

Show all work. Do your own work. Without supporting work, the slightest misstep leads to zero credit. Spread your work out! If you get stuck, start a fresh piece of paper. You can always *insert* more pages if you do it this way, but don't make me jump around, or cram your work into a small space with some misguided "Save the Trees!" thinking. WE save trees by doing our math well enough to get a job and *plant more trees*.

- (15 pts) The point $P(2, -2)$ lies on the graph of $f(x) = x^2 - 3x$. Estimate the slope of this curve at $x = 2$, by evaluating the average slope between P and the point $Q(x, x^2 - 3x)$, which is just another point on the graph of f . Use $x = 2.001$ and $x = 1.999$. I want your estimates to be accurate to the 4th decimal place.
- (5 pts) Tell me what you think the precise slope of f is, at $x = 2$.
- (5 pts) Based on your answer to #2, write the equation of the tangent line to $f(x) = x^2 - 3x$ at $x = 2$.
- (5 pts each) Evaluate the following limits, if they exist. When one does not exist, say so.
 - $\lim_{x \rightarrow 2} \frac{x^2 + 5x - 14}{2x^2 - 9x + 10}$
 - $\lim_{x \rightarrow 5^-} \frac{|x - 5|}{3x^2 - 11x - 20}$
 - $\lim_{x \rightarrow 5} \frac{|x - 5|}{3x^2 - 11x - 20}$
- (15 pts) Sketch the graph of the piecewise-defined function $f(x) = \begin{cases} 2(x+2)^2 - 3 & \text{if } x < -1 \\ 2x + 3 & \text{if } x \geq -1 \end{cases}$. On what interval(s) is it continuous?

Bonus (5 pts) What value of a will make $f(x) = \begin{cases} 2(x+2)^2 - 3 & \text{if } x < -1 \\ 2x + a & \text{if } x \geq -1 \end{cases}$ continuous?

6. Compute $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ for the following.

a. (10 pts) $f(x) = x^2 - 3x - 7$

b. (5 pts) $f(x) = \sqrt{x}$

7. (10 pts) Sketch a plausible graph of a function, f , that satisfies all of the properties listed.

a. $\lim_{x \rightarrow -5^-} f(x) = 3$

d. $\lim_{x \rightarrow 1^-} f(x) = -\infty$

g. $f(4) = 3$

b. $\lim_{x \rightarrow -5^+} f(x) = 1$

e. $\lim_{x \rightarrow 1^+} f(x) = \infty$

c. $f(-5) = 3$

f. $\lim_{x \rightarrow 4} f(x) = 2$

8. (10 pts) Prove that $\lim_{x \rightarrow 2} (3x - 2) = 4$. (This is the $\epsilon - \delta$ proof you're dying to do.)

See back side of this test!

9. (5 pts) Convince me that $f(x) = x^4 - 6x^3 + 2x^2 + 14x + 5$ has a zero in the interval $(2,3)$, without, you know, actually finding it.

Bonus Beyond this Point. Answer any 2 bonus problems for up to 20!? bonus points. I grade the first two I come to, so don't do 'em all and expect me to just grade the one(s) you got right. Teacher don't play dat.



Are you smarter than the average bear? *This bear is smarter than the average ranger!*

10. (10 pts) Prove that $\lim_{x \rightarrow 2} (x^2 - 3x) = -2$, using the precise definition of limit.

11. (10 pts) Simplify the difference quotient and pass to the limit as h approaches zero for the function

$$f(x) = \frac{1}{\sqrt{x}}.$$

12. (10 pts) See if you can squeeze out a convincing argument for the claim $\lim_{x \rightarrow 0} \left(x^2 \sin\left(\frac{1}{x}\right) \right) = 0$.

13. (10 pts) Write the definition of the piecewise-defined function from its graph.

