201 E4 Re-Take 4/27/15

① ⓐ $\int_1^3 (3x^2+1)\,dx$ (15 pts) KEY

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$

$x_k = a + k\Delta x = 1 + \frac{2k}{n} = \frac{2k}{n} + 1$

$\Delta x \sum_{k=1}^{n} f(x_k) = \frac{2}{n}\sum_{k=1}^{n}(3x_k^2+1) = \frac{2}{n}\sum\left(3\left(\frac{2k}{n}+1\right)^2+1\right)$

$= \frac{2}{n}\sum_{k=1}^{n}\left(3\left(\frac{4k^2}{n^2}+\frac{4k}{n}+1\right)+1\right) = \frac{2}{n}\sum_{k=1}^{n}\frac{12}{n^2}k^2 + \frac{2}{n}\sum_{k=1}^{n}\frac{12}{n}k + \frac{2}{n}\sum_{k=1}^{n}4$

$= \frac{2}{n}\cdot\frac{12}{n^2}\sum_{k=1}^{n}k^2 + \frac{2}{n}\cdot\frac{12}{n}\sum k + \frac{2}{n}\sum_{k=1}^{n}4$

$= \frac{24}{n^3}\cdot\frac{n^3+m}{3} + \frac{24}{n^2}\cdot\frac{n^2+m}{2} + \frac{2}{n}\cdot 4\cdot n$

$\xrightarrow{n \to \infty} \frac{24}{3} + \frac{24}{2} + 8 = 8 + 12 + 8 = 28$

ⓑ $\int_1^3 (3x^2+1)\,dx = \left[x^3+x\right]_1^3 = 3^3 + 3 - (1^3+1)$ (5pts)

$= 30 - 2 = 28$

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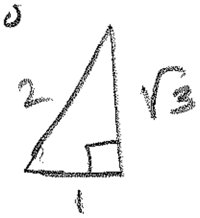
② ⓐ $\int_0^{\frac{\pi}{3}} \sec(x)\tan(x)\,dx$ 10 pts

$$= \left[\sec(x)\right]_0^{\frac{\pi}{3}} = \sec\left(\frac{\pi}{3}\right) - \sec(0) = 2 - 1 = \boxed{1}$$



ⓑ $\frac{d}{dx}\int_0^{x^2-3x}\left(\frac{\sin(t)-\pi t}{t^2+7}\,dt\right)$ 10 pts

$$= \left(\frac{\sin(x^2-3x) - \pi(x^2-3x)}{(x^2-3x)^2+7}\right)(2x-3)$$

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③ $f(t) = 3t^2 - t - 10 \longrightarrow$

ⓐ Net Displacement on $[0, 4]$ is

$$\int_0^4 (3t^2 - t - 10)\,dt = \left[t^3 - \tfrac{1}{2}t^2 - 10t\right]_0^4$$

$$= 4^3 - \tfrac{1}{2}(4)^2 - 10(4) = 64 - 8 - 40 = 64 - 48 = \boxed{16\ m}$$

10 pts

$$-10 - \frac{1}{12} = \frac{-120 - 1}{12}$$

ⓑ Total Distance Traveled =

10 pts

$$\int_0^4 |3t^2 - t - 10|\,dt$$ Need sign pattern for $3t^2 - t - 10$, in order to handle the absolute value.

$\boxed{m1}$

$(3t + 5)(t - 2)$

$(3)(-10) = -30$

$-6 + 5 = -1$

$3t^2 - 6t + 5t - 10$

$\boxed{m2}$

$a = 3, b = -1, c = -10$

$b^2 - 4ac = (-1)^2 - 4(3)(-10)$

$= 1 + 120 = 121$

$t = \dfrac{1 \pm \sqrt{121}}{2(3)} = \dfrac{1 \pm 11}{6}$ $\frac{12}{6} = 2$, $\frac{-10}{6} = -\frac{5}{3}$

$\boxed{m3}$

$3t^2 - t - 10 =$

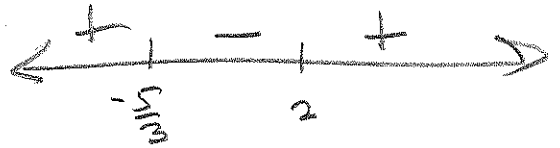
$3\left(t^2 - \frac{1}{3}t + \left(\frac{1}{6}\right)^2\right) - 10 - \frac{3}{36}$

$= 3\left(t - \frac{1}{6}\right)^2 - \frac{121}{12} \overset{\text{SET}}{=} 0$

$\left(t - \frac{1}{6}\right)^2 = \frac{121}{12} \cdot \frac{1}{3}$

$t - \frac{1}{6} = \pm \frac{11}{6}$

$t = \frac{1 \pm 11}{6}$ ✓



$$-\int_0^2 (3t^2 - t - 10)\,dt + \int_2^4 (3t^2 - t - 10)\,dt = -\left(t^3 - \tfrac{1}{2}t^2 - 10t\right)_0^2$$

$$+ \left(t^3 - \tfrac{1}{2}t^2 - 10t\right)_2^4 = -\left(2^3 - \tfrac{1}{2}(2)^2 - 10(2)\right) + \left[4^3 - \tfrac{1}{2}(4)^2 - 10(4) - (2^3 - \tfrac{1}{2}(2)^2\right.$$

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(3b) cont'd

$$-\int_0^2 (3t^2 - t - 10)\,dt + \int_2^4 (3t^2 - t - 10)\,dt$$

$$= -\left[t^3 - \frac{1}{2}t^2 - 10t\right]_0^2 + \left[t^3 - \frac{1}{2}t^2 - 10t\right]_2^4$$

$$= -\left[2^3 - \frac{1}{2}(2)^2 - 10(2) - (0)\right]$$

$$+ \left[4^3 - \frac{1}{2}(4)^2 - 10(4) - \left(2^3 - \frac{1}{2}(2)^2 - 10(2)\right)\right]$$

$$= -\left[8 - \frac{1}{2}(4) - 20\right] + \left[64 - \frac{1}{2}(16) - 40 - (8 - \frac{1}{2}(4) - 20\right]$$

$$= -[8 - 22] + [24 - 8 - (8 - 22)]$$

$$= -[-14] + 16 - (-14)$$

$$= 14 + 16 + 14 = \boxed{44}$$

201 E4

④ ⓐ $\int 3x\sqrt{3x-1}\,dx$

$u = 3x-1$ $\qquad 3x-1=u$

$du = 3dx$ $\qquad 3x = u+1$

$dx = \frac{du}{3}$ $\qquad x = \frac{u+1}{3}$

10 pts

$= \int \cancel{3}\left(\frac{u+1}{\cancel{3}}\right)(u)^{\frac{1}{2}}\left(\frac{du}{3}\right)$

$= \frac{1}{3}\int u^{\frac{1}{2}}(u+1)\,du = \frac{1}{3}\int\left(u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)du$

$= \frac{1}{3}\left(\frac{3}{5}u^{\frac{5}{2}}+\frac{2}{3}u^{\frac{3}{2}}\right)+C = \frac{2}{15}u^{\frac{5}{2}}+\frac{2}{9}u^{\frac{3}{2}}+C$

$= \frac{2}{15}(3x-1)^{\frac{5}{2}}+\frac{2}{9}(3x-1)^{\frac{3}{2}}+C$ OR $\frac{3}{15}\sqrt{(3x-1)^5}+\frac{2}{9}\sqrt{(3x-1)^3}+C$

ⓑ $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \csc(2x)\cot(2x)\,dx = \left[-\frac{1}{2}\csc(2x)\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

10 pts

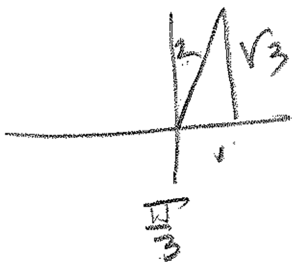
$u=2x,\ du = 2dx$

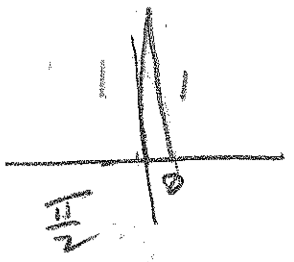
$dx = \frac{1}{2}du$

$= -\frac{1}{2}\left[\csc\left(\frac{\pi}{2}\right)-\csc\left(\frac{\pi}{3}\right)\right] = -\frac{1}{2}\left[1-\frac{2}{\sqrt{3}}\right] =$

$= -\frac{1}{2}\left[\frac{\sqrt{3}-2}{\sqrt{3}}\right] = \frac{2-\sqrt{3}}{2\sqrt{3}}$

OR $\frac{2\sqrt{3}-3}{6}$ OR $\frac{\sqrt{3}}{3}-\frac{1}{2}$





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5 B 5pts

$$\sqrt{36x^2+3x}+6x$$

$$=\left(\frac{\sqrt{36x^2+3x}+6x}{1}\right)\left(\frac{\sqrt{36x^2+3x}-6x}{\sqrt{36x^2+3x}-6x}\right)$$

$$=\frac{36x^2+3x-36x^2}{\sqrt{36x^2+3x}-6x}=\frac{3x}{\sqrt{36x^2}\sqrt{1+\frac{3x}{36x^2}}-6x}$$

(x is negative!)

$$=\frac{3x}{6|x|\sqrt{1+\frac{1}{12x}}-6x}=\frac{3x}{-6x\sqrt{1+\frac{1}{12x}}-6x}$$

$$=\frac{3x}{-6x\left(\sqrt{1+\frac{1}{12x}}+1\right)}=\frac{1}{-2\left(\sqrt{1+\frac{1}{12x}}+1\right)}$$

$$\xrightarrow{x\longrightarrow-\infty}-\frac{1}{2(1+1)}=\boxed{-\frac{1}{4}}$$

201 E4 RE-TAKE

6B $f(x) = \frac{x-2}{x+4}$

$\mathcal{D} = \mathbb{R} \setminus \{-4\}$ No holes

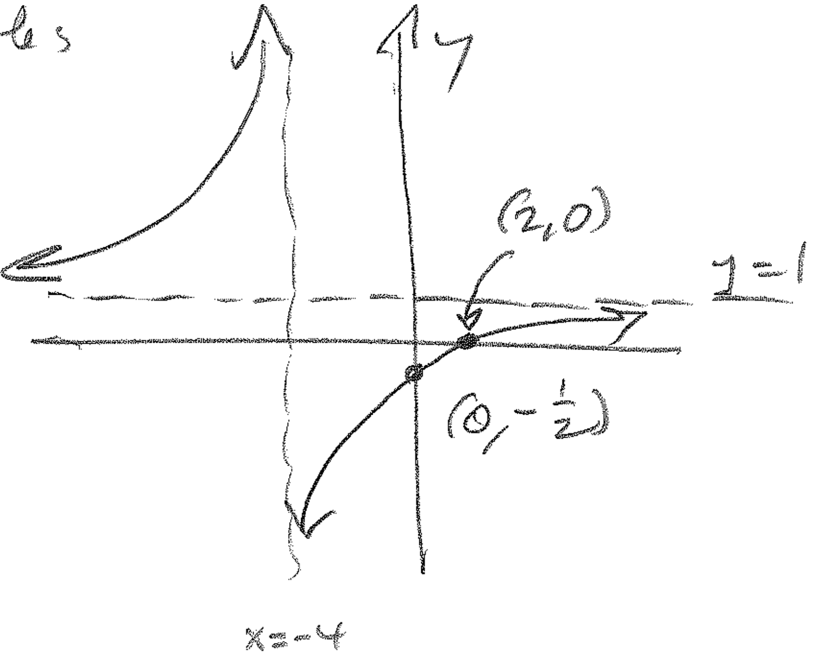
V.A.: $\boxed{x = -4}$

H.A.: $y = \frac{x}{x} = \boxed{1 = y}$

$\boxed{\text{x-int: } (2,0)}$

$\boxed{\text{y-int: }} \frac{-2}{4} = -\frac{1}{2}$

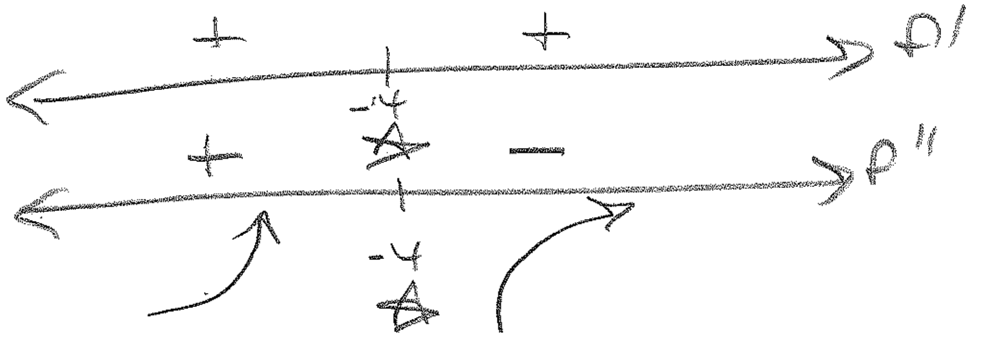
$\rightsquigarrow \boxed{(0, -\frac{1}{2})}$



$$f'(x) = \frac{1(x+4) - (x-2)(1)}{(x+4)^2}$$

$$= \frac{x+4-x+2}{(x+4)^2} = \frac{6}{(x+4)^2} = f'(x)$$

$$f''(x) = -12(x+4)^{-3}$$



201 E4 RE-TAKE

⑦ Ⓑ (5pts)

$$\boxed{3x+15} \rightarrow y = 3x+5 \text{ is oblique asymptote for } f(x)$$

$$\begin{array}{r|l} & 3x^3 + 0x^2 - 5x + 6 \\ x^2 - 5x + 6 & \\ & -\left(3x^3 - 15x^2 + 18x\right) \\ \hline & \qquad 15x^2 - 23x + 6 \end{array}$$

$$f(x) = \frac{3x^3 - 5x + 6}{x^2 - 5x + 6}$$