Test 4 Review

1. (20 pts) Evaluate $\int_1^4 \left(x^2 - 2\right) dx$, by the limit definition of the definite integral.

$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$. $\quad x_k = a + k\Delta x = 1 + k \cdot \frac{3}{n} = \frac{3k}{n} + 1$

where'd the -2 go?

missing the -2

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} (x_k^2 - 2)\frac{3}{n} = \frac{3}{n}\sum_{k=1}^{n}\left(\frac{3k}{n}+1\right)^2 = \frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2}{n^2} + \frac{6k}{n} + \cancel{+2}\right)$$

-1

$$\left(\frac{3k}{n}+1\right)^2 = \left(\frac{3k}{n}\right)^2 + 2\left(\frac{3k}{n}\right)(1) + 1^2 \qquad \left(\frac{3k}{n}\right)^2 = \frac{3^2k^2}{n^2}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \frac{3}{n}\left(\frac{9}{n^2}\sum_{k=1}^{n} k^2 + \frac{6}{n}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1\right) \quad -\sum 1$$

$$= \frac{3}{n}\left(\frac{9}{n^2}\cdot\frac{n^3 + \text{smaller}}{3} + \frac{6}{n}\cdot\frac{n^2+\text{smaller}}{2} - n\right)$$

$$= \frac{27}{3n^3}\left(n^3 + \text{smaller}\right) + \frac{18}{2n^2}\left(n^2 + \text{smaller}\right) - 3$$

$$\xrightarrow{n\to\infty} \frac{27}{3} + \frac{18}{2} + 3 = \frac{27}{3} + 9 - 3 = 9 + 9 - 3 = 15$$

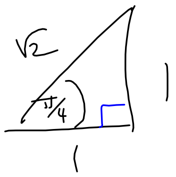
(10 pts) Evaluate $\int_0^{\frac{\pi}{4}} \left(\sec^2(x) - 2\right) dx$ using the Fundamental Theorem of Calculus.

$$= \Big[\tan x \quad - 2x\Big]_0^{\frac{\pi}{4}}$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$= \tan\left(\tfrac{\pi}{4}\right) - 2\left(\tfrac{\pi}{4}\right) - \Big(\tan(0) - 2(0)\Big)$$



$$= \boxed{1 - \frac{\pi}{2}}$$

(10 pts) Evaluate $\dfrac{d}{dx}\displaystyle\int_0^{\sin(x)} \left(\dfrac{\sec^2(t)+12t}{t^2-7}\right) dt$ by the Fundamental Theorem.

$$= \left(\frac{\sec^2(\sin x) + 12\sin(x)}{\sin^2(x) - 7}\right) \cdot \cos(x)$$

3. The velocity of a particle, in meters per second, is given by $f(t)=t^2-5t+6$, where t = time, in seconds.

   Give *exact* answers to the following.

   a. (10 pts) Find the net displacement of the particle, from time $t=0$ to time $t=3$.

   b. (10 pts) Find the total distance travelled, from time t = 0 to time t = 3.

(a) $\int_0^3 (t^2-5t+6)\,dt = \left[\frac13 t^3-\frac52 t^2+6t\right]_0^3$

$$= \frac13(3^3)-\frac52(3)^2+6(3)-\left(\frac13(0)^3-\frac52(0)^2+6(0)\right)$$

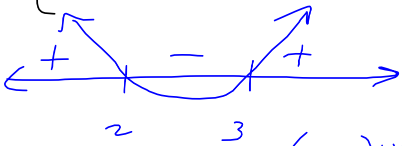
$$= 9-\frac{45}{2}+18 = 27-\frac{45}{2}=\frac{54}{2}-\frac{45}{2}=\boxed{\frac92}$$

$$\int_0^4 \cdots = \frac{16}{3}$$

(b) $\int_0^3 |t^2-5t+6|\,dt$

$$|t^2-5t+6| = \begin{cases} t^2-5t+6 & \text{if } t^2-5t+6\ge 0\\ -(t^2-5t+6) & \text{if } t^2-5t+6<0\end{cases}$$

$(t-3)(t-2)$



$$= \begin{cases} t^2-5t+6 & \text{if } x\le 2 \text{ or } x\ge 3 & (-\infty,2)\cup(3,\infty)\\ -(t^2-5t+6) & \text{if } 2<x<3 & (2,3)\end{cases}$$

$$= \int_0^2 (t^2-5t+6)\,dt - \int_2^3 (t^2-5t+6)\,dt$$

$$\int_0^4 |f| = \int_0^2 f - \int_2^3 f + \int_3^4 f$$

$$\left[\frac13t^3-\frac52t^2+6t\right]_0^2 - \left[\frac13t^3-\frac52t^2+6t\right]_2^3$$

$$=\left[\frac13(2)^3-\frac52(2)^2+6(2)-(0-0+0)\right]-\left[\frac13(3)^3-\frac52(3)^2+6(3)-\left(\frac13(2)^3-\frac52(2)^2+6(2)\right)\right]$$

$$2\left[\frac83-10+12\right]-\left[9-\frac{45}{2}+18\right] = 2\left[\frac83+\frac63\right]-\left[-\frac{45}{2}+\frac{54}{2}\right]$$

$$=2\left[\frac{14}{3}\right]-\left[\frac92\right]=\frac{28}{3}-\frac92=\frac{56-27}{6}=\boxed{\frac{29}{6}}$$

a. (10 pts) $\int \left(\frac{dx}{\left(\sqrt{x}+1\right)^3} \right) =$

$u = \sqrt{x} + 1 = x^{\frac{1}{2}} + 1$

$du = \frac{1}{2} x^{-\frac{1}{2}} dx \implies dx = \frac{du}{\frac{1}{2}x^{-\frac{1}{2}}} = 2x^{\frac{1}{2}} du$

$= \int \frac{2x^{\frac{1}{2}} du}{u^3} = 2\int \frac{(u-1)du}{u^3}$

$u = x^{\frac{1}{2}} + 1$

$u - 1 = x^{\frac{1}{2}}$

$\frac{u-1}{u^3} = u^{-3}(u-1) = u^{-2} - u^{-3}$

$\frac{u-1}{u^3} = \frac{u}{u^3} - \frac{1}{u^3} = \frac{1}{u^2} - \frac{1}{u^3} = u^{-2} - u^{-3}$

$= 2\int (u^{-2} - u^{-3}) du$

$\frac{u^3}{u-1} = \frac{u^3}{u} - \frac{u^3}{1}$ No!

$= 2\left[\frac{u^{-1}}{-1} - \frac{u^{-2}}{-2}\right] + C$

$\frac{1}{4} = \frac{1}{3+1} = \frac{1}{3} + \frac{1}{1} = \frac{1}{3} + 1 = \frac{4}{3}$

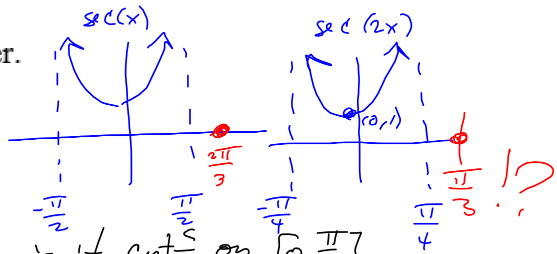
$= -\frac{2}{u} + \frac{1}{u^2} + C = \boxed{\frac{-2}{\sqrt{x}+1} + \frac{1}{(\sqrt{x}+1)^2} + C}$

$\sqrt{x+1}$

$\sqrt{x} + 1$

b. (10 pts) $\boxed{\int_0^{\frac{\pi}{6}} \sec^2(2x)\, dx}$. I want an *exact* answer.

$$\boxed{\int_0^{\frac{\pi}{3}} \sec^2(2x)\,dx}$$



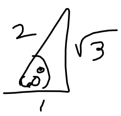
$\frac{\pi}{3}\cdot 2 = \frac{2\pi}{3} > \frac{\pi}{2}$, so sec(2x) isn't cont on $[0, \frac{\pi}{3}]$,

so FTC D.N.A.

$u = 2x$
$du = 2dx$

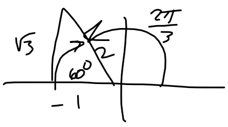
$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \sec^2(2x)\cdot 2dx = \frac{1}{2}\Big[\tan(2x)\Big]_0^{\frac{\pi}{6}} = \frac{1}{2}\Big[\tan\left(\frac{\pi}{3}\right) - \tan(0)\Big]$$

$$= \frac{1}{2}\Big[\sqrt{3}\Big] = \frac{\sqrt{3}}{2}$$



$$\int_0^{\frac{\pi}{3}} \sec^2(2x)\,dx = -\frac{\sqrt{3}}{2}$$

$$= \frac{1}{2}\Big[\tan(2x)\Big]_0^{\frac{\pi}{3}} = \frac{1}{2}\Big[\tan\left(\frac{2\pi}{3}\right) - \tan(0)\Big] = \frac{1}{2}\Big[-\sqrt{3}\Big] = -\frac{\sqrt{3}}{2}$$



Negative ?!

(5 pts) Evaluate $\lim_{x\to -\infty}\left(\sqrt{49x^2+3x}+7x\right)$.

$$a^2-b^2$$

$$\left(\frac{\sqrt{49x^2+3x}+7x}{1}\right)\left(\frac{\sqrt{49x^2+3x}-7x}{\sqrt{49x^2+3x}-7x}\right)=\frac{49x^2+3x-49x^2}{\sqrt{49x^2+3x}-7x}$$

$$=\frac{3x}{\sqrt{49x^2\left(1+\frac{3x}{49x^2}\right)}-7x}=\frac{3x}{7|x|\sqrt{1+\frac{3}{49x}}-7x}=\frac{3x}{-7x\sqrt{1+\frac{3}{49x}}-7x}$$

$|x|=-x$ when $x\longrightarrow -$Anything

$$=\frac{3x}{-7x\left(\sqrt{1+\frac{3}{49x}}+1\right)}$$

$$=\frac{3}{-7\left(\underbrace{\sqrt{1+\frac{3}{49x}}}_{1}+\underbrace{1}_{}\right)}\xrightarrow{x\to-\infty}\frac{3}{-7(1+1)}=\boxed{-\frac{3}{14}}$$

6. (5 pts) Find all vertical and horizontal asymptote of $f(x)=\frac{x-3}{x+2}$, and use them, together with intervals of increase and decrease, and concavity to sketch the graph of f. (Show work!)

$\mathcal{D} = \mathbb{R} \setminus \{-2\}$

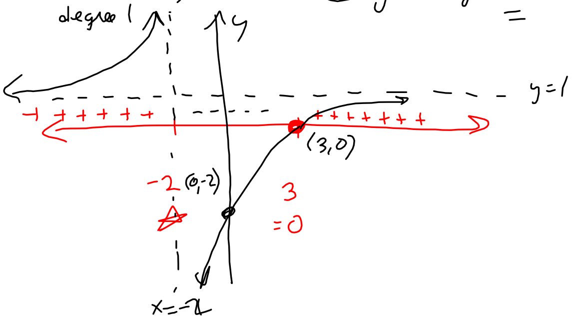
V.A.: $x = -2$

y-int: $-\frac{3}{2} \rightsquigarrow \boxed{(0,-\frac{3}{2})}$

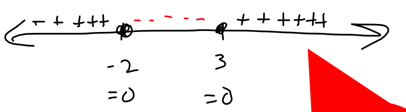
x-int: $f(x)=0 \Rightarrow x-3=0 \Rightarrow x=3 \rightsquigarrow \boxed{(3,0)}$

H.A.: $f(x) \xrightarrow{x\to\pm\infty} \frac{x}{x} = \boxed{1=y}$

$\frac{\text{degree } 1}{\text{degree } 1}$ is tie. Look @ highest-degree terms.



$(x-3)'(x+2)' = x^2 +$ smaller



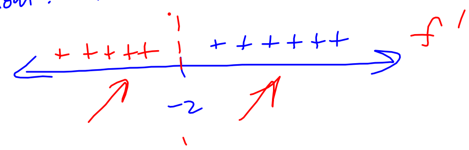
Calculus

$f(x) = \frac{x-3}{x+2}$

$f'(x) = \frac{1(x+2)-(x-3)(1)}{(x+2)^2}$

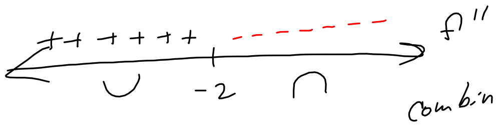
$= \frac{x+2-x+3}{(x+2)^2} = \frac{5}{(x+2)^2} = 0$ Never!

denom: $(x+2)^2 = 0 \Rightarrow x=-2$



$f'(x) = 5/(x+2)^2 = 5(x+2)^{-2}$

$\Rightarrow f''(x) = -10(x+2)^{-3} = \frac{-10}{(x+2)^3}$



combine

7. (5 pts) Find the equation of the oblique asymptote for $f(x)=\dfrac{2x^3-5x+6}{x^2-2x}$

$$
\begin{array}{r l}
 & 2x+4 \\
x^2-2x \,\big)\! & 2x^3+0x^2-5x+6 \\
 & -(2x^3-4x^2) \\
\hline
 & +4x^2-5x+6 \\
 & 4x^2-10x \\
\hline
\end{array}
$$

$\Rightarrow y=2x+4$ is oblique.