MAT 201 Final Test – Spring, 2015 Name_____

200 Points Comprehensive

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Show all work. Do your own work. Submit problems in the proper order. Spread your work out! If you get stuck, **start a fresh piece of paper**. You can always *insert* more pages if you do it this way. Only your *name* should be on this cover sheet. Test is 1 hour, 50 minutes. Start a 12:10. End at 2:00.

1. Let $f(x)=2x^2-3$. Find $\frac{df}{dx}$ in two ways:
   a. (10 pts) the limit definition. b. (5 pts) the easy way.
2. Let $f(x)=2x^2-3$.
   a. (5 pts) Find an equation of the tangent line to f at $x = 2$.
   b. (5 pts) Sketch a graph of f and the tangent line you obtained in part a.
   c. (5 pts) Use your tangent line to approximate $f(2.5)$.
3. Evaluate the following limits.
   a. (5 pts) $\lim_{x\to 3}\left(\frac{2x^2-11x+15}{3x^2-7x-6}\right)$ b. (5 pts) $\lim_{x\to \infty}\left(\frac{2x^2-11x+15}{3x^2-7x-6}\right)$ c. (5 pts) $\lim_{x\to 3^-}\left(\frac{|x-3|}{x^2-x-6}\right)$
4. (5 pts) Prove that $\lim_{x\to 3}(2x-5)=1$.
5. (5 pts) Convince me – without *solving* – that $f(x)=x^3-x^2-16x+16$ has a zero in the interval $(0,2)$. I suggest use of a major theorem.
6. Sketch the graph of $f(x)=x^3-x^2-16x+16$, showing all extremes and inflection points. Be smart about the time spent on calculations (a lot) versus points available for doing so (very little).
   a. (5 pts) *x*-values corresponding to max/min. (Corresponding *y*-value: 0 points)
   b. (5 pts) *x*-values corresponding to inflection points. (Corresponding *y*-value: 0 points)
   c. (5 pts) Sign pattern on $f'(x)$ and $f''(x)$.
   d. (5 pts) *x*-intercepts and *y*-intercept.
   e. (5 pts) Sketch, showing extremes, inflection point, and "shape" (concavity).
7. Find $\frac{dy}{dx}$:
   a. (5 pts) $y=-\frac{1}{\sqrt[5]{x^2}}+5x^2-4$
   b. (5 pts) $y=2x^3\cos(x^2-3)$

c. (5 pts) $x^2y^2 - 3xy - 3x = 2y^2 + xy$

d. (10 pts) $y = \int_0^{x^3} \frac{t^2 \sin(t)}{10 - \cos^2(t)}\,dt$

8. (10 pts) Use a differential to estimate the change in volume when a sphere's radius increases from 10 cm to 10.1 cm.

9. Let $f(x) = x^3 - x^2 - 16x + 16$, once again.

   a. (5 pts) Since it's a polynomial, it satisfies the hypotheses of the Mean Value Theorem on $[0, 2]$. What are the hypotheses of the Mean Value Theorem, again?

   b. (5 pts) Find all values c satisfying the conclusion of the theorem. That is, find c such that $f'(c) = m_{avg}$ on $[0, 2]$.

10. (10 points) This is a good place for the *other* Mean Value Theorem: Find all c in $(0,1)$ such that $g(c) = g_{avg}$ on $[0,1]$, for $g(x) = 3x^2 - 2x - 7$.

11. Let $h(x) = 2\sin(x)\cos(x) + x$.

   a. (5 pts) Find all values x, where $h'(x) = 0$ in $(0, 2\pi)$.

   b. (5 pts) Find all values x, where $h''(x) = 0$.

12. Evaluate the indefinite integrals:

   a. (10 pts) $\int \csc^2(x)dx$ b. (10 pts) $\int \frac{dx}{\left(\sqrt{x}+1\right)^3}$ c. (10 pts) $\int x\csc^2(4x^2)dx$

13. (5 pts) Write – but do not evaluate – the integral that gives the area bounded by $y = 2x$ and $y = 8 - x^2$.

14. Write – but do not evaluate – the integral that gives the volume of the solid of revolution obtained when the region bounded by $y = 16 - x^2, x = 0,$ and $y = 0$ is rotated around the x-axis in 2 ways:

   a. (10 points) Using the disc method.

   b. (10 points) Using the shell method.

Bonus. Answer ONE of the following for 10 points.

15. (10 points Bonus) Evaluate the integral: $\int_0^5 |x^2 - 16|\,dx$

16. (10 points Bonus) Prove that $\lim_{x\to 3}(x^2 - 3x - 10) = -10$

17. (10 pts) Prove that $\lim_{x\to 3}(x^2 - 3x - 10) = -10$