MAT 201 Test 3 – Fall, 2013 Name________________

100 Points Covers Chapter 4 NO GRAPHING CALCULATORS!!!

There are up to 10 bonus points available. I grade the first 2 bonus problems I come to and ignore the rest. So be sure to tell me which Bonuses you want to do.

1. (10 pts) Use the limit definition of the definite integral to evaluate $\int_0^3 (x^2 - 2x)dx$.

2. (15 pts) Use the Fundamental Theorem of Calculus (Part II) to evaluate $\int_0^3 (x^2 - 2x)dx$

B1 (Bonus 5 pts) What would the x_k be if the interval were [1, 3] instead of [0, 3], in Problem #1? I just want to see the x_k. Don't do any more than that.

3. 20 pts ~~(10 pts)~~ Show that $\int_0^1 y^4\,dy = \int_0^1 (1 - x^{1/4})dx$ by evaluating each, separately.

B2 (Bonus 5 pts) Draw a picture and explain what's going on in the previous problem.

4. (10 pts) Evaluate the definite integral. $\int_0^6 |x - 4|\,dx$

B3 (Bonus 5 pts) Evaluate the definite integral $\int_{-\pi/3}^{\pi/4} |\sec x \tan x|\,dx$

5. (10 pts) Evaluate the indefinite integral. It will involve a u-substitution. $\int \frac{x}{\sqrt{2x+1}}dx$

6. 15 pts ~~(10 pts)~~ Evaluate the indefinite integral. $\int \frac{\sin x}{\cos^2 x}dx$.

B4 (Bonus 5 pts) Find an upper bound and a lower bound for the definite integral $\int_1^3 \frac{1}{x}dx$. You do *not* know how to evaluate this integral, using only this semester's worth of training. But you *can* put a ceiling and a floor on it.

7. (10 pts) Suppose the velocity (meters per second) of a particle moving in a straight line is $v(t) = -t^2 + 4t + 5$. Tell me what the two integrals represent.

   a. $\int_0^6 v(t)dt$ b. $\int_0^6 |v(t)|dt$

B5 (Bonus 5 pts) Evaluate the 2nd integral in the previous problem: $\int_0^6 |v(t)|dt$

8. (10 pts) Perform the indicated differentiation:

   a. $\frac{d}{dx}\int_{\pi/6}^{x} \frac{t^2 - 3t}{\csc^3(t)}dt$. Assume $\frac{\pi}{6} < x < \pi$.

   b. $\frac{d}{dx}\int_{\pi/6}^{x^3} \frac{t^2 - 3t}{\csc^3(t)}dt$. Assume $\frac{\pi}{6} < x^3 < \pi$

① $\int_0^3 (x^2-2x)\,dx \qquad \Delta x = \frac{3-0}{n} = \frac{3}{n}$

$x_k = a + \frac{3k}{n} = \frac{3k}{n}$

$$\frac{3}{n}\sum_{k=1}^{n} (x_k^2 - 2x_k) = \frac{3}{n}\sum_{k=1}^{n}\left(\left(\frac{3k}{n}\right)^2 - 2\left(\frac{3k}{n}\right)\right)$$

$$= \frac{3}{n}\cdot\frac{9}{n^2}\sum k^2 - \frac{3}{n}\cdot 2\cdot\frac{3}{n}\sum k$$

$$= \frac{27}{n^3}\cdot\frac{n^3 + \cdots}{3} - \frac{18}{n^2}\cdot\frac{n^2 + \cdots}{2}$$

$$\xrightarrow{n\to\infty} \frac{27}{3} - \frac{18}{2} = 9 - 9 = 0 \quad !?$$

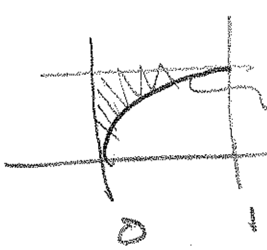
② $\int_0^3 (x^2-2x)\,dx = \left[\frac{x^3}{3} - x^2\right]_0^3 = \frac{27}{3} - 9 = 0$

③ $\int_0^1 y^4\,dy = \left.\frac{y^5}{5}\right]_0^1 = \frac{1}{5}$

$\int_0^1 \left(1 - x^{\frac{1}{4}}\right)dx = \left[x - \frac{4}{5}x^{\frac{5}{4}}\right]_0^1 = 1 - \frac{4}{5} = \frac{1}{5}$

B1 $[a,b] = [1,3] \longrightarrow \boxed{x_k = 1 + \frac{3k}{n}}$

B2



$y = x^{\frac{1}{4}} \longrightarrow x = y^4$

$\int_0^1 \left(1 - x^{\frac{1}{4}}\right) = \int_0^1 y^4\,dy$ = Shaded area.

201 TEST 3

(4) $\int_0^6 |x-4|\,dx = \int_0^4 -(x-4)\,dx + \int_4^6 (x-4)\,dx$

$= -\left[\frac{x^2}{2}-4x\right]_0^4 + \left[\frac{x^2}{2}-4x\right]_4^6$

$= -\left[\frac{4^2}{2}-4(4)-(0-0)\right] + \left[\frac{6^2}{2}-4(6)-\left(\frac{4^2}{2}-4(4)\right)\right]$

$= -\left[\frac{16}{2}-16\right]+\left[\frac{36}{2}-24-\left(\frac{16}{2}-16\right)\right]$

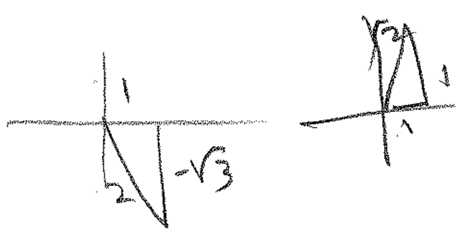
$= -[-8]+[18-24-(-8)]$

$= 8+2 = 10$

B3 $\int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} |\sec x\tan x|\,dx = -\int_{-\frac{\pi}{3}}^{0} \sec x\tan x\,dx + \int_0^{\frac{\pi}{4}}\sec x \tan x\,dx$

$= -\left[\sec x\right]_{-\frac{\pi}{3}}^{0} + \left[\sec x\right]_0^{\frac{\pi}{4}} = -\left[\sec 0 - \sec\left(-\frac{\pi}{3}\right)\right] + \left[\sec\frac{\pi}{4} - \sec 0\right]$

$= -[1-2] + [\sqrt{2} - 1] = 1+\sqrt{2}-1 = \boxed{\sqrt{2}}$



201 TEST 3

5) $\int \frac{x}{\sqrt{2x+1}}\,dx$ $\quad u = 2x+1 \quad du = 2dx$ $\quad \longrightarrow u-1 = 2x \Longrightarrow \frac{(u-1)}{2} = x$

$= \frac{1}{2}\int \frac{x}{\sqrt{2x+1}} \cdot 2dx = \frac{1}{2}\int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\,du$

$= \frac{1}{4}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du = \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) + C$

$= \boxed{\frac{1}{6}\sqrt{2x+1}^{\,3} - \frac{1}{2}\sqrt{2x+1} + C}$

6) $\int \frac{\sin x}{\cos^2 x}\,dx = \int \sec x \tan x \, dx = \boxed{\sec x + C}$

B4) $\frac{1}{x}$ is decreasing on $[1, 3]$

$m = \frac{1}{3},\ M = 1 \Longrightarrow$

$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$

$\frac{1}{3}(3-1) \le \int_1^3 \frac{dx}{x} \le 1(3-1)$

$\boxed{\frac{2}{3} \le \int_1^3 \frac{dx}{x} \le 2}$

7) a) $\int_0^5 v(t)dt$ = NET DISPLACEMENT / CHANGE.

b) $\int_0^5 |v(t)|dt$ = Total displacement / distance.

201 TEST 3

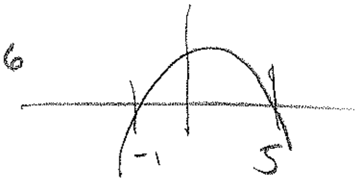
B5 $\int_0^6 |-t^2+4t+5|\,dt$

$-t^2+4t+5=0$

$\Rightarrow t^2-4t-5=0$

$\Rightarrow (t-5)(t+1)=0$

$t=-1,5$



$$= \int_0^5 (-t^2+4t+5)\,dt + \int_5^6 (t^2-4t-5)\,dt$$

$$= \left[-\tfrac{1}{3}t^3+2t^2+5t\right]_0^5 + \left[\tfrac{1}{3}t^3-2t^2-5t\right]_5^6$$

$$= \left[-\tfrac{1}{3}(5)^3+2(5)^2+5(5)-(0+0+0)\right]$$
$$+\left[\tfrac{1}{3}(6)^3-2(6)^2-5(6)-\left(\tfrac{1}{3}(5)^3-2(5)^2-5(5)\right)\right]$$

$$= \left[-\frac{125}{3}+50+25\right] + \left[\frac{216}{3}-72-30-\left(\frac{125}{3}-50-25\right)\right]$$

$$= \frac{-125+216-125}{3}+75-102+75$$

$$= \frac{-34}{3}+48 = \frac{-34+144}{3} = \boxed{\frac{110}{3}} = \boxed{36.\overline{6}}$$

36.667234

201 TEST 3 C4

8 a $\dfrac{d}{dx}\displaystyle\int_{\frac{\pi}{6}}^{x} \frac{t^2-3t}{\csc^3 t}\,dt = \frac{x^2-3x}{\csc^3 x}$

b $\dfrac{d}{dx}\displaystyle\int_{\frac{\pi}{6}}^{x^3} \frac{t^2-3t}{\csc^3 t}\,dt = \frac{x^6-3x^3}{\csc^3(x^3)}\cdot 3x^2$