201 TEST 3 Spring '15

(1) $f(x) = 3x^5 - 5x^3 + 2$ on $[-2, 2]$

10pts local & abs max/min.

$f(-2) = -54$

$$\begin{array}{r|rrrrrr} -2 & 3 & 0 & -5 & 0 & 0 & 2 \\ & & -6 & 12 & -14 & 28 & -56 \\ \hline & 3 & -6 & 7 & -14 & 28 & -54 \end{array}$$

$f(2) = 58$

$\boxed{(2, 58)}$

$$\begin{array}{r|rrrrrr} 2 & 3 & 0 & -5 & 0 & 0 & 2 \\ & & 6 & 12 & 14 & 28 & 56 \\ \hline & 3 & 6 & 7 & 14 & 28 & 58 \end{array}$$

$f'(x) = 15x^4 - 15x^2 = 15x^2(x^2 - 1) \overset{SET}{=} 0 \longrightarrow$

$x = 0, \pm 1$

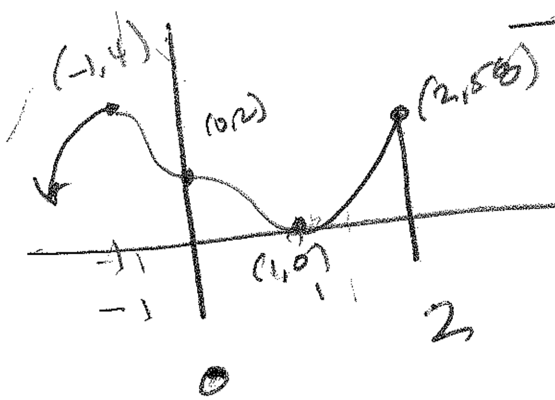
$$\begin{array}{ccccccc} & + & & - & & - & & + \\ \longleftarrow & & -1 & & 0 & & 1 & & \longrightarrow \end{array}$$

$f(-1) = 4$

$$\begin{array}{r|rrrrrr} -1 & 3 & 0 & -5 & 0 & 0 & 2 \\ & & -3 & 3 & 2 & -2 & 2 \\ \hline & 3 & -3 & -2 & 2 & -2 & 4 \end{array}$$

$f(1) = 0$

$$\begin{array}{r|rrrrrr} 1 & 3 & 0 & -5 & 0 & 0 & 2 \\ & & 3 & 3 & -2 & -2 & -2 \\ \hline & 3 & 3 & -2 & -2 & -2 & 0 \end{array}$$



(0,2)

(-1,4) ∉ Graph

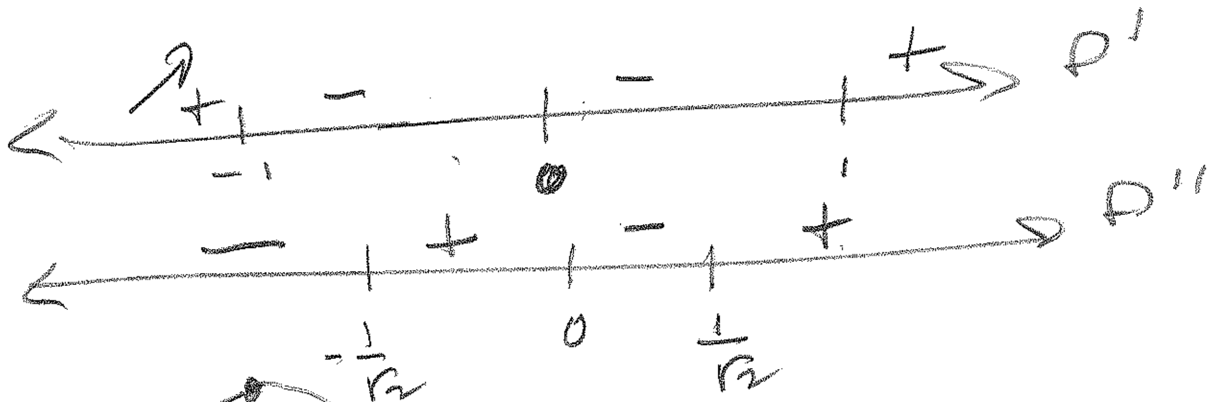
$\boxed{\begin{array}{l} (1,0) \text{ local \& ABS MIN} \\ (2,58) \text{ MAX} \end{array}}$

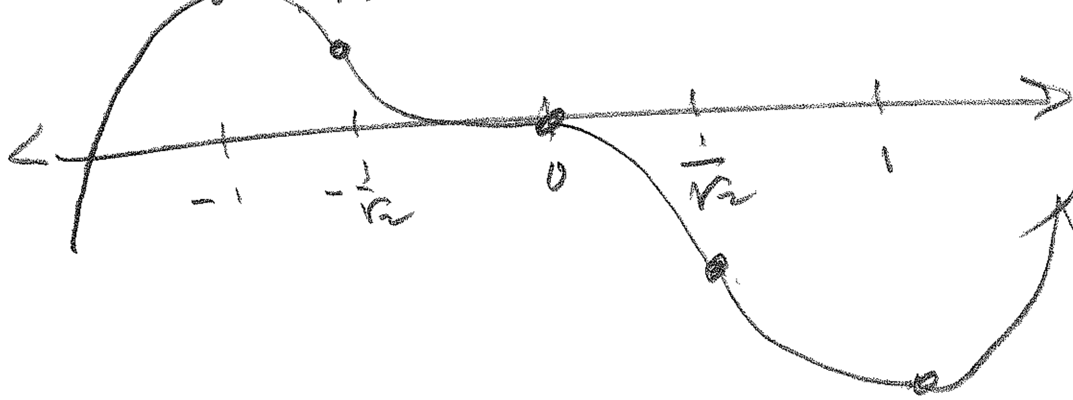
E3

① $f''(x) = 60x^3 - 30x$

$$= 30x(2x^2 - 1) \overset{SET}{=} 0$$

$$x = 0, \pm \frac{1}{\sqrt{2}}$$





201 E5

(2) $f(x)$ is polynomial $\Rightarrow$ Diff'bl & Cont $\forall x$.

$f(x) = x^3 - 3x^2 - 45x + 47$ on $[0, 3]$

$f(3) = -88$

$f(0) = 47$

10 pts

3	1	-3	-45	47
		3	0	-135
	1	0	-45	-88

$$\frac{-88-47}{3-0} = -\frac{135}{3} = -45 = m_{AVG}$$

$f'(x) = 3x^2 - 6x - 45$ SET -45

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0$ ✗ $\boxed{x = 2}$ $\rightarrow (2, -47)$

2	1	-3	-45	47
		2	-2	-94
	1	-1	-47	-47

Check: $f'(2) = 3(4) - 6(2) - 45$

$= 12 - 12 - 45 = -45$ ✓

(3) $f(x) = x^3 - 3x^2 - 45x + 47$

$f'(x) = 3x^2 - 6x - 45 = 3(x^2 - 2x - 15)$ SET 0

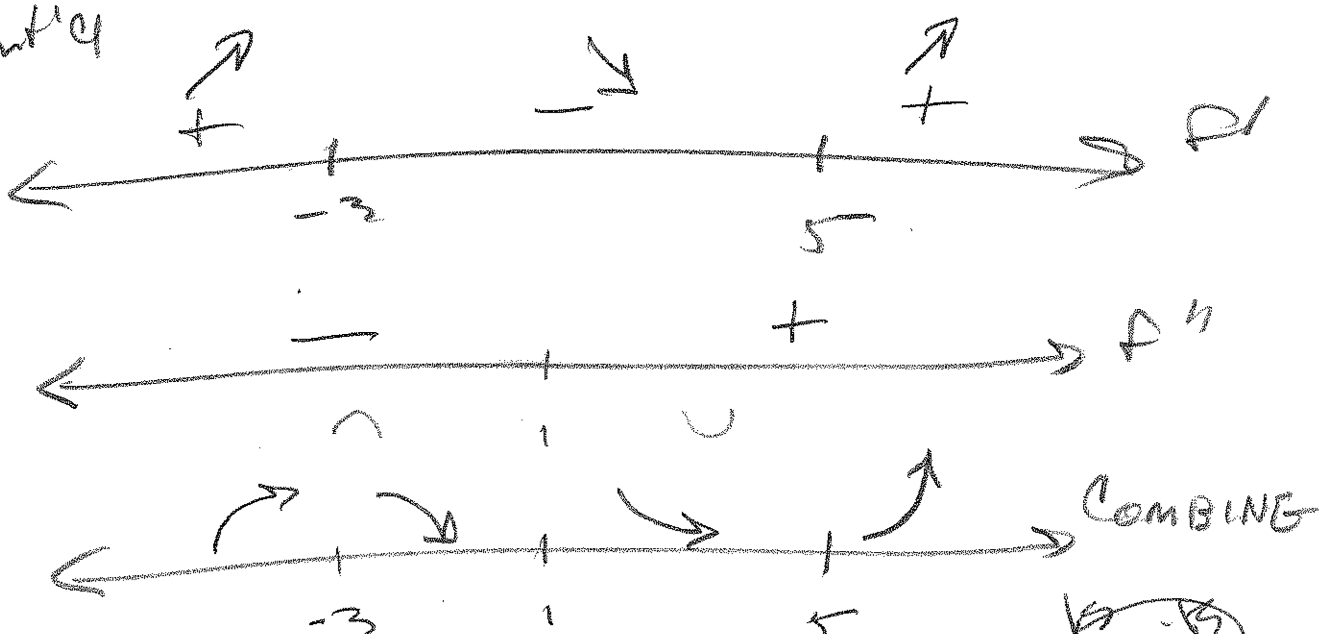
$\rightarrow (x-5)(x+3) = 0 \Rightarrow x \in \{-3, 5\}$

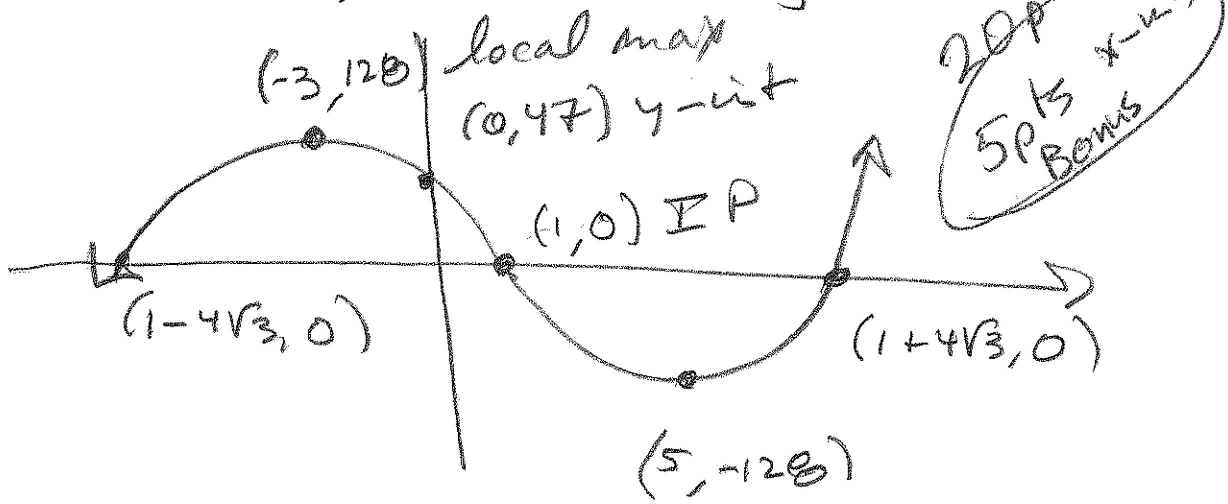
$f''(x) = 6x - 6$ SET 0 $\Rightarrow x = 1$

f'

201 E3

3 cnt'd





$$
\begin{array}{r|rrrr}
-3 & 1 & -3 & -45 & 47 \\
 & & -3 & 18 & 81 \\
\hline
 & 1 & -6 & -27 & 128
\end{array}
$$

$$
\begin{array}{r|rrrr}
1 & 1 & -3 & -45 & 47 \\
 & & 1 & -2 & -47 \\
\hline
 & 1 & -2 & -47 & 0
\end{array}
$$

201 E3

4 cont'd

$\boxed{5}$	1	-3	-45	47
		5	10	-175
	1	2	-35	-128

#8 work

Bonus Work

$f(x) = (x-1)(x^2-2x-47) \overset{\text{SET}}{=} 0 \Rightarrow$

$x = 1$ ✓ $\quad x^2-2x-47=0$

$x^2-2x+1^2 = 47+1$

$(x-1)^2 = 48$

$x = 1 \pm \sqrt{48} = 1 \pm 4\sqrt{3}$

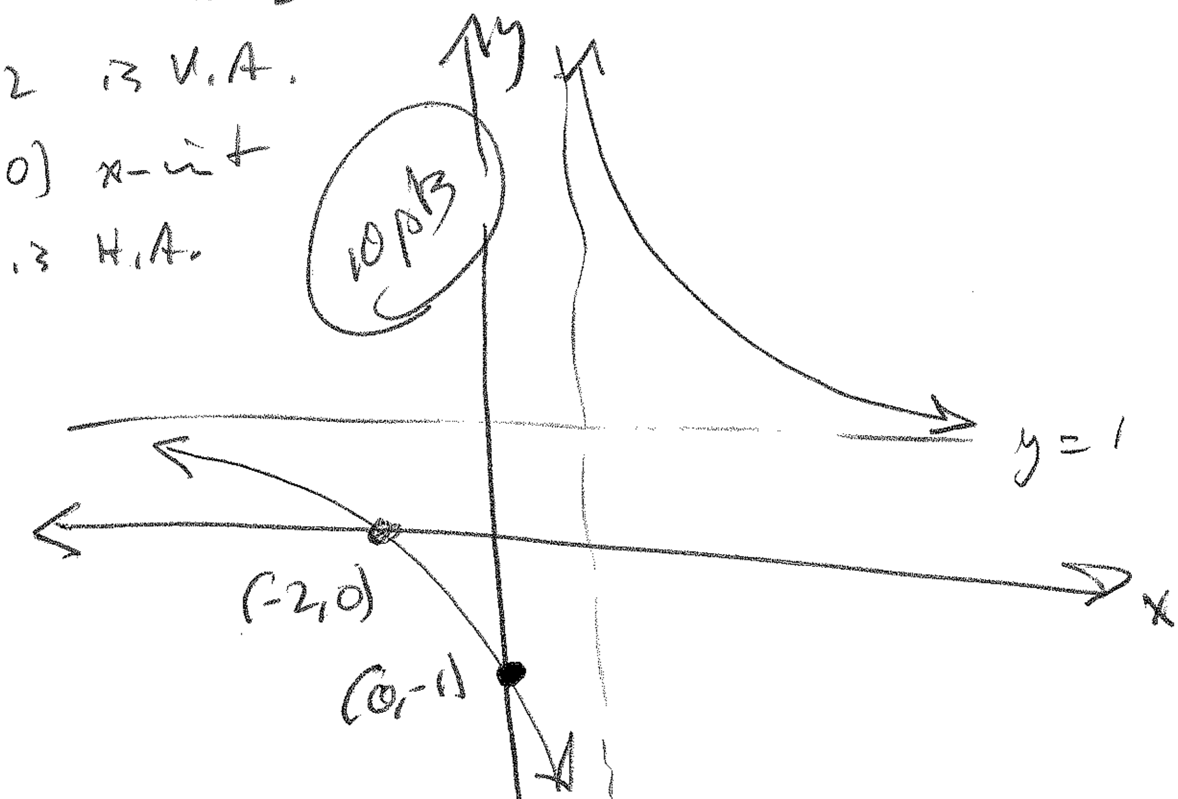
201 E3

(4) $f(x) = \dfrac{x+2}{x-2}$ $\mathcal{D} = \mathbb{R} \setminus \{2\}$

$x = 2$ is V.A.

$(-2,0)$ x-int

$y = 1$ is H.A.

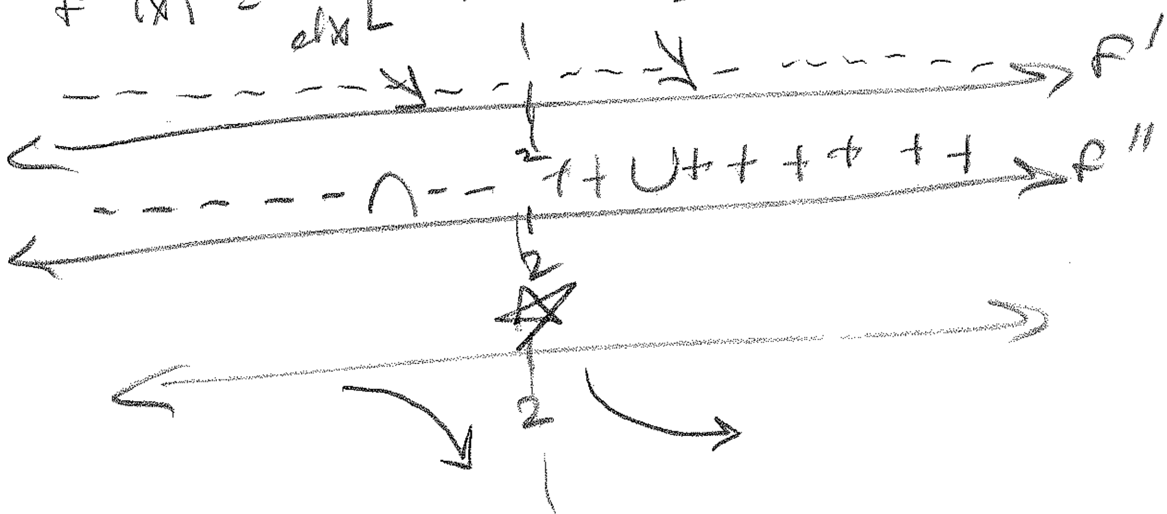


$x=2$

$$f'(x) = \frac{(x-2)-(x+2)}{(x-2)^2}$$

$$= \frac{-4}{(x-2)^2} < 0 \quad \forall x \neq 2$$

$$f''(x) = \frac{d}{dx}\left[-4(x-2)^{-2}\right] = 8(x-2)^{-3} = \frac{8}{(x-2)^3}$$



201 E3

⑤ $\sqrt{25x^2-7x} + 5x$

$25x^2$

$= \dfrac{\sqrt{25x^2-7x} + 5x}{1} \cdot \dfrac{\sqrt{25x^2-7x} - 5x}{\sqrt{25x^2-7x} - 5x}$

10 pts

$= \dfrac{25x^2-7x-25x^2}{\sqrt{25x^2-7x}-5x} = \dfrac{-7x}{-5x\left(\sqrt{1-\frac{7}{25x}}+1\right)}$

$\sqrt{25x^2} = 5\sqrt{x^2} = 5|x|$ @ $-\infty$ $\Rightarrow |x| = -x!$

$x \longrightarrow -\infty$ $\dfrac{-7}{-5(1+1)} = \dfrac{-7}{-10} = \dfrac{7}{10}$

⑥ Oblique Asymptote $\dfrac{2x^3-5x^2+7}{x^2-3}$

$$\begin{array}{r|l} & 2x-15 \\ \hline x^2-3 & 2x^3-15x^2+0x+7 \\ & -(2x^3 \quad\quad -6x) \\ \hline & -15x^2+6x+7 \end{array}$$

$y = 2x-5$ is O.A.

10 pts

201 E3 #5 in more detail

$$\lim_{x \to -\infty} \left(\sqrt{25x^2 - 7x} + 5x\right)$$

$x \to -\infty$ means all x's are negative &

$|x| = -x$ when x is negative.

$$\sqrt{25x^2 - 7x} + 5x = \left(\frac{\sqrt{25x^2 - 7x} + 5x}{1}\right)\left(\frac{\sqrt{25x^2 - 7x} - 5x}{\sqrt{25x^2 - 7x} - 5x}\right)$$

$$= \frac{\left(\sqrt{25x^2 - 7x}\right)^2 - (5x)^2}{\sqrt{25x^2 - 7x} - 5x} = \frac{25x^2 - 7x - 25x^2}{\sqrt{25x^2\left(1 - \frac{7x}{25x^2}\right)} - 5x}$$

$$= \frac{-7x}{\sqrt{25x^2}\sqrt{1 - \frac{7x}{25x^2}} - 5x} = \frac{-7x}{\sqrt{25}\sqrt{x^2}\sqrt{1 - \frac{7}{25x}} - 5x}$$

$$= \frac{-7x}{5|x|\sqrt{1 - \frac{7}{25x}} - 5x} = \frac{-7x}{-5x\sqrt{1 - \frac{7}{25x}} - 5x}$$

$|x| = -x$ when $x < 0$!

$$= \frac{-7x}{-5x\left(\sqrt{1 - \frac{7}{25x}} + 1\right)}$$

Factor out $-5x$ from denom

$$= \frac{-7}{-5\left(\sqrt{1 - \frac{7}{25x}} + 1\right)} \xrightarrow{x \to -\infty} \frac{-7}{-5(\sqrt{1} + 1)} = \frac{-7}{-10}$$

$$= \frac{7}{10}!$$

201 E3

(7) $f''(x) = 12x^2 + 12x - 6, \quad f'(1) = 8, \quad f(1) = -1$

$f'(x) = \frac{12}{3}x^3 + \frac{12}{2}x^2 - 6x + C$

$= 4x^3 + 6x^2 - 6x + C$

10pts

$f'(1) = 4 + 6 - 6 + C = 8$

$C + 4 = 8$

$C = 4$

$f'(x) = 4x^3 + 6x^2 - 6x + 4$

$f(x) = x^4 + 2x^3 - 3x^2 + 4x + d$

$f(1) = 1 + 2 - 3 + 4 + d = -1$

$d + 4 = -1$

$d = -5$

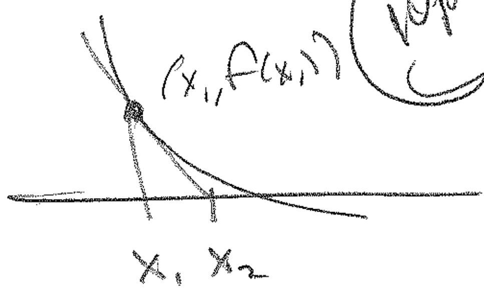
$$\boxed{f(x) = x^4 + 2x^3 - 3x^2 + 4x - 5}$$

2191 F3

(8) Fill in details of #3 (5pts Bonus)

(a) We find the zero of $f(x)$ by Newton's method, starting with a guess: $x = x_1$.

10pts



Tangent line thru $(x_1, f(x_1))$ is

$$y = m(x - x_1) + y_1$$
$$= f'(x_1)(x - x_1) + f(x_1).$$

x_2 is obtained by finding its intersection w/ x-axis:

$$y = f'(x_1)(x - x_1) + f(x_1) \overset{\text{SET}}{=} 0 \implies$$

$$f'(x_1)(x - x_1) = -f(x_1) \longrightarrow$$

$$x - x_1 = -\frac{f(x_1)}{f'(x_1)} \longrightarrow$$

$$x = x_1 - \frac{f(x_1)}{f'(x_1)} = x_2.$$ In general,

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$