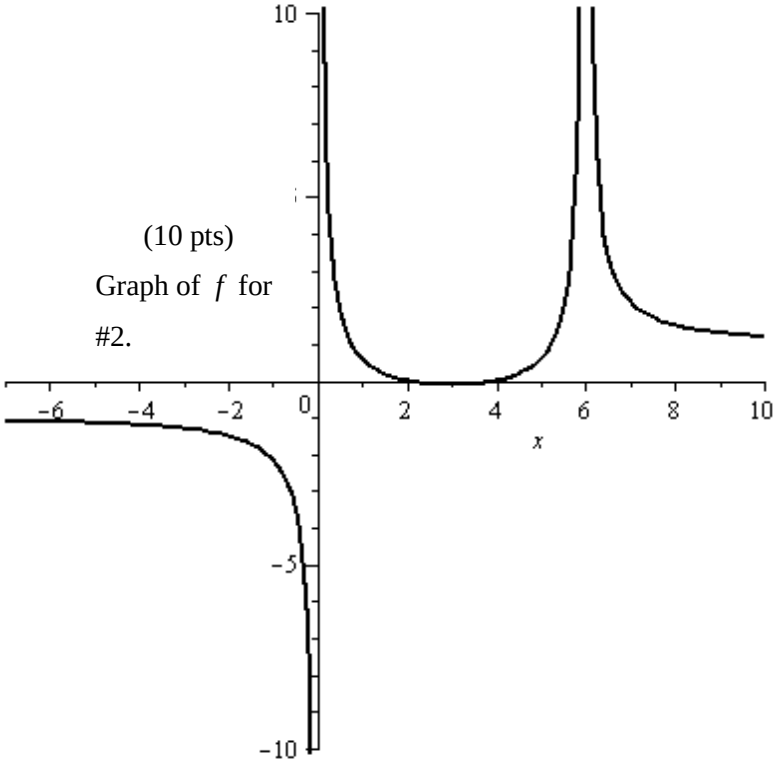
MAT 201 Test 2 – Spring, 2015 Name_____

100 Points Covers Chapter 2 NO GRAPHING CALCULATORS!!!

Show all work. Do your own work. Without supporting work, the slightest misstep leads to zero credit. Spread your work out! If you get stuck, start a fresh piece of paper. You can always *insert* more pages if you do it this way. No work should be on this cover sheet, except the graph for #2.

(10 pts) Graph of f for #2.



1. Let $f(x)=\sqrt{x+1}$.
   a. (5 pts) Find an equation of the tangent line to f at the point $(3, 2)$.
   b. (5 pts) Sketch a graph showing f and the tangent line to f at the point $(3, 2)$.
2. (10 pts) The graph of a function f is given on the right. On the same set of axes, sketch a graph of f'.
3. (5 pts each) Differentiate the following with respect to the independent variable.
   a. $f(x)=x^5-6x^{\frac{7}{3}}+6\sqrt[3]{x^7}+4x^{\frac{2}{5}}-\frac{3}{2}x^{-\frac{2}{3}}$
   b. $h(\omega)=\left(\omega^2+3\omega+13\right)\left(\omega^3-7\omega^2\right)$
   c. $H(t)=\dfrac{t^2+3t}{t^3+6t-11}$
   d. $g(x)=\left(x^2+3x+13\right)^3\left(x^3-7x^2\right)^{-5}$
   e. $r(x)=\dfrac{\left(x^2+3x\right)^3}{\left(x^3-7x^2\right)^5}$
   f. $Q(t)=\dfrac{\sin\left(t^2-3t\right)}{\cos(5t)}$
   g. $R(x)=\dfrac{\csc^3(5x)}{\tan(\pi x)}$
4. (10 pts) Show that $f(x)=x^3-6x^2+15x-7$ has no tangent line with a slope of $m=-2$.
5. Consider the relation $y\sin(2x)=x\cos(2y)$.
   a. (5 pts) Use implicit differentiation to find $y'=\dfrac{dy}{dx}$.
   b. (5 pts) Find an equation of the tangent line to the curve at the point $\left(\dfrac{\pi}{2}, \dfrac{\pi}{4}\right)$.

6. (10 pts) A lighthouse is located on a small island *exactly* $\sqrt{3}$ km from the nearest point P on a straight shoreline. The light makes 5 revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km away from P ?

7. (10 pts) The radius of a sphere is 3 cm, with a possible error in measurement of 0.1 cm.
   a. Use differentials to estimate the error in the volume calculated from this measurement of the radius.

      $\left(\text{Hint: The volume of a sphere is given by } V = \frac{4}{3}\pi r^3\right)$.

   b. What is the relative error?
   c. What is the percent error?

Work up to 2 Bonus questions for up to 10 points extra.

Bonus (5 pts) Show, using implicit differentiation, that any tangent line to a circle, at a point $P = (x_1, y_1)$ on the circle, is perpendicular to the radius QP, where $Q = (h,k)$ is the center of the circle.

Bonus (5 pts) Prove that $\lim_{x\to 3}\left(x^2 - 2x + 1\right) = 4$.

Bonus (5 pts) Give a rough sketch of the graph of $y = 3(2x-7)^{2/3} - 3$, by transforming the graph of a basic function. Include x- and y-intercepts.

Bonus (5 pts) Convince me that $f(x) = x^4 - 3x^3 - 22x^2 + 78x - 60$ has a zero in the interval $(4,5)$, without, you know, actually finding it.

Bonus (5 pts) Approximate $\sin\left(48^0\right)$ using the linearization.