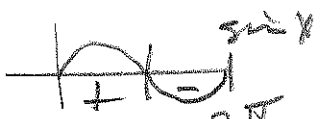
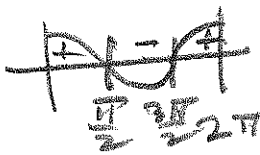
201 E3 TAKE-HOME

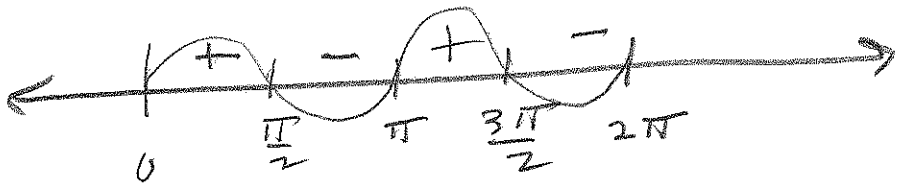
① We graph $f(x) = \sin(x)\cos(x)$ on $[0, 2\pi]$

$\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$







When zeros are easy, rough sketch with intercepts is nice.

$f'(x) = \cos x \cos x + \sin x(-\sin x)$

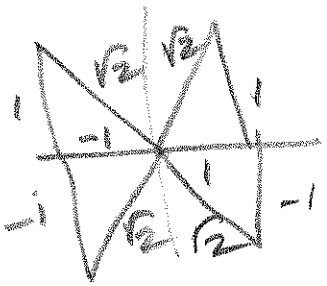
$= \cos^2 x - \sin^2 x$

$= \cos^2 x - (1 - \cos^2 x)$

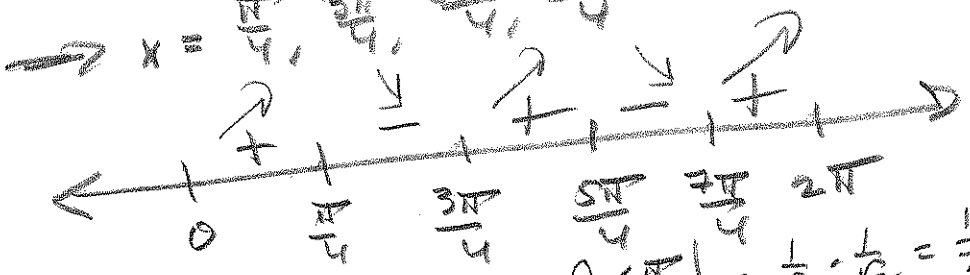
$= 2\cos^2 x - 1 \overset{\text{SET}}{=} 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$



$\Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



$2\cos^2\left(\frac{\pi}{4} + .01\right) \approx -.02$

$2\cos^2\left(\frac{3\pi}{4} + .01\right) \approx .02$

$2\cos^2\left(\frac{5\pi}{4} + .01\right) \approx -.02$

$2\cos^2\left(\frac{7\pi}{4} + .01\right) \approx .02$

$f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$ MAX $\left(\frac{\pi}{4}, \frac{1}{2}\right)$

$f\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}} \cdot \left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$ MIN $\left(\frac{3\pi}{4}, -\frac{1}{2}\right)$

$f\left(\frac{5\pi}{4}\right) = \frac{1}{2}$ $\left(\frac{5\pi}{4}, \frac{1}{2}\right)$ MAX

$f\left(\frac{7\pi}{4}\right) = -\frac{1}{2}$ $\left(\frac{7\pi}{4}, -\frac{1}{2}\right)$ MIN

201 E3 TAKE-HOME

(1) cont'd

$f''(x) = 4\cos(x)(-\sin(x)) = \boxed{-4\sin x \cos x = f''(x)}$

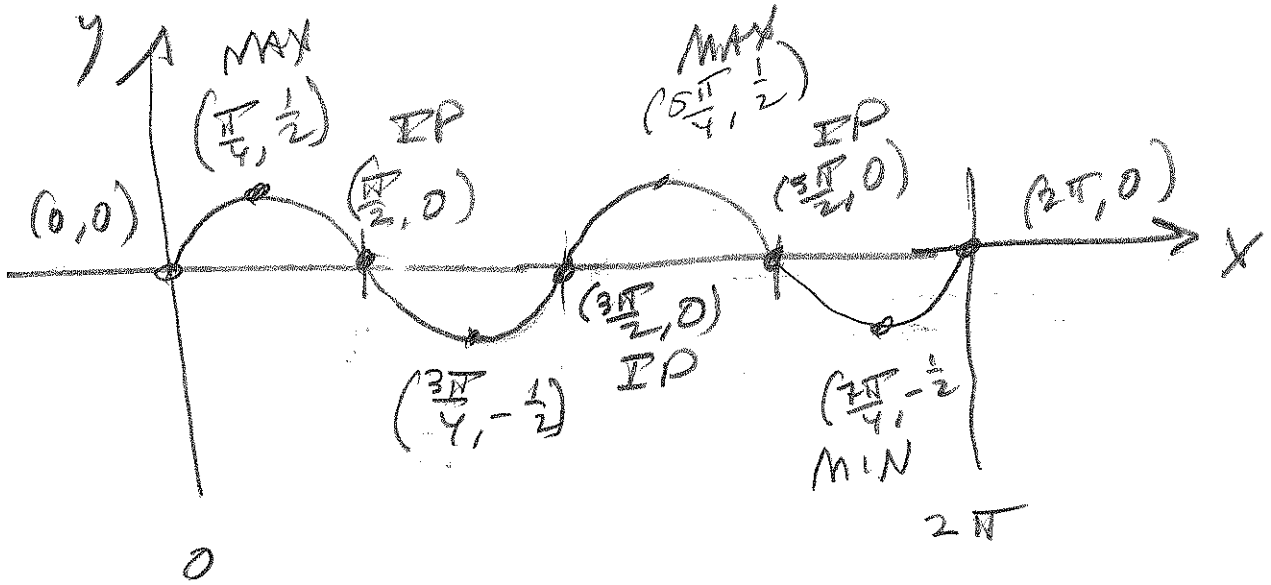
has same zeros as f'.

Still looking at graphs of sin(x) & cos(x) to analyze for pattern.

- on $(0, \frac{\pi}{2})$ b/c sin(x), cos(x) > 0 there

Alternates rest of the way.

So, here's what it looks like



201 E3 ☐H

(3) We find the zero of $f(x) = \cos(x) - x$ by Newton's Method, accurate to 6 decimal places.

$f'(x) = -\sin x - 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\cos(x_n) - x_n}{-\sin(x_n) - 1}$$

$$= x_n + \frac{\cos(x_n) - x_n}{\sin(x_n) + 1}$$

n	x_n
1	2
2	.7345361689
3	.7390897242
4	.7390851332
5	.7390851332

$x \approx .739085$

n	x_n
1	5
2	-109.82
3	15.96
4	-6.615
5	4.600
6	-743.61
7	

On a take-home, GRAPH IT! That'll get you close!