MAT 201 Final Test – Fall, 2013 Name_________________

100 Points Covers Chapters 1 thru 5 NO GRAPHING CALCULATORS!!!

1. (5 pts) Evaluate $\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16}$.
2. Evaluate each of the following by factoring and simplifying. One exists. The other doesn't.
   a. (5 pts) $\lim_{x \to 5} \frac{2x^2 - 13x + 15}{x^2 - 3x - 10}$
   b. (5 pts) $\lim_{x \to 5} \frac{2x^2 + 13x + 15}{x^2 - 3x - 10}$
3. (5 pts) Prove that $\lim_{x \to 2} (5x - 7) = 3$ (This is the $\varepsilon - \delta$ proof you're dying to do.)
4. (5 pts) Compute the derivative of $f(x) = 2x^2 - 3x + 5$ by the definition of derivative. This means taking the limit of a difference quotient.
5. (5 pts each) Compute the derivatives of each of the following. Do not simplify your answer.
   a. $y = (\sin(x^2 - 3x))(\cos(x))$
   b. $y = \frac{\tan(x^2 - 3x)}{x^2 - 3x}$
6. (5 pts) Find an equation of the tangent line to $f(x) = \cos(x)$ at $x = \frac{\pi}{4}$.
7. (5 pts) Use your result from the previous problem to approximate $\cos(48^0)$
8. Let $f(x) = x + 2\cos(x)$ on $[0, 2\pi]$. (Bonus: Same question, with $f(x) = x + \cos(2x)$)
   a. (5 pts) Find all critical values of f on $(0, 2\pi)$. Find the corresponding points on the graph of *f*. Report them as ordered pairs, for now.
   b. (5 pts) Find all inflection points of f on $(0, 2\pi)$. Report *these* as ordered pairs.
   c. (5 pts) Based on your work in *a* and *b*, above, provide a nice, neatly labeled sketch of the graph of f on the interval $(0, 2\pi)$. Labels and the *qualities* of the graph are more important than a slavish adherence to tickmarks on an axis.
9. (10 pts) Find $\frac{dy}{dx}$, given that $\csc(x) + \cos(y) = 2x^2y^3 - 3x^2y^2$
10. (5 pts) Use the Intermediate Value Theorem to show that $f(x) = 2x^3 - x^2 - 83x + 154$ has a zero in the interval [3, 6].
11. (5 pts) Show that it is fruitless to find a spot on the graph of $f(x) = \frac{1}{x}$ on the interval (-1, 1), where the instantaneous slope of f is the same as the average slope of f on [-1, 1]. *Then* explain why this doesn't mean the Mean Value Theorem is a lie.
12. Consider the region bounded by $y = (x-1)^3$, $x = 1$, and $x = 2$
    a. (5 pts) Sketch this region and find its area.
    b. (5 pts) Sketch the solid obtained by rotating this region about the line $x = -1$. Include a representative cylinder on your graph and write the integral for finding its volume by the shell method.Sketch the region.
    c. (5 pts) Sketch the solid obtained by rotating this region about the line $x = -1$. Include a representative washer on your graph and write the integral for finding its volume by the washer method.