201 TEST 1 KEY

1 15pts

$$\frac{f(2.001)-f(2)}{2.001-2} = \boxed{m_{sec} \approx 1.001}$$

$(x_1,y_1)=(2,-2)$

$(x_2,y_2)=(2.001, f(2.001))$

$$\frac{f(1.999)-f(2)}{1.999-2} = \boxed{m_{sec} \approx .999}$$

$(x_2,y_2)=(1.999, f(1.999))$

2 5pts $\boxed{m_{tan} = 1}$ is my guess.

3 5pts $y = m(x-x_1)+y_1$

$\boxed{y = 1(x-2)-2} = x-2-2 = x-4 = y$

4

$$\frac{x^2+5x-14}{2x^2-9x+10} = \frac{(x+7)(x-2)}{(2x-5)(x-2)} = \frac{x+7}{2x-5}$$

$(x \neq 2)$

2 5pts

$$\xrightarrow{x \to 2} \frac{2+7}{2(2)-5} = \frac{9}{-1} \boxed{=-9}$$

5 5pts

$$\frac{|x-5|}{3x^2-11x-20} = \frac{-(x-5)}{(3x+4)(x-5)} \text{ if } x<5$$

$$= \frac{-1}{3x+4} \xrightarrow{x \to 5^-} \frac{-1}{3(5)+4} = \frac{-1}{19} = \boxed{\frac{-1}{19}}$$

201 TEST 1 KEY

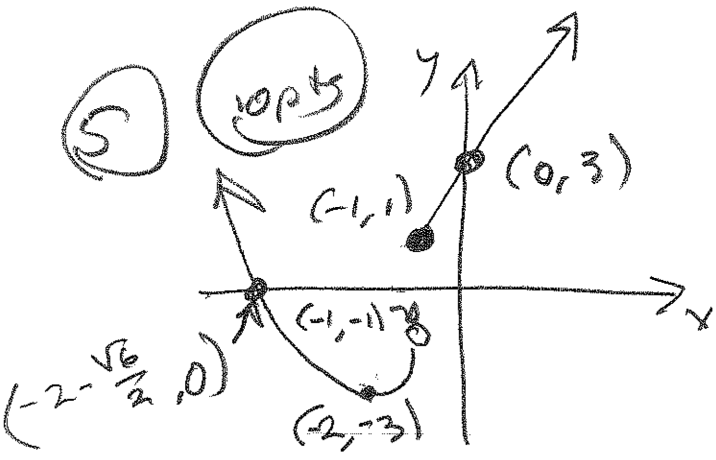
(c) For $\lim_{x \to 5} \frac{|x-5|}{3x^2-11x-20}$, we need to look @ $\lim_{x \to 5^+} f(x)$. When $x > 5$, we have

$$\frac{x-5}{(3x+4)(x-5)} = \frac{1}{3x+4} \xrightarrow{x \to 5^+} \frac{1}{3(5)+4} = \frac{1}{19}$$

So, the limit $\nexists$, since right-hand & left-hand don't agree, i.e.

$$\lim_{x \to 5^-} f(x) = -\frac{1}{19} \neq +\frac{1}{19} = \lim_{x \to 5^+} f(x)$$

(5) 10pts



$2(-1+2)^2 - 3$
$= 2(1)^2 - 3$
$= 2 - 3 = -1 \rightsquigarrow (-1,-1)$
$(h, k) = (-2, -3)$

$2(-1) + 3 = -2 + 3 = 1$
$\rightsquigarrow (-1, 1)$

$2(x+2)^2 = 3$

$(x+2)^2 = \frac{3}{2}$

$x = -2 \pm \sqrt{\frac{3}{2}} = -2 \pm \frac{\sqrt{6}}{2}$

$\boxed{\text{f is cnts on } (-\infty, -1) \cup (-1, \infty)}$ 5pts

201 TEST 1 KEY

BONUS

5 pts

$\lim_{x \to -1^-} f(x) = -1$, by previous.

Want $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (2x+a) = 2(-1)+a$

$= -2+a$ to be same ∴

$-2+a = -1 \implies$

$\boxed{a=1}$

10 pts

6 $x^2-3x-7 = f(x) \implies$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-3(x+h)-7-(x^2-3x-7)}{h}$$

$$= \frac{x^2+2xh+h^2-3x-3h-7-x^2+3x+7}{h}$$

$$= \frac{2xh+h^2-3h}{h} = \frac{h(2x+h-3)}{h}$$

$$= 2x+h-3 \xrightarrow{h \to 0} \boxed{2x-3}$$

$(h \neq 0)$

2101 TEST 1 KEY

6 b $f(x) = \sqrt{x} \implies$ Difference Quotient

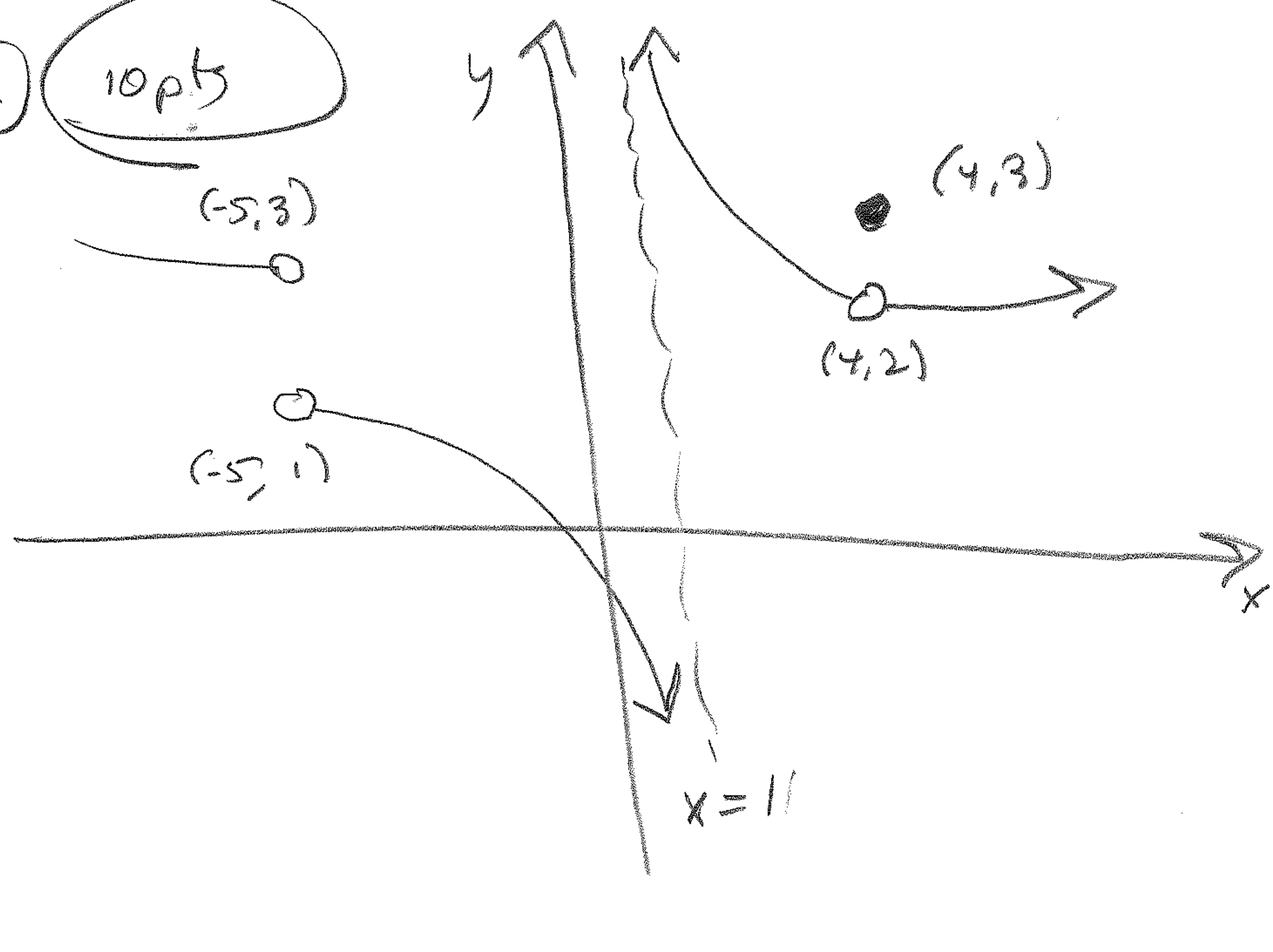
10 pts

is $\dfrac{\sqrt{x+h} - \sqrt{x}}{h} = \dfrac{(\sqrt{x+h} - \sqrt{x})}{h}\left[\dfrac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right]$

$= \dfrac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \dfrac{h}{h(\sqrt{x+h} + \sqrt{x})} = \dfrac{1}{\sqrt{x+h} + \sqrt{x}}$ $(h \neq 0)$

$\xrightarrow{h \to 0} \dfrac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\dfrac{1}{2\sqrt{x}}}$

7 10 pts



201 TEST 1 KEY

(8) $\lim_{x \to 2} (3x-2) = 4$

$\boxed{\text{Pf}}$ Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{3}$.

Then $0 < |x-2| < \delta \longrightarrow$

$$|(3x-2)-4| = |3x-6| = 3|x-2| < 3\delta$$

$$= 3 \cdot \frac{\varepsilon}{3} = \varepsilon \blacksquare$$

(9) $f(x) = x^4 - 6x^3 + 2x^2 + 14x - 5$

$f(2) = 9$

$$\begin{array}{r|rrrrr} 2 & 1 & -6 & 2 & 14 & +5 \\ & & 2 & -8 & -12 & 4 \\ \hline & 1 & -4 & -6 & 2 & 9 \end{array}$$

$f(3) = -16$

$$\begin{array}{r|rrrrr} 3 & 1 & -6 & 2 & 14 & +5 \\ & & 3 & -9 & -21 & -21 \\ \hline & 1 & -3 & -7 & -7 & -16 \end{array}$$

$f(2) > 0 > f(3)$

$\longrightarrow \exists\, c \in (2,3)$

$\ni f(c) = 0$, by IVT & continuity of polynomials!

201 TEST 1 KEY

(10) CLAIM: $\lim_{x\to 2}(x^2-3x) = -2$

Scratch

$x^2-3x = -2$

$x^2-3x+2=0$

$|(x-2)(x-1)| < |x-1|\,\delta$

Assume $\delta \le 1$!

$1 \le x \le 3$

$0 \le x-1 \le 2$ $\longrightarrow \frac{\varepsilon}{2}$!

Proof Let $\varepsilon>0$ be given

Define $\delta = \min\{1, \frac{\varepsilon}{2}\}$. Then

$0<|x-2|<\delta \Longrightarrow$

$|x^2-3x-(-2)| = |x^2-3x+2| = |x-1||x-2|$

$\le 2|x-2| < 2\delta \le 2\cdot\frac{\varepsilon}{2} = \varepsilon$ ∎

201 TEST 1 KEY

(1) $f(x) = \frac{1}{\sqrt{x}} \Longrightarrow$

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right]$$

$$= \frac{1}{h}\left[\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right]$$

$$= \frac{1}{h}\left[\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right]\left[\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}\right]$$

$$= \frac{1}{h}\left[\frac{x - x - h}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)}\right]$$

$$= \frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)} \quad (h \neq 0)$$

$$\xrightarrow{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x}\left(\sqrt{x} + \sqrt{x}\right)} = \frac{-1}{x(2\sqrt{x})} = \boxed{-\frac{1}{2x\sqrt{x}}}$$

201 TEST 1 KEY

12. $-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$, since

$-1 \le \sin\left(\frac{1}{x}\right) \le 1$ & $x^2 \ge 0$, so,

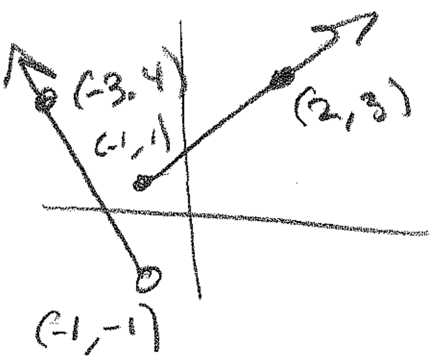
$$\lim_{x \to 0} (-x^2) \le \lim_{x \to 0} x^2 \sin\left(\tfrac{1}{x}\right) \le \lim_{x \to 0} x^2$$

but this just says

$$0 \le \lim_{x \to 0} x^2 \sin\left(\tfrac{1}{x}\right) \le 0, \text{ i.e.,}$$

$$\lim_{x \to 0} x^2 \sin\left(\tfrac{1}{x}\right) = 0 \quad \blacksquare$$

13.



$(-3,4)$, $(-1,-1)$

$$m = \frac{-1-4}{-1+3} = \frac{-5}{2}$$

$(-1,1)$, $(2,3)$

$$m = \frac{3-1}{2+1} = \frac{2}{3}$$

$$\boxed{f(x) = \begin{cases} -\frac{5}{2}(x+1) - 1 & \text{if } x < -1 \\ \frac{2}{3}(x+1) + 1 & \text{if } x \ge -1 \end{cases}}$$