Suppose the graph of f is given. Write equations for the graphs that are obtained from the graph of f as follows.

(a) Shift 3 units upward.

(b) Shift 3 units downward.

(c) Shift 3 units to the right.

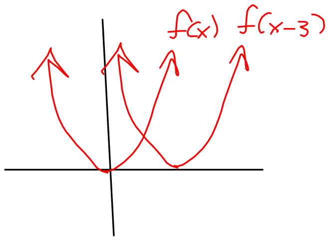
(d) Shift 3 units to the left.

(e) Reflect about the x-axis.

(f) Reflect about the y-axis.

(g) Stretch vertically by a factor of 3.

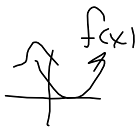
(h) Shrink vertically by a factor of 3.

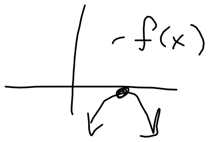


(a) $f(x)+3$ (b) $f(x-3)$

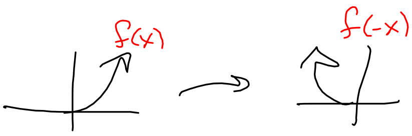
(c) $f(x)-3$ (d) $f(x+3)$

(e) reflect about the x-axis : $-f(x)$





(f) $f(-x)$



$f(-x)$ is reflection about the y-axis

(g) $3f(x)$ vertical stretch (h) $\frac{1}{3}f(x)$ shrink by factor of 3.

2. Explain how each graph is obtained from the graph of $y = f(x)$.

(a) $y = f(x) + 8$ (b) $y = f(x + 8)$

(c) $y = 8f(x)$ (d) $y = f(8x)$

(e) $y = -f(x) - 1$ (f) $y = 8f(\frac{1}{8}x)$

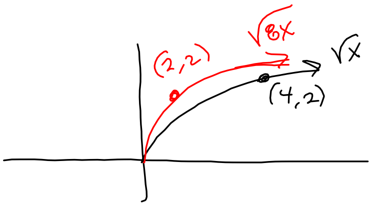
(2) $y = f(x) \longrightarrow$ ...

... (a) $f(x)+8$ is 8 up

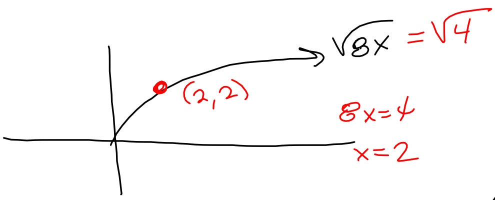
... (b) $f(x+8)$ is 8 left

... (c) $8f(x)$ is stretch by factor of 8 / Vertical

... d



... (d) $f(8x)$ is horizontal shrink by factor of 8.



$8x = 4$

$x = 2$

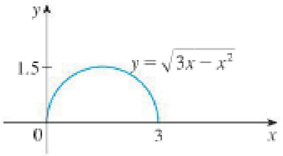
(e) $y = -f(x) - 1$

vertical flip 1st.

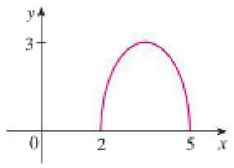
Then down 1.

(8) $8f(\frac{1}{8}x)$ is Horiz & vert. STRETCH by a factor of 8.

6–7 The graph of $y = \sqrt{3x - x^2}$ is given. Use transformations to create a function whose graph is as shown.



6.



$\boxed{2f(x-3)}$

① Vertical stretch, factor of 2

② Right 3

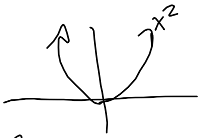
Put vertical stretch BEFORE vertical shift.

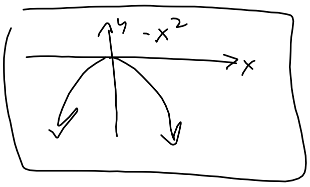
9–24 Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions given in Section 1.2, and then applying the appropriate transformations.

9. $y = -x^2$

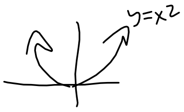
$= -f(x)$

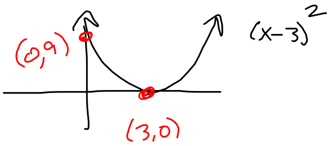
for $f(x) = x^2$



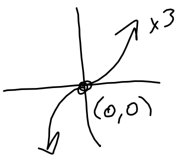


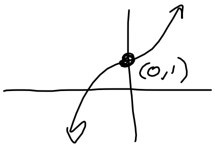
10. $y = (x-3)^2$ $= f(x-3)$ for $f(x) = x^2$



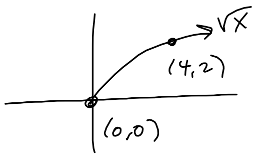


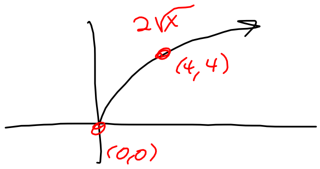
11. $y = x^3 + 1$ $= f(x) + 1$ for $f(x) = x^3$

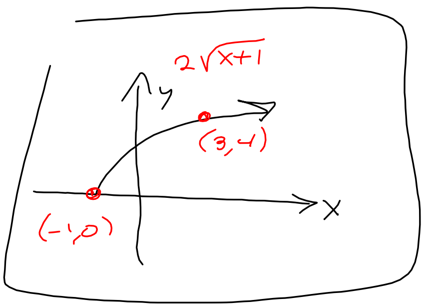




14. $y = 2\sqrt{x+1}$ $= 2f(x+1)$ for $f(x) = \sqrt{x}$







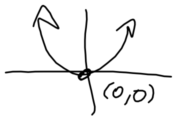
15. $y = x^2 - 4x + 5$ Complete the square!

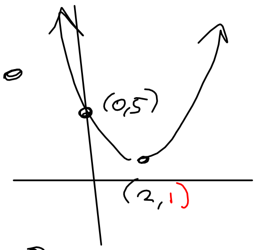
$= x^2 - 4x + 2^2 - 4 + 5$

$\frac{4}{2} = 2 \to 2^2$

$= (x-2)^2 + 1$

Right 2 up 1

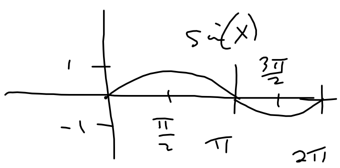


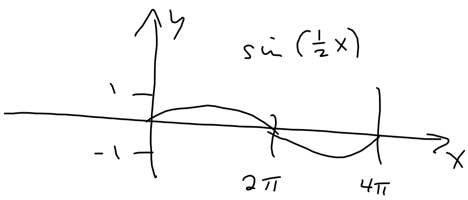


$(x-2)^2 + 1 = 0$

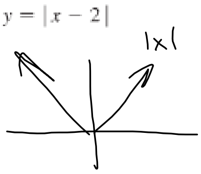
$(x-2)^2 =$

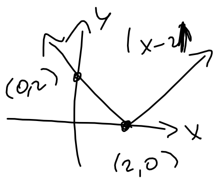
19. $y = \sin(\frac{1}{2}x)$





21. $y = |x - 2|$





25. The city of New Orleans is located at latitude 30°N. Use Figure 9 to find a function that models the number of hours of daylight at New Orleans as a function of the time of year. To check the accuracy of your model, use the fact that on March 31 the sun rises at 5:51 AM and sets at 6:18 PM in New Orleans.

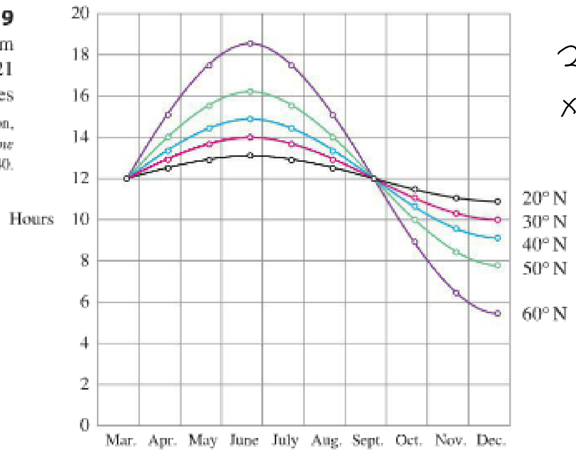
midline : $y = 12$

High : $y = 14 \Rightarrow$ Amp is 2

Period : 365 days

FIGURE 9
Graph of the length of daylight from March 21 through December 21 at various latitudes

Source: Adapted from L. Harrison, *Daylight, Twilight, Darkness and Time* (New York: Silver, Burdett, 1935), 40.



March 31^{st} is Day $90 = x$

$2\cos\left(\frac{2\pi}{365}(x - 151)\right) + 12$

$x =$ # days into the year.

J	31
F	28
M	31
A	30
M	31
J	

$bx = 2\pi$

when $x = 365$

$b = \frac{2\pi}{365}$

12.995026577814361230854305496912935073974377960270772347 45...

5:51 rise

$$\begin{array}{r} 6{:}18 \text{ set} \\ -\,5{:}51 \\ \hline :27 \end{array}$$

12 hrs 27 min.

So < 12.5, when the model predicts > 12.5, almost 13 hrs.

26. A variable star is one whose brightness alternately increases and decreases. For the most visible variable star, Delta Cephei, the time between periods of maximum brightness is 5.4 days, the average brightness (or magnitude) of the star is 4.0, and its brightness varies by ±0.35 magnitude. Find a function that models the brightness of Delta Cephei as a function of time.

#5.

31–32 Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g and state their domains.

31. $f(x) = x^3 + 2x^2$, $g(x) = 3x^2 - 1$

Polynomials!

(a) $(f+g)(x) = f(x)+g(x) = x^3+2x^2+(3x^2-1)$
$= \boxed{x^3+5x^2-1 = f+g}$ & $\boxed{\mathcal{D} = \mathbb{R}}$

(b) $(f-g)(x) = f(x)-g(x) = x^3+2x^2-(3x^2-1) = \boxed{x^3-x^2+1 = f-g}$
$\boxed{\mathcal{D} = \mathbb{R}}$

juxtaposition

(c) $(fg)(x) = f(x)g(x) = (x^3+2x^2)(3x^2-1)$ & $\boxed{3x^5-3x^3+6x^4-2x^2 = fg \quad \mathcal{D} = \mathbb{R}}$

$\mathcal{D}(f+g) = \mathcal{D}(f-g) = \mathcal{D}(fg)$
$= \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)\}$
$= \mathcal{D}(f) \cap \mathcal{D}(g)$ = overlap/intersection of $\mathcal{D}(g)$ & $\mathcal{D}(f)$

$\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \underline{\text{ and }} g(x) \neq 0\}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^3+2x^2}{3x^2-1}$

$\mathcal{D}(f) = \mathcal{D}(g) = \mathbb{R}$. Need $3x^2-1 \neq 0$

$3x^2-1=0$
$3x^2=1$
$x^2=\frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ is bad

So, $\mathcal{D}\left(\frac{f}{g}\right) = \mathbb{R} \setminus \{\pm\frac{1}{\sqrt{3}}\}$ OR $\mathbb{R} \setminus \{\pm\frac{\sqrt{3}}{3}\}$
$= (-\infty, -\frac{\sqrt{3}}{3}) \cup (-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$
$= \{x \mid x \in \mathbb{R} \text{ and } x \neq \pm\frac{\sqrt{3}}{3}\}$

32. $f(x) = \sqrt{3 - x}$, $g(x) = \sqrt{x^2 - 1}$ $\Rightarrow$

(a) $(f+g)(x) = f(x) + g(x) = \sqrt{3-x} + \sqrt{x^2-1}$

$\mathcal{D}(f)$: Need $3 - x \geq 0$

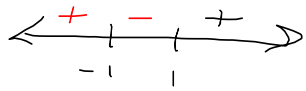
$-x \geq -3$

$\frac{-x}{-1} \leq \frac{-3}{-1}$

$x \leq 3$

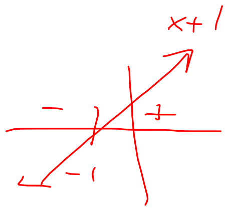
$\mathcal{D}(g)$: Need $x^2 - 1 \geq 0$

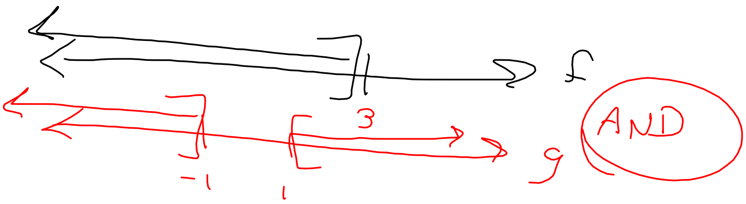
$(x-1)(x+1) \geq 0$

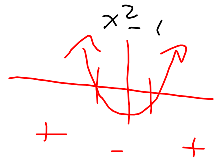


$\mathcal{D}(g) =$ Y Y Y N Y Y

$(-\infty, -1] \cup [1, \infty)$







$(-\infty, -1] \cup [1, 3] = \mathcal{D}(f+g) = \mathcal{D}(f-g) = \mathcal{D}(fg)$

(b) $\boxed{f - g = \sqrt{3-x} - \sqrt{x^2-1}}$

(c) $(fg) = \sqrt{3-x}\sqrt{x^2-1} = \sqrt{(3-x)(x^2-1)}$

(d) $\frac{f}{g} = \frac{\sqrt{3-x}}{\sqrt{x^2-1}}$ $\mathcal{D}\left(\frac{f}{g}\right)$ also need to exclude where $g = 0$

$x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$ to exclude

$\boxed{(-\infty, -1) \cup (1, \infty) = \mathcal{D}\left(\frac{f}{g}\right)}$

33–38 Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$ and their domains.

33. $f(x) = 3x + 5$, $g(x) = x^2 + x$

f composed with g.
feed g to f.

(a) $(f \circ g)(x) = f(g(x)) = f(x^2+x) = 3(x^2+x)+5$
$= 3x^2+3x+5 = f \circ g$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$
$= \mathbb{R}$, b/c $\mathcal{D}(f) = \mathcal{D}(g) = \mathbb{R}$.

(b) $(g \circ f)(x) = g(f(x)) = (f(x))^2 + f(x) = \boxed{(3x+5)^2 + 3x+5}$ Unsimplified Ans.
$= 9x^2+30x+25+3x+5$
$= 9x^2+33x+30$

Test! often I'll say "Don't Simplify."

$\boxed{\mathcal{D} = \mathbb{R}.}$

(c) $(f \circ f)(x) = f(f(x)) = 3f(x)+5$
$= 3(3x+5)+5$
$= 9x+15+5$
$\boxed{= 9x+20 = f \circ f}$ $\boxed{\mathcal{D} = \mathbb{R}.}$

(d) $(g \circ g)(x) = g(g(x)) = g^2 + g = (x^2+x)^2 + (x^2+x)$
$= x^4+2x^3+x^2+x^2+x$
$\boxed{= x^4+2x^3+2x^2+x}$ $\boxed{\mathcal{D} = \mathbb{R}}$

37. $f(x) = x + \frac{1}{x}$, $g(x) = \frac{x+1}{x+2}$ Need $x+2 \neq 0$, so $x \neq -2$, so $\mathcal{D} = \mathbb{R} \setminus \{-2\}$

$\mathcal{D}(f) = \mathbb{R} \setminus \{0\}$

(a) $f \circ g = \left(g(x) + \frac{1}{g(x)}\right) = \frac{x+1}{x+2} + \frac{1}{\frac{x+1}{x+2}}$

$= \frac{x+1}{x+2} + \frac{x+2}{x+1} = \frac{(x+1)^2 + (x+2)^2}{(x+1)(x+2)} = \frac{x^2+2x+1 + x^2+4x+4}{\text{LCD}}$

$= \frac{2x^2+6x+5}{(x+1)(x+2)}$ Looks like $\mathbb{R} \setminus \{-1,-2\}$

FORMALLY:

$\{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\} = \{x \mid x \neq -2 \text{ and } \frac{x+1}{x+2} \neq 0\}$

$= \{x \mid x \neq -2 \text{ and } x \neq -1\} = \mathbb{R} \setminus \{-1,-2\}$

$\frac{x+1}{x+2} = 0 \Rightarrow x+1 = 0 \Rightarrow x = -1$

(b) $g \circ f = g(f(x)) = \frac{f(x)+1}{f(x)+2} = \frac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2} = \frac{\frac{x^2+1+x}{x}}{\frac{x^2+1+2x}{x}}$

$= \frac{x^2+x+1}{x^2+2x+1}$ $\mathcal{D} = \mathbb{R} \setminus \{-1\}$ by inspection,

$\mathcal{D} = \{x \mid x \in \mathcal{D}(f) \text{ \& } f(x) \in \mathcal{D}(g)\}$

$\{x \mid x \neq 0 \text{ \& } x + \frac{1}{x} \neq -2\}$

$= \{x \mid x \neq 0 \text{ \& } x \neq -\frac{5}{3}\}$

$= \boxed{\mathbb{R} \setminus \{-\frac{5}{3}, 0\}}$

$\frac{x+1}{x+2} = -2$

$\frac{x+1}{x+2} = \frac{-2(x+2)}{x+2} = \frac{-2x-4}{x+2}$

$x+1 = -2x-4$

$3x = -5$

$x = -\frac{5}{3}$ is also bad

38. $f(x) = \frac{x}{1+x}$, $g(x) = \sin 2x$

$f \circ g = f(g(x))$

$\mathcal{D}(f) = \mathbb{R} \setminus \{-1\}$

$\boxed{f \circ g,\ g \circ f}$, $f \circ f$, $g \circ g$

$\mathcal{D}(g) = \mathbb{R}$

$$\boxed{\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}}$$

$= \{x \mid x \in \mathbb{R}$ and $\sin(2x) \neq -1$

No restrict

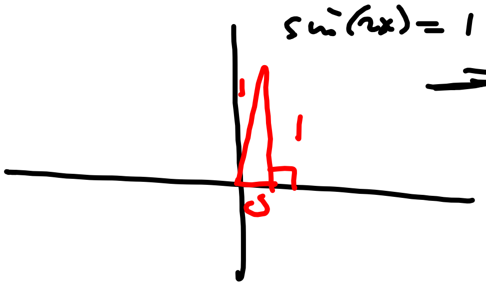
The work.

$$(f \circ g)(x) = f(g(x)) = f(\sin(2x))$$

$$= \frac{\sin(2x)}{1 + \sin(2x)}$$

Need $\sin(2x) \neq -1$

$\sin(2x) = 1$



$\Rightarrow \quad 2x = 90^\circ = \frac{\pi}{2}$

$2x = 90^\circ + 360^\circ n$

$x = 45^\circ + 180^\circ n$

$$\mathcal{D} = \mathbb{R} \setminus \{45^\circ + 180^\circ n \mid n \in \mathbb{Z}\}$$

$$= \mathbb{R} \setminus \{\tfrac{\pi}{4} + n\pi \mid n \in \mathbb{Z}\}$$

39–42 Find $f \circ g \circ h$.

39. $f(x) = 3x - 2$, $g(x) = \sin x$, $h(x) = x^2$

$\mathcal{D}$ = Domain = $\{x \mid f(x)$ can eat$\}$
$= \mathcal{D}(f)$

$\mathbb{R} = (-\infty, \infty) = \{x \mid x$ is real$\}$

$\exists$ - "There is" or "There exists"

$\ni$ - "such that" or "so that"

$\forall$ - "for all," "for each," or "for every"

$A \Rightarrow B$ - "A implies B"

$A \Leftrightarrow B$ - "A implies B and B implies A."

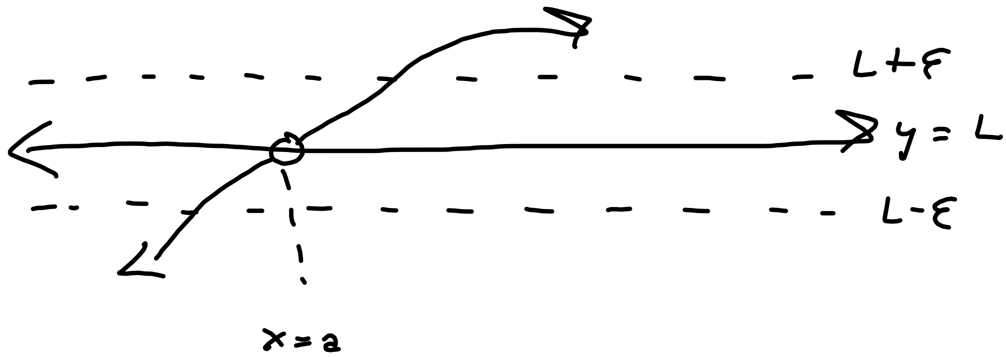
A iff B - Same deal. "A holds if and only if B holds"

I can reason forward & back, so iff.

$\lim_{x \to a} f(x) = L$, means.

$\forall\ \varepsilon > 0, \exists\ \delta > 0 \ni |x - a| < \delta \Rightarrow$
$|f(x) - L| < \varepsilon.$

For any ε greater than zero, I can find a δ " " zero, such that if x is within δ units of a, then $f(x)$ is within ε units of L.



$$\frac{f(x+h) - f(x)}{h}$$

$3x + 2 = 5$

$\iff 3x = 3$

I ♡ u $\forall\, t \in \mathbb{R}$

43–48 Express the function in the form $f \circ g$.

43. $F(x) = (2x + x^2)^4$

45. $F(x) = \dfrac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$

46. $G(x) = \sqrt[3]{\dfrac{x}{1 + x}}$

49–51 Express the function in the form $f \circ g \circ h$.

50. $H(x) = \sqrt[8]{2 + |x|}$

52. Use the table to evaluate each expression.

(a) $f(g(1))$ (b) $g(f(1))$ (c) $f(f(1))$

(d) $g(g(1))$ (e) $(g \circ f)(3)$ (f) $(f \circ g)(6)$

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

53. Use the given graphs of f and g to evaluate each expression, or explain why it is undefined.

(a) $f(g(2))$ (b) $g(f(0))$ (c) $(f \circ g)(0)$