201 E4

1. $\int_1^4 (x^2-2)\,dx$ $\quad \Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \boxed{\frac{3}{n} = \Delta x}$

$x_k = a + k\Delta x = 1 + k\cdot\frac{3}{n} = \boxed{\frac{3k}{n} + 1 = x_k}$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}(x_k^2-2)\cdot\frac{3}{n} = \frac{3}{n}\sum_{k=1}^{n}\left(\left(\frac{3}{n}k+1\right)^2-2\right)$$

$$= \frac{3}{n}\sum_{k=1}^{n}\left(\left(\frac{3k}{n}\right)^2 + 2\left(\frac{3k}{n}\right)(1) + 1^2 - 2\right)$$

$$= \frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2}{n^2} + \frac{6k}{n} - 1\right)$$

$$= \frac{3}{n}\sum_{k=1}^{n}\frac{9k^2}{n^2} + \frac{3}{n}\sum_{k=1}^{n}\frac{6k}{n} - \frac{3}{n}\sum_{k=1}^{n}1$$

$$= \frac{27}{n^3}\sum k^2 + \frac{18}{n^2}\sum_{k=1}^{n}k - \frac{3}{n}\sum_{k=1}^{n}1$$

$$= \frac{27}{n^3}\cdot\frac{n^3+\text{smaller}}{3} + \frac{18}{n^2}\cdot\frac{n^2+\text{smaller}}{2} - \frac{3}{n}\cdot n$$

$$\xrightarrow{n\to\infty} \frac{27}{3} + \frac{18}{2} - 3$$

$$= 9 + 9 - 3 = \boxed{15}$$

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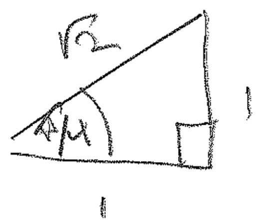
2 a $\displaystyle\int_0^{\frac{\pi}{4}} (\sec^2(x) - 2)\,dx$

$= \Big[\tan x - 2x\Big]_0^{\frac{\pi}{4}} = \tan\left(\frac{\pi}{4}\right) - 2\left(\frac{\pi}{4}\right) - \Big(\tan(0) - 2(0)\Big)$

$= \boxed{1 - \frac{\pi}{2}}$



b $\displaystyle\frac{d}{dx}\int_0^{\sin(x)} \frac{\sec^2(t) + 12t}{t^2 + 7}\,dt$

$= \dfrac{\sec^2(\sin x) + 12\sin x}{(\sin(x))^2 + 7} \cdot \cos x$

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(3) $f(t)$ = Velocity of t = $t^2 - 5t + 6$ in meters per seco

(a) Net Displacement, from $t=0$ to $t=4$

$$\int_0^4 (t^2 - 5t + 6)\,dt = \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_0^4$$

$$= \frac{1}{3}(4)^3 - \frac{5}{2}(4)^2 + 6(4) - \left(\frac{1}{3}(0)^3 - \frac{5}{2}(0)^2 + 6(0)\right)$$

$$= \frac{64}{3} - \frac{5}{2}(16) + 24 = \frac{64}{3} - 40 + \frac{24}{3}$$

$$= \frac{64 - 120 + 72}{3} = \frac{136 - 120}{3} = \boxed{\frac{16}{3}\text{ m}} = 5.\overline{3}$$

(b) Total Distance $(t-2)(t-3)$

$+ \quad | \quad - \quad | \quad +$

$\qquad 2 \qquad\quad 3$

$$= \int_0^4 |t^2 - 5t + 6|\,dt$$

$$= \int_0^2 (t^2 - 5t + 6)\,dt - \int_2^3 (t^2 - 5t + 6)\,dt + \int_3^4 (t^2 - 5t + 6)\,dt$$

$$= \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_0^2 - \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_2^3 + \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_3^4$$

$$= \frac{1}{3}(2)^3 - \frac{5}{2}(2)^2 + 6(2) - (0 + 0 + 0)$$

$$- \left[\left(\frac{1}{3}(3)^3 - \frac{5}{2}(3)^2 + 6(3)\right) - \left(\frac{1}{3}(2)^3 - \frac{5}{2}(2)^2 + 6(2)\right)\right]$$

$$+ \left[\left(\frac{1}{3}(4)^3 - \frac{5}{2}(4)^2 + 6(4)\right) - \left(\frac{1}{3}(3)^3 - \frac{5}{2}(3)^2 + 6(3)\right)\right]$$

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3b Total Distance is

$$\int_0^4 |t^2-5t+6|\,dt$$

Analyze $|t^2-5t+6|$:

$|(t-2)(t-3)|$

$+ \quad | \quad - \quad | \quad +$

$\quad\ \ 2 \qquad\ 3$

So, $|t^2-5t+6| = \begin{cases} t^2-5t+6 & \text{if } x \in (-\infty, 2] \cup [3, \infty) \\ -(t^2-5t+6) & \text{if } x \in (2,3) \end{cases}$

$\therefore \int_0^4 |f(t)|\,dt = \int_0^2 f(t)\,dt - \int_2^3 f(t)\,dt + \int_3^4 f(t)\,dt$

$= \int_0^2 (t^2-5t+6)\,dt - \int_2^3 (t^2-5t+6)\,dt + \int_3^4 (t^2-5t+6)\,dt$

$= \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_0^2 - \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_2^3 + \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_3^4$

$= \left[\frac{1}{3}(2)^3 - \frac{5}{2}(2)^2 + 6(2) - (0)\right]$

$- \left[\frac{1}{3}(3)^3 - \frac{5}{2}(3)^2 + 6(3) - \left(\frac{1}{3}(2)^3 - \frac{5}{2}(2)^2 + 6(2)\right)\right]$

$+ \left[\frac{1}{3}(4)^3 - \frac{5}{2}(4)^2 + 6(4) - \left(\frac{1}{3}(3)^3 - \frac{5}{2}(3)^2 + 6(3)\right)\right]$

$= \frac{17}{3} = 5.\overline{6}$

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3b Cont'd

$$= 2\left[\frac{1}{3}(8) - \frac{5}{2}(4) + 12\right]$$

2 of 'em

$$- \left[\frac{1}{3}(27) - \frac{5}{2}(9) + 18 - \left(\frac{1}{3}(8) - \frac{5}{2}(4) + 12\right)\right]$$

2 of 'em

$$+ \left[\frac{1}{3}(64) - \frac{5}{2}(16) + 24 - \left(\frac{1}{3}(27) - \frac{5}{2}(9)\right) + 18\right]$$

$$= 2\left(\frac{8}{3} - 10 + 12\right) - 2\left(9 - \frac{45}{2} + 18\right) + \left(\frac{64}{3} - 40 + 24\right)$$

$$= 2\left(\frac{8+6}{3}\right) - 2\left(\frac{54}{2} - \frac{45}{2}\right) + \left(\frac{64-48}{3}\right)$$

$$= 2\left(\frac{14}{3}\right) - 2\left(\frac{9}{2}\right) + \left(\frac{16}{3}\right)$$

$$= \frac{28}{3} - 9 + \frac{16}{3} = \frac{44}{3} - \frac{27}{3} = \boxed{\frac{17}{3}}$$

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4 (a) $\int \frac{dx}{(\sqrt{x}+1)^3} =$

$\boxed{u = \sqrt{x}+1} = x^{\frac{1}{2}}+1 \longrightarrow \boxed{x^{\frac{1}{2}} = u-1}$

$du = \frac{1}{2}x^{-\frac{1}{2}}dx$

$\Longrightarrow \boxed{dx = 2x^{\frac{1}{2}}du}$

This gives $\int \frac{2x^{\frac{1}{2}}du}{u^3} = \int \frac{2(u-1)du}{u^3}$

$= 2\int\left(\frac{u}{u^3} - \frac{1}{u^3}\right)du = 2\int(u^{-2} - u^{-3})\,du$

$= 2\left[\frac{u^{-1}}{-1} - \frac{u^{-2}}{-2}\right] + C = u^{-2} - 2u^{-1} + C$

$= (\sqrt{x}+1)^{-2} - 2(\sqrt{x}+1)^{-1} + C$ OR

$$\boxed{\frac{1}{(\sqrt{x}+1)^2} - \frac{2}{\sqrt{x}+1} + C}$$

Go back & double-check scoring on #4a ✓ DONE

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4b

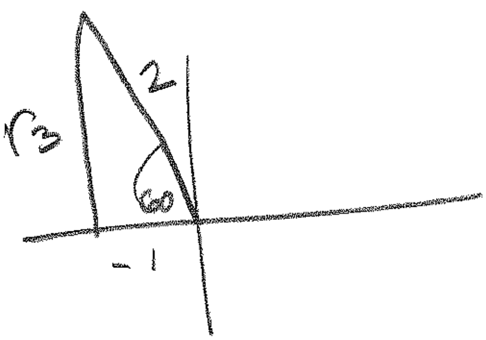
$$\int_0^{\frac{\pi}{3}} \sec^2(2x)\,dx$$ $\sec^2(2x)$ Doesn't satisfy FTC II on $[0, \frac{\pi}{3}]$

$u = 2x \Rightarrow du = 2dx \longrightarrow dx = \frac{du}{2}$

$$\int_{0=x}^{x=\frac{\pi}{3}} \sec^2(u)\left(\tfrac{1}{2}du\right)$$

$$= \tfrac{1}{2}\Big[\tan(u)\Big]_{0=x}^{x=\frac{\pi}{3}} = \tfrac{1}{2}\tan(2x)\Big]_{x=0}^{\frac{\pi}{3}} = \tfrac{1}{2}\tan\left(\tfrac{2\pi}{3}\right)$$

$$- \tfrac{1}{2}\tan(0)$$



$$= \tfrac{1}{2}\left(-\sqrt{3}\right) - \tfrac{1}{2}(0)$$

$$= -\frac{\sqrt{3}}{2}$$

201 E4

(5) BONUS $\lim_{x \to -\infty} \left(\sqrt{49x^2+3x} + 7x \right)$

$$\left(\frac{\sqrt{49x^2+3x} + 7x}{1} \right) \left(\frac{\sqrt{49x^2+3x} - 7x}{\sqrt{49x^2+3x} - 7x} \right)$$

$$= \frac{49x^2 + 3x - 49x^2}{\sqrt{49x^2\left(1 + \frac{3}{49x}\right)} - 7x}$$

$$= \frac{3x}{-7x\sqrt{1 + \frac{3}{49x}} - 7x} = \frac{3x}{-7x\left(\sqrt{1+\frac{3}{49x}} + 1\right)}$$

$|49x^2| = -7x$ when $x < 0$

$$= \frac{3}{-7\left(\sqrt{1+\frac{3}{49x}} + 1\right)} \xrightarrow{x \to -\infty} \frac{3}{-7(1+1)} = -\frac{3}{14}$$

2.01 ~~8~~ E4

6 $\quad \dfrac{x-3}{x+2} = f(x)$

H.A.: $y=1$

V.A.: $x=-2$

x-int: $(3,0)$

y-int: $(0,-\frac{3}{2})$

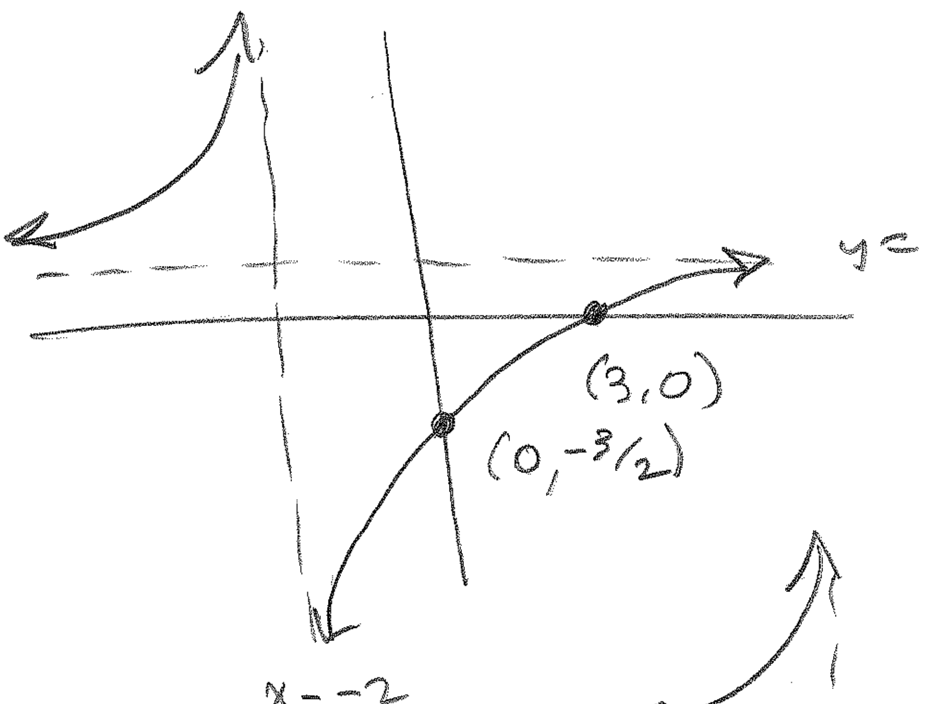
$$f'(x) = \frac{1(x+2)-(x-3)(1)}{(x+2)^2}$$

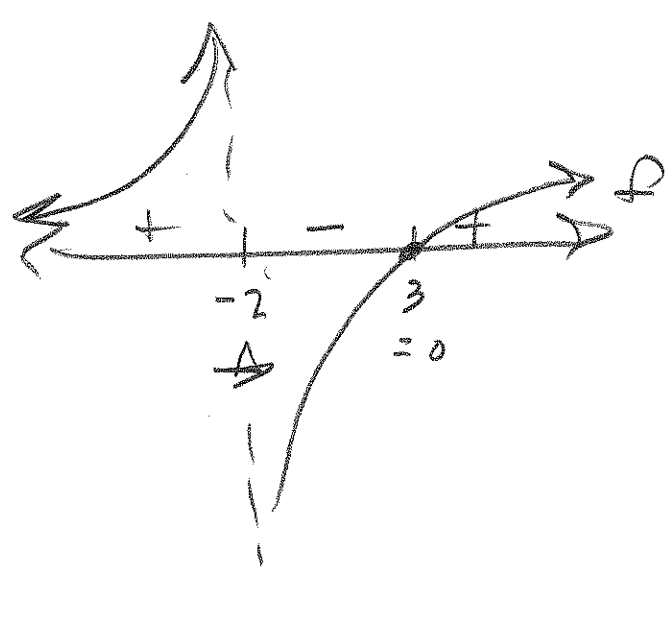
$$= \frac{x+2-x+3}{(x+2)^2} = \frac{5}{(x+2)^2} \Longrightarrow$$

f': + | + at -2; ↗ ↗

$$f''(x) = -10(x+2)^{-3} = \frac{-10}{(x+2)^3}$$

+ | − at -2





201 E4

6 $f(x) = \dfrac{2x^3 - 5x + 6}{x^2 - 2x}$

$\boxed{y = 2x + 4}$

$$\begin{array}{r|l} & \quad 2x + 4 \\ \hline x^2 - 2x & 2x^3 + 0x^2 - 5x + 6 \\ & -(2x^3 - 4x^2) \\ \hline & \qquad 4x^2 - 5x + 6 \end{array}$$

<u>Done</u> for oblique Asymptote question.

I finish the long division and interpret.

$$\begin{array}{r|l} & \quad 2x + 4 \quad r\ 3x + 6 \\ \hline x^2 - 2x & 2x^3 + 0x^2 - 5x + 6 \\ & -(2x^3 - 4x^2) \\ \hline & \qquad 4x^2 - 5x + 6 \\ & \quad -(4x^2 - 8x) \\ \hline & \qquad\qquad 3x + 6 \end{array}$$

So,

$$2x^3 - 5x + 6 = (x^2 - 2x)(2x + 4) + 3x + 6$$

OR

$$\frac{2x^3 - 5x + 6}{x^2 - 2x} = 2x + 4 + \frac{3x + 6}{x^2 - 2x} \xrightarrow{|x| \to \text{BIG}} 2x + 4$$