F4 Take-Home

① $f(x) = \sqrt{16-4x^2}$ $x = -1$ to $x=2$

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$\boxed{n = 8 \rightarrow \Delta x = \frac{3}{8}}$

$x_k = a + k\Delta x = -1 + k\left(\frac{3}{n}\right) = \frac{3k}{n} - 1$

$x_1 = \frac{3}{n} - 1,\ x_2 = \frac{6}{n} - 1,\ x_3 = \frac{9}{n} - 1, \ldots, x_8 = \frac{24}{n} - 1$

$f(x_k) = \sqrt{16 - 4x_k^2} = \sqrt{16 - 4\left(\frac{3k}{n} - 1\right)^2}$

$= \sqrt{16 - 4\left(\frac{9k^2}{n^2} - \frac{6k}{n} + 1\right)} = \sqrt{16 - \frac{36k^2}{n^2} + \frac{24k}{n} - 4}$

$= \sqrt{-\frac{36k^2}{n^2} + \frac{24k}{n} + 12}$

$A \approx \sum_{k=1}^{n} f(x_k)\,\Delta x = \frac{3}{8}\sum_{k=1}^{8}\sqrt{-\frac{36k^2}{n^2} + \frac{24k}{n} + 12}$

$\approx \frac{3}{32}\sqrt{231} + \frac{9}{16}\sqrt{7} + \frac{3}{32}\sqrt{255} + \frac{3}{8}\sqrt{15}$

$+ \frac{9}{32}\sqrt{23} + \frac{3}{16}\sqrt{39} + \frac{3}{32}\sqrt{87} + 0$

$\approx 1.424876639 + 1.488235112 + 1.497067446$

$+ 1.452368755 + 1.348827616 + 1.170937125$

$+ 0.874417862 + 0 = \boxed{9.256754479 \approx A}$

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(2) $f(x) = 4x^3 - 12x^2 + 6x + 4$

$$\begin{array}{r|rrrr} 2 & 4 & -12 & 6 & 4 \\ & & 8 & -8 & -4 \\ \hline & 4 & -4 & -2 & 0 \end{array}$$

$(x-2)(4x^2-4x-2)$

$a=4, b=-4, c=-2$

$b^2-4ac = (-4)^2-4(4)(-2)$

$= 16+32 = 48$

$\rightsquigarrow \sqrt{48} = \sqrt{16\cdot 3} = 4\sqrt{3}$

So, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{4 \pm 4\sqrt{3}}{2(4)} = \dfrac{4(1 \pm \sqrt{3})}{2(4)}$

$= \dfrac{1 \pm \sqrt{3}}{2}$

$f(x) = 4(x-2)\left(x - \left(\frac{1+\sqrt{3}}{2}\right)\right)\left(x - \left(\frac{1-\sqrt{3}}{2}\right)\right)$

$f'(x) = 12x^2 - 24x + 6 \overset{\text{SET}}{=} 0$

$\Rightarrow 2x^2 - 4x + 1 = 0$

$a=2, b=-4, c=1$

$b^2-4ac = (-4)^2 - 4(2)(1)$

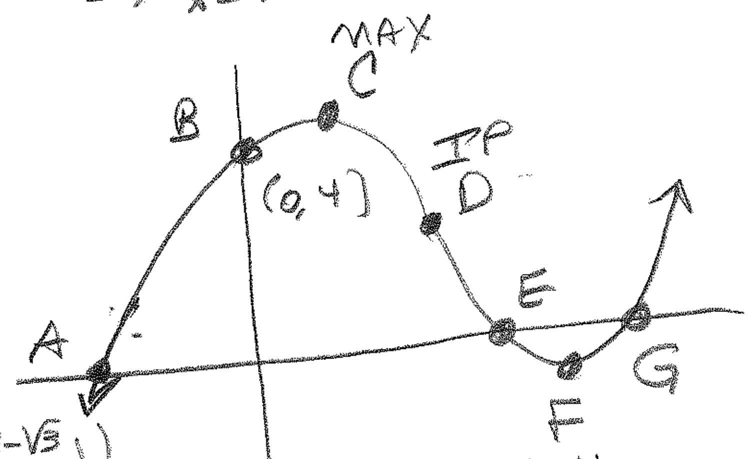
$= 16 - 8 = 8$

$\rightsquigarrow \sqrt{8} = 2\sqrt{2}$

$x = \dfrac{4 \pm 2\sqrt{2}}{2(2)} = \dfrac{2 \pm \sqrt{2}}{2} = x$

$f''(x) = 4x - 4 \overset{\text{SET}}{=} 0$

$\rightarrow x = 1$



$A = \left(\frac{1-\sqrt{3}}{2}, 0\right) \approx (-0.3660254, 0)$

$B = (0,4)$

$C = \left(\frac{2-\sqrt{2}}{2}, 2+2\sqrt{2}\right) \approx (0.29893, 4.828427)$

$D = (1,2)$ IP

$E = \left(\frac{1+\sqrt{3}}{2}, 0\right) \approx (1.366025, 0)$

$F = \left(\frac{2+\sqrt{2}}{2}, 2-2\sqrt{2}\right)$

$\approx (1.707107, -0.828427)$

$G = (2,0)$

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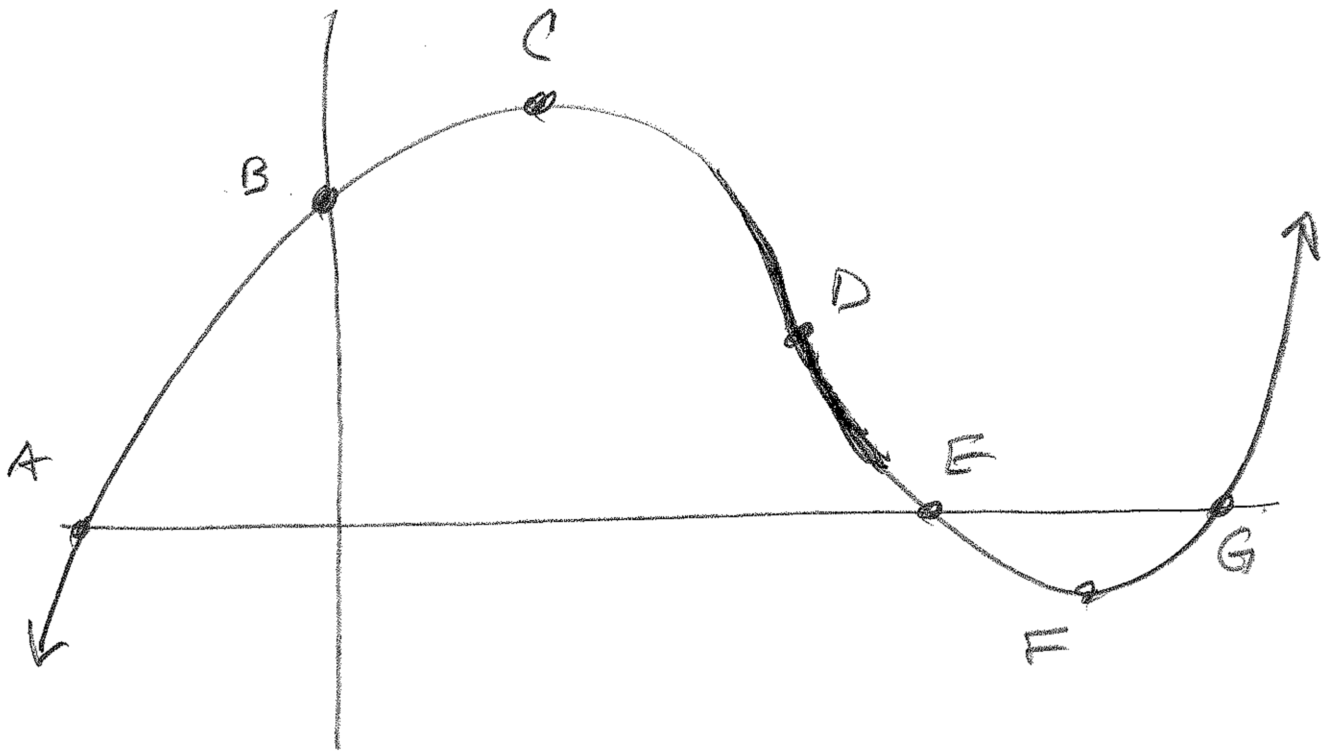
(2)



$A = \left(\frac{1-\sqrt{3}}{2}, 0\right) \approx (-.3660254, 0)$ x-int

$B = (0,4)$ y-int

$C = \left(\frac{2-\sqrt{2}}{2}, 2+2\sqrt{2}\right) \approx (0.2989322, 4.828427\text{-}1)$ MAX

$D = (1,2)$ I.P.

$E = \left(\frac{1+\sqrt{3}}{2}, 0\right) \approx (1.3660254, 0)$ x-int

$F = \left(\frac{2+\sqrt{2}}{2}, 2-2\sqrt{2}\right) \approx (1.7071068, -0.828427\text{-}1)$ MIN

$G = (2,0)$

$f := x \rightarrow \mathrm{sqrt}(16 - 4 \cdot x^2)$

$$x \rightarrow \sqrt{16 - 4\,x^2} \qquad \textbf{(1)}$$

$a := -1$

$$-1 \qquad \textbf{(2)}$$

$b := 2$

$$2 \qquad \textbf{(3)}$$

$deltax := \dfrac{(b - a)}{n}$

$$\frac{3}{n} \qquad \textbf{(4)}$$

$xk := k \rightarrow a + k \cdot deltax$

$$k \rightarrow a + k\, deltax \qquad \textbf{(5)}$$

$xk(1)$

$$-1 + \frac{3}{n} \qquad \textbf{(6)}$$

$xk(2)$

$$-1 + \frac{6}{n} \qquad \textbf{(7)}$$

$\displaystyle\sum_{k=1}^{8} f(xk(k)) \cdot deltax$

$$\frac{6\sqrt{4 - \left(-1 + \frac{3}{n}\right)^2}}{n} + \frac{6\sqrt{4 - \left(-1 + \frac{6}{n}\right)^2}}{n} + \frac{6\sqrt{4 - \left(-1 + \frac{9}{n}\right)^2}}{n} + \frac{6\sqrt{4 - \left(-1 + \frac{12}{n}\right)^2}}{n} + \frac{6\sqrt{4 - \left(-1 + \frac{15}{n}\right)^2}}{n} + \frac{6\sqrt{4 - \left(-1 + \frac{18}{n}\right)^2}}{n} + \frac{6\sqrt{4 - \left(-1 + \frac{21}{n}\right)^2}}{n} + \frac{6\sqrt{4 - \left(-1 + \frac{24}{n}\right)^2}}{n} \qquad \textbf{(8)}$$

$n := 8$

$$8 \qquad \textbf{(9)}$$

$\displaystyle\sum_{k=1}^{8} f(xk(k)) \cdot deltax$

$$\frac{3}{32}\sqrt{231} + \frac{3}{32}\sqrt{252} + \frac{3}{32}\sqrt{255} + \frac{3}{32}\sqrt{240} + \frac{3}{32}\sqrt{207} + \frac{3}{32}\sqrt{156} + \frac{3}{32}\sqrt{87} \qquad \textbf{(10)}$$

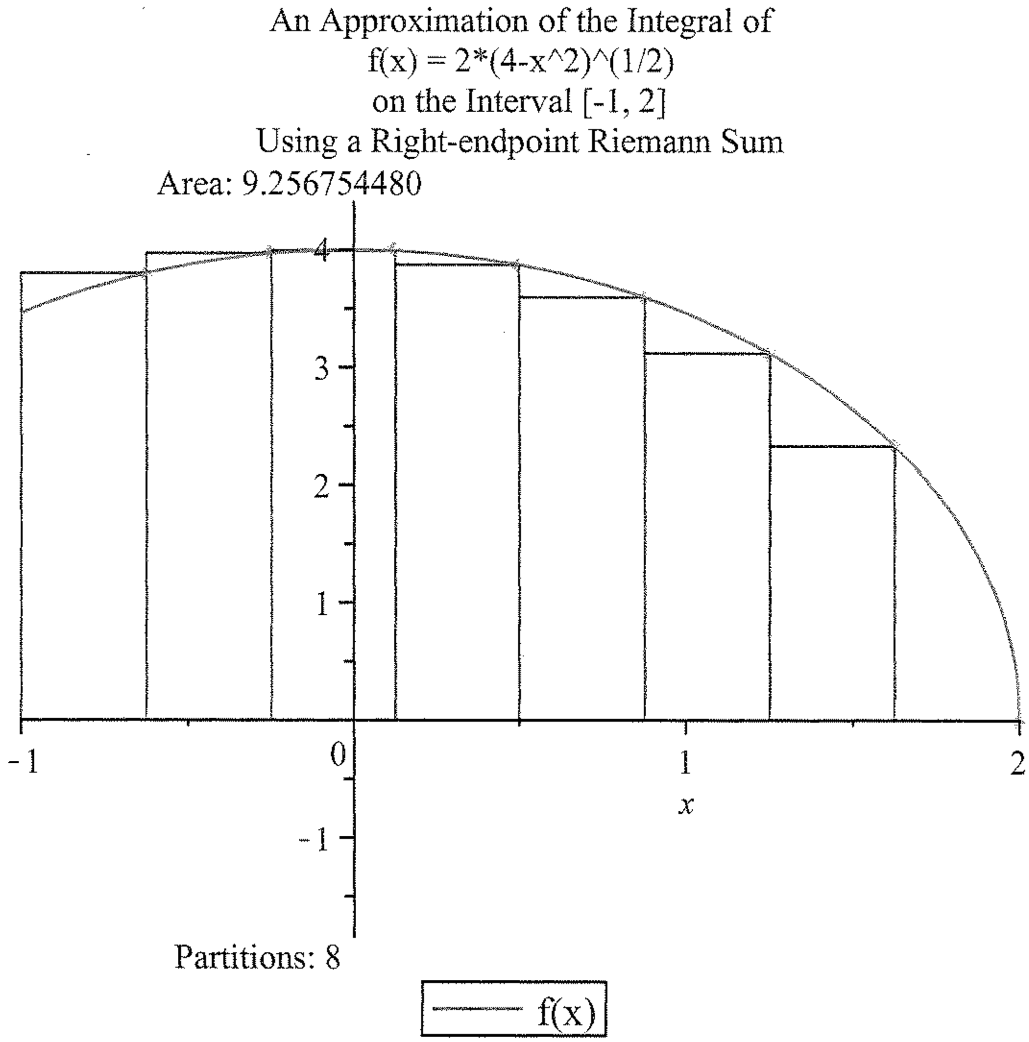
$evalf(\%)$

$$9.256754479 \qquad \textbf{(11)}$$

evalf(%)

$$9.256754480 \tag{15}$$

ApproximateInt(2 * (4 - x^2)^(1/2), -1 ..2, 'partition' = 8, 'method' = right, 'partitiontype' = normal, 'output' = 'plot','showarea'='true');



$f(xk(1)) \cdot deltax$

$$\frac{3}{32}\sqrt{231} \tag{16}$$

$f(xk(2)) \cdot deltax$

$$\frac{9}{16}\sqrt{7} \tag{17}$$

$f(xk(3)) \cdot deltax$

$$\frac{3}{32}\sqrt{255} \tag{18}$$

$f(xk(4)) \cdot deltax$

$$\frac{3}{8}\sqrt{15} \tag{19}$$

$f(xk(5)) \cdot deltax$

$$\frac{9}{32}\sqrt{23} \tag{20}$$

$f(xk(6)) \cdot deltax$

$$\frac{3}{16}\sqrt{39} \tag{21}$$

$f(xk(7)) \cdot deltax$

$$\frac{3}{32}\sqrt{87} \tag{22}$$

$f(xk(8)) \cdot deltax$

$$0 \tag{23}$$

$mysummands := evalf([f(xk(1)) \cdot deltax, f(xk(2)) \cdot deltax, f(xk(3)) \cdot deltax, f(xk(4)) \cdot deltax, f(xk(5)) \cdot deltax, f(xk(6)) \cdot deltax, f(xk(7)) \cdot deltax, f(xk(8)) \cdot deltax])$

$$[1.424876639, 1.488235112, 1.497067446, 1.452368755, 1.348827616, 1.170937125, 0.8744417862, 0.] \tag{24}$$

$\sum_{k=1}^{8} mysummands[k]$

$$9.256754479 \tag{25}$$

$mysummands(1)$

$$[1.424876639, 1.488235112, 1.497067446, 1.452368755, 1.348827616, 1.170937125, 0.8744417862, 0.] \tag{26}$$

$mysummands[1]$

$$1.424876639 \tag{27}$$