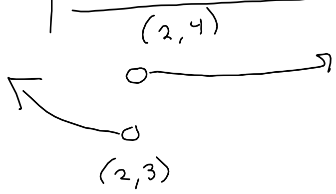
Silent on "f(2)"

SCalc8 1.5.002. (3354190) (Remove) -- view

Explain what it means to say that

$$\lim_{x \to 2^-} f(x) = 3 \quad \text{and} \quad \lim_{x \to 2^+} f(x) = 4.$$

In this situation is it possible that $\lim_{x \to 2} f(x)$ exists? Explain.



No. For the (2-sided) limit to exist, we need the left and right limits to agree!

Jump Discontinuity

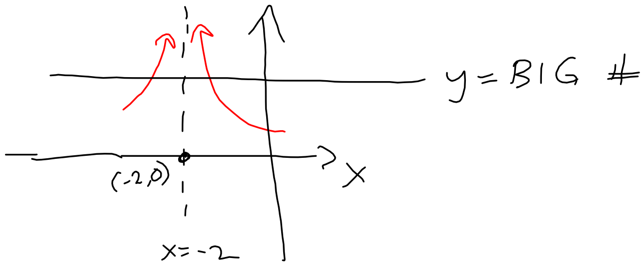
$$\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$$

SCalc8 1.5.003. (3413006) (Add) -- view

Explain the meaning of each of the following.

(a) $\lim_{x \to -2} f(x) = \infty$

(b) $\lim_{x \to 3^+} f(x) = -\infty$

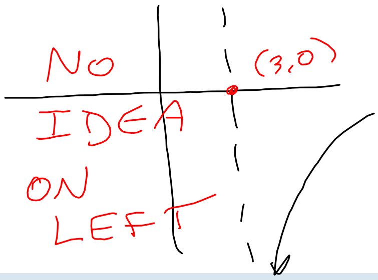


(a) $\lim_{x \to -2} f(x) = \infty$ means that I can make f(x) arbitrarily large by taking x arbitrarily close to -2. Give me a big positive number. I can take x close enough to x = -2 to make f(x) BIGGER than your positive number AND REMAIN bigger, anywhere closer to x = -2.

(b) $\lim_{x \to 3^+} f(x) = -\infty$ means that I can make f(x) come in BELOW any negative number by taking x sufficiently close to x = 3, coming in from the right.

Negative # with BIG absolute value.

(Large negative number).



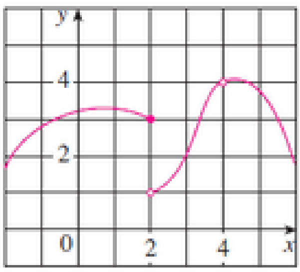
x = 3

SCalc8 1.5.004. (3354294) (Add) -- view

Comment: slightly modified, not randomized

3m

Use the given graph of f to state the value of each quantity, if it exists. (If an answer does not exist, enter DNE.)



(a) $\lim_{x \to 2^-} f(x) = 3$

(b) $\lim_{x \to 2^+} f(x) = 1$

(c) $\lim_{x \to 2} f(x)$ $\nexists$

(d) $f(2) = 3$

(e) $\lim_{x \to 4} f(x) = 4$

(f) $f(4)$ $\nexists$

$$\lim_{x \to 2^+} \frac{1}{x-2} = \infty$$

Give me a big number

$y = 1000$

I can make

$\left|\frac{1}{x-2}\right| > 1000$, easily

Want $\frac{1}{|x-2|} > 1000$

$$1 > |x-2|\,1000$$

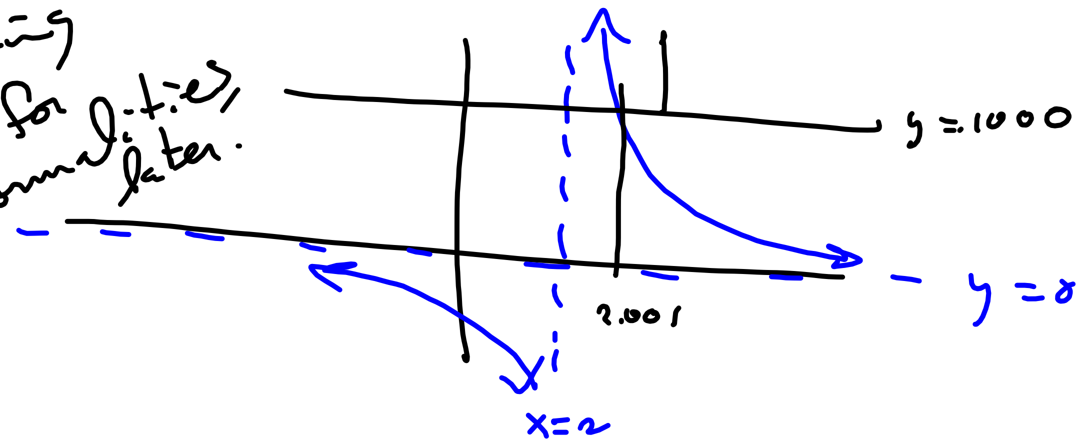
$$1000\,|x-2| < 1$$

$$|x-2| < \frac{1}{1000}$$

$$-\frac{1}{1000} < x-2 < \frac{1}{1000}$$

$$1.999 < x < 2.001$$

Planting seeds for the formalities later.



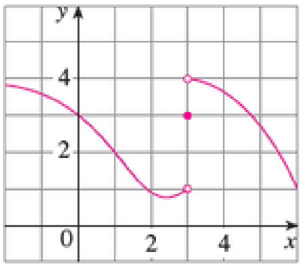
$M = 1000$

Let $2 < x < 2 + \frac{1}{M}$

SCalc8 1.5.005. (3354534) (Add) -- view

Comment: slightly modified, not randomized

For the function f whose graph is given, state the value of each quantity, if it exists. (If an answer does not exist, enter DNE.)



(a) $\lim_{x \to 1} f(x)$ $= 2$

(b) $\lim_{x \to 3^-} f(x)$ $= 1$

(c) $\lim_{x \to 3^+} f(x)$ $= 4$

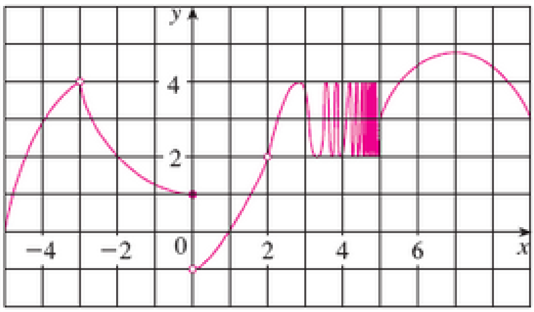
(d) $\lim_{x \to 3} f(x)$ $\nexists$ (Jump Discontinuity.)

(e) $f(3)$ $= 3$

SCalc8 1.5.006. (3354147) (Add) -- view

Comment: not randomized

For the function h whose graph is given, state the value of each quantity, if it exists. (If an answer does not exist, enter DNE.)



(a) $\lim_{x \to -3^-} h(x)$

(b) $\lim_{x \to -3^+} h(x)$

(c) $\lim_{x \to -3} h(x)$

(d) $h(-3)$

(e) $\lim_{x \to 0^-} h(x)$

(f) $\lim_{x \to 0^+} h(x)$

(g) $\lim_{x \to 0} h(x)$

(h) $h(0)$

(i) $\lim_{x \to 2} h(x)$

(j) $h(2)$

(k) $\lim_{x \to 5^+} h(x)$

(l) $\lim_{x \to 5^-} h(x)$

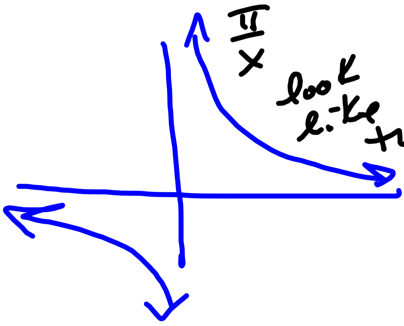
Topologist's Sine Curve.

$f(x) = \sin\left(\frac{1}{x}\right)$

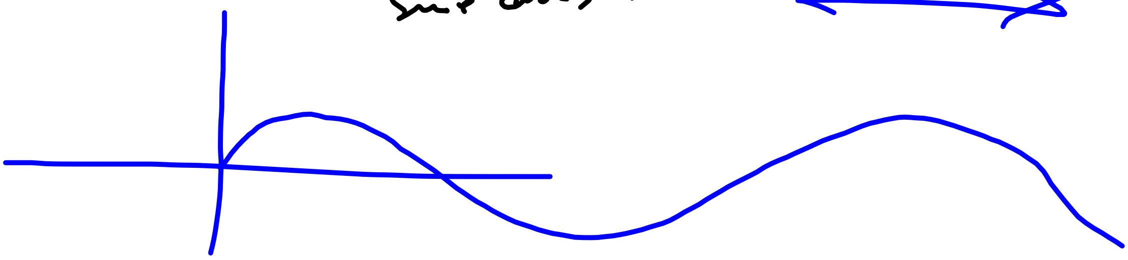
OR

$f(x) = \sin\left(\frac{\pi}{x}\right)$

$\frac{1}{x}$ OR $\frac{\pi}{x}$ look like this



sin does this



$\sin\left(\frac{1}{x}\right)$

The whole graph of sin x, from 1 to ∞ is packed into $(0,1]$!