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① $\dfrac{x^3-64}{x^2-16} = \dfrac{(x-4)(x^2+4x+16)}{(x-4)(x+4)} = \dfrac{x^2+4x+16}{x+4}$

$\xrightarrow{x \to 4} \dfrac{16+16+16}{8} = \dfrac{48}{8} = \boxed{6}$

(2a) $\dfrac{2x^2-13x+15}{x^2-3x-10} = \dfrac{(2x-3)(x-5)}{(x-5)(x+2)} = \dfrac{2x-3}{x+2}$

$\xrightarrow{x \to 5} \dfrac{7}{7} = \boxed{1}$

(2b) $\dfrac{2x^2+13x+15}{x^2-3x-10} = \dfrac{(2x+3)(x+5)}{(x-5)(x+2)} \xrightarrow{x \to 5} \boxed{\nexists}$

③ Claim: $\lim_{x \to 2} (5x-7) = 3$

Proof: Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{5}$.

Then $0 < |x-2| < \delta \implies$

$$|(5x-7)-3| = |5x-10| = 5|x-2| < 5\delta = 5\frac{\varepsilon}{5} = \varepsilon$$

④ $\dfrac{2(x+h)^2-3(x+h)+5-(2x^2-3x+5)}{h}$

$= \dfrac{2(x^2+2xh+h^2)-3x-3h+5-2x^2+3x-5}{h}$

$= \dfrac{2x^2+4xh+2h^2-3h-2x^2}{h} = \dfrac{4xh+2h^2-3h}{h} = 4x+2h-3 \xrightarrow{h \to 0} \boxed{4x-3} = f'(x)$

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(5a) $\frac{d}{dx}\left[(\sin(x^2-3x))(\cos(x))\right]$

$= (\cos(x^2-3x))(2x-3)\cos(x) + (\sin(x^2-3x))(-\sin(x))$

(5b) $\frac{d}{dx}\left[\frac{\tan(x^2-3x)}{x^2-3x}\right] = \frac{((\sec^2(x^2-3x))(2x-3))(x^2-3x)-(\tan(x^2-3x))(2x-3)}{(x^2-3x)^2}$

(6) $y = f(x) = \cos x$ @ $x = \frac{\pi}{4}$:

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \rightsquigarrow (x_0, y_0) = (\frac{\pi}{4}, \frac{\sqrt{2}}{2})$

$f'(x) = -\sin x$

$f'(\frac{\pi}{4}) = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\rightsquigarrow m = -\frac{\sqrt{2}}{2}$

$y = m(x-x_0) + y_0$

$\boxed{y = -\frac{\sqrt{2}}{2}(x-\frac{\pi}{4}) + \frac{\sqrt{2}}{2}} \approx .707(x-.785) + .707$

(7) $\cos(48^\circ) = \cos(45^\circ + 3^\circ)$

$x_0 = 45^\circ = \frac{\pi}{4}$ rad

$\Delta x = 3^\circ = \frac{3\pi}{180} = \frac{\pi}{60}$ rad

$f(x+\Delta x) \approx -\frac{\sqrt{2}}{2}(\frac{\pi}{4}+\frac{\pi}{60}-\frac{\pi}{4}) + \frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{2}(\frac{\pi}{60}) + \frac{\sqrt{2}}{2} = \boxed{-\frac{\sqrt{2}\pi}{120} + \frac{\sqrt{2}}{2}} \approx .6700827567$

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8. $f(x) = x + 2\cos x$

a) $f'(x) = 1 - 2\sin x \overset{\text{SET}}{=} 0$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

$$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 2\cos\frac{\pi}{6} = \frac{\pi}{6} + 2\cdot\frac{\sqrt{3}}{2} = \frac{\pi}{6} + \sqrt{3}$$

$$f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + 2\cos\frac{5\pi}{6} = \frac{5\pi}{6} - 2\frac{\sqrt{3}}{2} = \frac{5\pi}{6} - \sqrt{3}$$

$$\boxed{A = \left(\frac{\pi}{6}, \frac{\pi}{6} + \sqrt{3}\right) \quad B = \left(\frac{5\pi}{6}, \frac{5\pi}{6} - \sqrt{3}\right)}$$

MAX MIN

b) $f''(x) = -2\cos x \overset{\text{SET}}{=} 0 \longrightarrow$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + 2\cos\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \rightsquigarrow$ C = $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ IPs

$f\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} + 2\cos\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} \rightsquigarrow$ D = $\left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

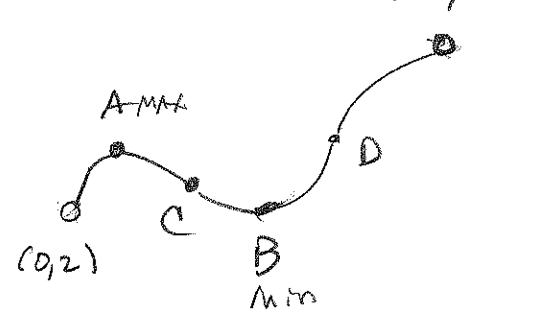
f': $+$ | $\frac{\pi}{6}$ | $-$ | $\frac{5\pi}{6}$ | $+$

$\nearrow$ $\frac{\pi}{6}$ $\searrow$ $\frac{5\pi}{6}$ $\nearrow$

$f'(0) = 1$, $f''(0) = -2$

f'': $-$ | $\frac{\pi}{2}$ | $+$ | $\frac{3\pi}{2}$ | $-$

$\frown$ $\frac{\pi}{2}$ $\smile$ $\frac{3\pi}{2}$ $\frown$



$A = \left(\frac{\pi}{6}, \frac{\pi}{6} + \sqrt{3}\right) \approx \left(\frac{\pi}{6}, 2.2556\right)$

$B = \left(\frac{5\pi}{6}, \frac{5\pi}{6} - \sqrt{3}\right) \approx \left(\frac{5\pi}{6}, .8859\right)$

$C = \left(\frac{\pi}{2}, \frac{\pi}{2}\right) \approx \left(\frac{\pi}{2}, 1.5708\right)$, $D = \left(\frac{3\pi}{2}, 4.7124\right)$

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(9) $\csc x + \cos y = 2x^2y^3 - 3x^2y^2$

$-\csc x \cot x - (\sin y)y' = 4xy^3 + 6x^2y^2y' - 6xy^2 - 6x^2yy'$

$(-\sin y - 6x^2y^2 - 6x^2y)\,y' = \csc x \cot x + 4xy^3 - 6xy^2$

$$\boxed{y' = \frac{\csc x \cot x + 4xy^3 - 6xy^2}{-\sin y - 6x^2y^2 - 6x^2y}}$$

(10) $f(x) = 2x^3 - x^2 - 83x + 154$ has a zero in $(3,6)$

3	2	-1	-83	154
		6	15	-204
	2	5	-68	$\boxed{-50 = f(3)}$

6	2	-1	-83	154
		12	66	-102
	2	11	-17	$\boxed{52 = f(6)}$

$f(3) < 0$
$f(6) > 0$
f is polynomial (hence cont's) $\Rightarrow$

$\exists\, c \in (3,6) \ni f(c) = 0$ by IVT.

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(11) $f(x) = \frac{1}{x} = x^{-1} \Rightarrow$

$f'(x) = -\frac{1}{x^2}$

$f(-1) = \frac{1}{-1} = -1 \rightsquigarrow (-1,-1)$

$f(1) = \frac{1}{1} = 1 \rightsquigarrow (1,1)$ $\longrightarrow$

m_{AVG} on $[-1,1]$ is $m = \frac{-1-1}{-1-1} = \frac{-2}{-2} = 1 = m_{AVG}$.

But $-\frac{1}{x^2} = 1 \Rightarrow$

$x^2 = -1 \Rightarrow$

$x = \pm i$! Not real! No real solution, i.e.

$\therefore$ f has no $c \in (-1,1) \ni f'(c) = \frac{f(1)-f(-1)}{1-(-1)}$.

But thats no violation or invalidation of MVT, which requires f is cnt^s on $[-1,1]$, which it is not, and requires f be difb^l on $(-1,1)$, which it is not.

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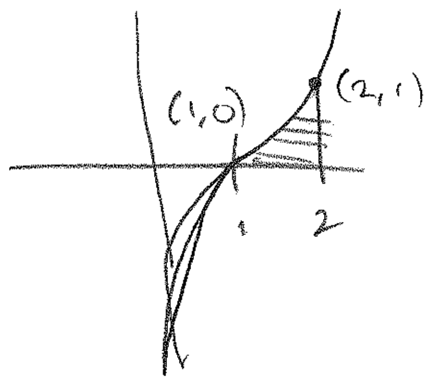
12. $y = (x-1)^3,\ x=1,\ x=2$

(a)



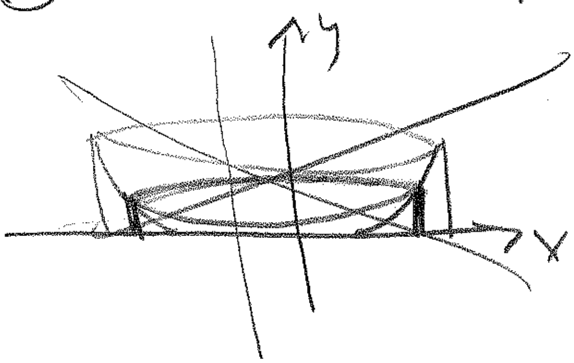
$A = \int_1^2 (x-1)^3\,dx$

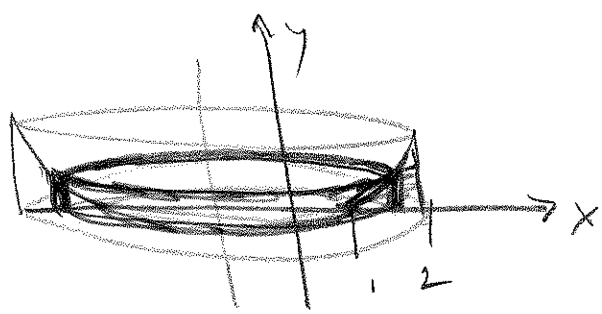
$u = x-1 \quad x=2 \to u = 2-1 = 1$

$du = dx \quad x = 1 \to u = 1-1 = 0$

This gives $\int_0^1 u^3\,du = \frac{1}{4}u^4\Big]_0^1 = \boxed{\frac{1}{4} = \text{Area}}$

(b) About $x=-1$ by cylindrical shells





$V = 2\pi \int_1^2 \underbrace{x}_{\text{radius}}\ \overbrace{f(x)}^{\text{height}}\,dx = 2\pi \int_1^2 (x-(-1))(x-1)^3\,dx$

$= \frac{7\pi}{5}$

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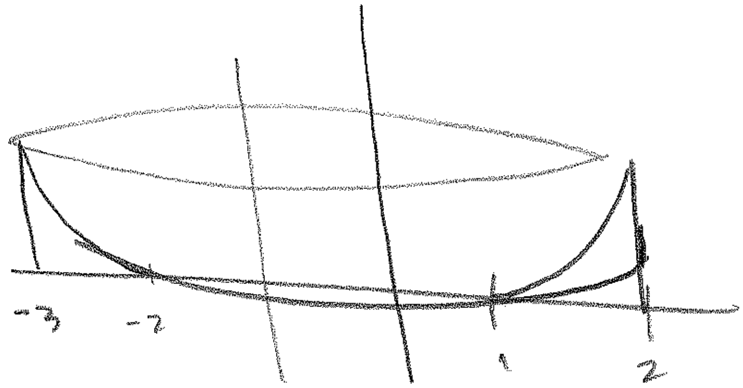
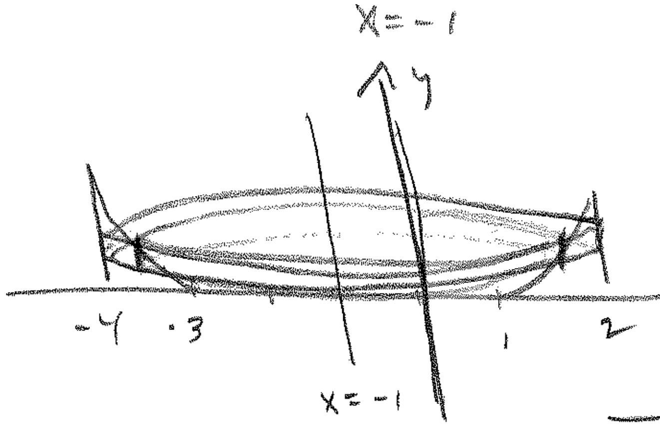
12 c Washers





$\pi \int \text{outer}^2 - \text{inner}^2 \, dy$

$\pi \int_0^1 \, dy$

outer = $2 - (-1) = 3$

inner: $y = (x-1)^3$

$\boxed{\text{Vol} = \pi \int_0^1 \left(3^2 - (\sqrt[3]{y} + 2)^3\right) dy}$

$= \frac{7\pi}{5}$

$\sqrt[3]{y} = x - 1$

$\sqrt[3]{y} + 1 = x$

$(\sqrt[3]{y} + 1) - (-1) = \sqrt[3]{y} + 2 = \cancel{x}$

Right − Left

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BONUS $f(x) = x + \cos(2x)$

(a) $f'(x) = 1 - 2\sin(2x) \overset{\text{SET}}{=} 0$

$\sin(2x) = \frac{1}{2}$

$2x = \frac{\pi}{6}$ $\quad$ $2x = \frac{5\pi}{6}$

$x = \frac{\pi}{12}$ $\quad$ $x = \frac{5\pi}{12}$

$2x = \frac{\pi}{6} + 2\pi$ $\quad$ $2x = \frac{5\pi}{6} + 2\pi$

$x = \frac{\pi}{12} + \pi$ $\quad$ $x = \frac{5\pi}{12} + \pi$

$= \frac{\pi + 12\pi}{12}$ $\quad$ $= \frac{5\pi + 12\pi}{12}$

$= \frac{13\pi}{12} \in (0, 2\pi)$ ✓ $\quad$ $= \frac{17\pi}{12} \in (0, 2\pi)$ ✓

CVS: $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

$f\left(\frac{\pi}{12}\right) = \frac{\pi}{12} + \cos\left(\frac{\pi}{6}\right) = \frac{\pi}{12} + \frac{\sqrt{3}}{2} \approx 1.127824792$

$f\left(\frac{5\pi}{12}\right) = \frac{5\pi}{12} + \cos\frac{5\pi}{6} = \frac{5\pi}{12} - \frac{\sqrt{3}}{2} \approx .4429715352$

$f\left(\frac{13\pi}{12}\right) = \frac{13\pi}{12} + \cos\frac{13\pi}{6} = \frac{13\pi}{12} + \frac{\sqrt{3}}{2} \approx 4.269417445$

$f\left(\frac{17\pi}{12}\right) = \frac{17\pi}{12} + \cos\left(\frac{17\pi}{6}\right) = \frac{17\pi}{12} + \frac{\sqrt{3}}{2} \approx 3.584564190$

$+ \quad - \quad + \quad - \quad +$

$\frac{\pi}{12} \quad \frac{5\pi}{12} \quad \frac{13\pi}{12} \quad \frac{17\pi}{12}$

$f'(0) = 1 - 0 = 1$ $+$

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BONUS

(b) $f''(x) = -4\cos(2x) \overset{\text{SET}}{=} 0$

$2x = \frac{\pi}{2}, \frac{3\pi}{2},$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\frac{\pi}{2} + 2\pi = 2x$

$\frac{\pi}{4} + \pi = x \in (0, 2\pi$

$\frac{5\pi}{4} = x$

$\frac{3\pi}{2} + 2\pi = 2x$

$x = \frac{3\pi}{4} + \pi$

$= \frac{3\pi + 4\pi}{4}$

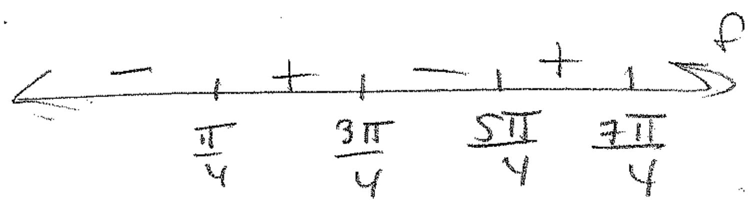
$= \frac{7\pi}{4} \in (0, 2\pi)$

$f(\frac{\pi}{4}) = \frac{\pi}{4} + \cos\frac{\pi}{2} = \frac{\pi}{4} \approx .785398$

$f(\frac{3\pi}{4}) = \frac{3\pi}{4} + \cos\frac{3\pi}{2} = \frac{3\pi}{4} \approx 2.356194$

$f(\frac{5\pi}{4}) = \frac{5\pi}{4} \approx 3.92699$

$f(\frac{7\pi}{4}) = \frac{7\pi}{4} \approx 5.497787144$



$f''(0) = -4 \quad -$

$A \approx (\frac{\pi}{12}, 1.128)$

$B \approx (\frac{\pi}{4}, .785)$

$C \approx (\frac{5\pi}{12}, .443)$

$D \approx (\frac{3\pi}{4}, 2.356)$

$E \approx (\frac{13\pi}{12}, 4.269)$

$F \approx (\frac{5\pi}{4}, 3.927)$

$G \approx (\frac{17\pi}{12}, 3.585)$

$H \approx (\frac{7\pi}{4}, 5.498)$