201 Test 2 Take-Home

(1) $f(x) = -2\sin x \cos x - x$ on $[0, 2\pi]$

$f(0)=0,\ f(2\pi) = -2\pi \longrightarrow (0,0),\ (2\pi, -2\pi)$

$(2\pi, -6.283)$

$f'(x) = -2\left[\cos^2 x - \sin^2 x\right] - 1$

$= -2\left[\cos^2 x - 1 + \cos^2 x\right] - 1$

$= -4\cos^2 x + 2 - 1 = f'(x)$

$= -4\cos^2 x + 1 \overset{\text{SET}}{=} 0$

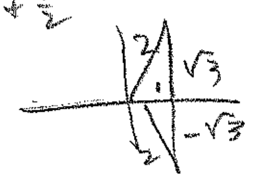
$\longrightarrow 4\cos^2 x = 1$

$\frac{9\pi}{3} = \frac{3\pi}{2}\ \ \frac{3}{2}\pi$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm\frac{1}{2}$

$+\frac{1}{2}$

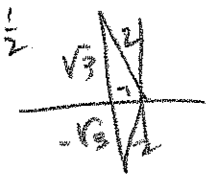


$\frac{\pi}{3}$, $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$f(\frac{\pi}{3}) = -2\sin\frac{\pi}{3}\cos\frac{\pi}{3} - \frac{\pi}{3}$

$= -2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \frac{\pi}{3}$

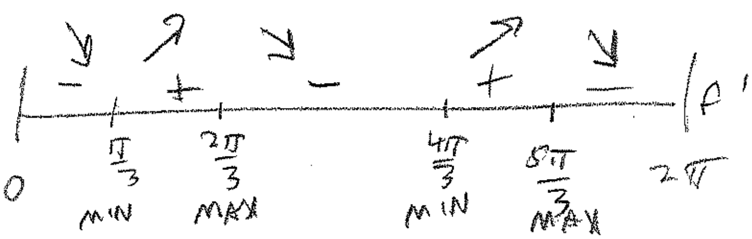
$-\frac{1}{2}$



$\frac{2\pi}{3},\ \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$= -\frac{\sqrt{3}}{2} - \frac{\pi}{3}$

$\rightsquigarrow \left(\frac{\pi}{3}, -\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right) \approx \left(\frac{\pi}{3}, -1.913\right)$



$f(\frac{2\pi}{3}) = -2\sin\frac{2\pi}{3}\cos\frac{2\pi}{3} - \frac{2\pi}{3}$

$= -2\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) - \frac{2\pi}{3}$

$= \frac{\sqrt{3}}{2} - \frac{2\pi}{3}$

$\rightsquigarrow \left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right) \approx \left(\frac{2\pi}{3}, -1.228\right)$

$f'(\frac{\pi}{4}) = -4\cos^2\frac{\pi}{4} + 1 = -1$

$f'(\frac{\pi}{2}) = 0 + 1 = 1$

$f'(\pi) = -4\cos^2(\pi) + 1 = -3$

$f'(\frac{3\pi}{2}) = 1$

$f'(\frac{11\pi}{6}) = -2$

$f(\frac{4\pi}{3}) = -2\sin\frac{4\pi}{3}\cos\frac{4\pi}{3} - \frac{4\pi}{3}$

$= -2\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) - \frac{4\pi}{3} = -\frac{\sqrt{3}}{2} - \frac{4\pi}{3}$

$\approx \left(\frac{4\pi}{3}, -5.055\right)$

$f(\frac{5\pi}{3}) \approx -4.370$

$\left(\frac{5\pi}{3}, -4.370\right)$

201 TEST 2 TAKE-HOME

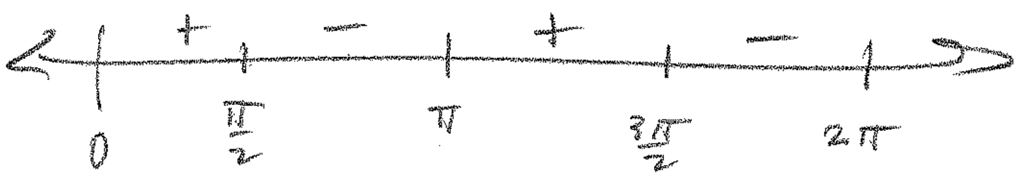
① cntd

$f'(x) = -4\cos^2 x + 1 \Longrightarrow$

$f''(x) = (-8\cos x)(-\sin x)$

$= 8\sin x \cos x \overset{\text{SET}}{=} 0$

$\Longrightarrow x = 0, \frac{\pi}{2}, \frac{\pi}{1}, \frac{3\pi}{2}, 2\pi$

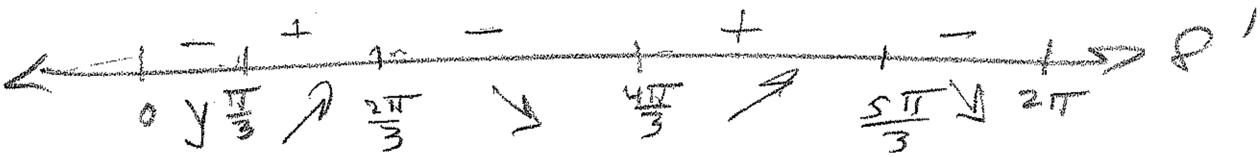


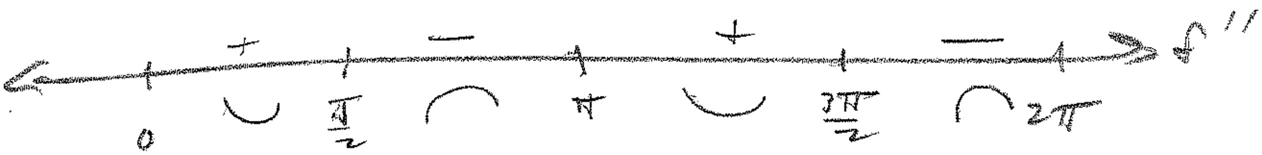
Test $\frac{\pi}{4}$: +

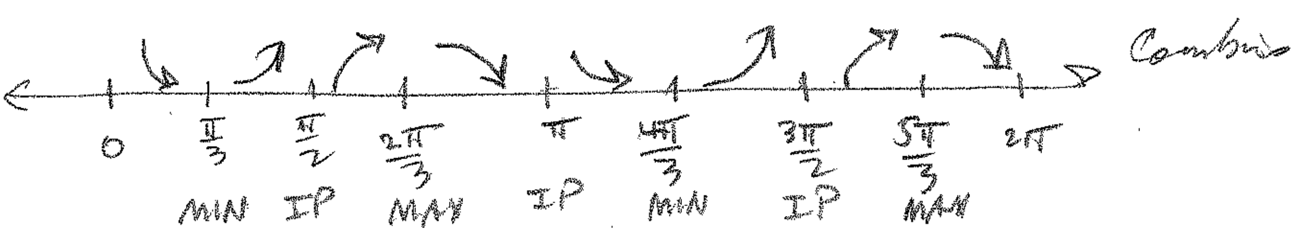
$\frac{2\pi}{3}$: −

$\frac{4\pi}{3}$: +

$\frac{7\pi}{4}$: −







201 TEST 2 TAKE-HOME

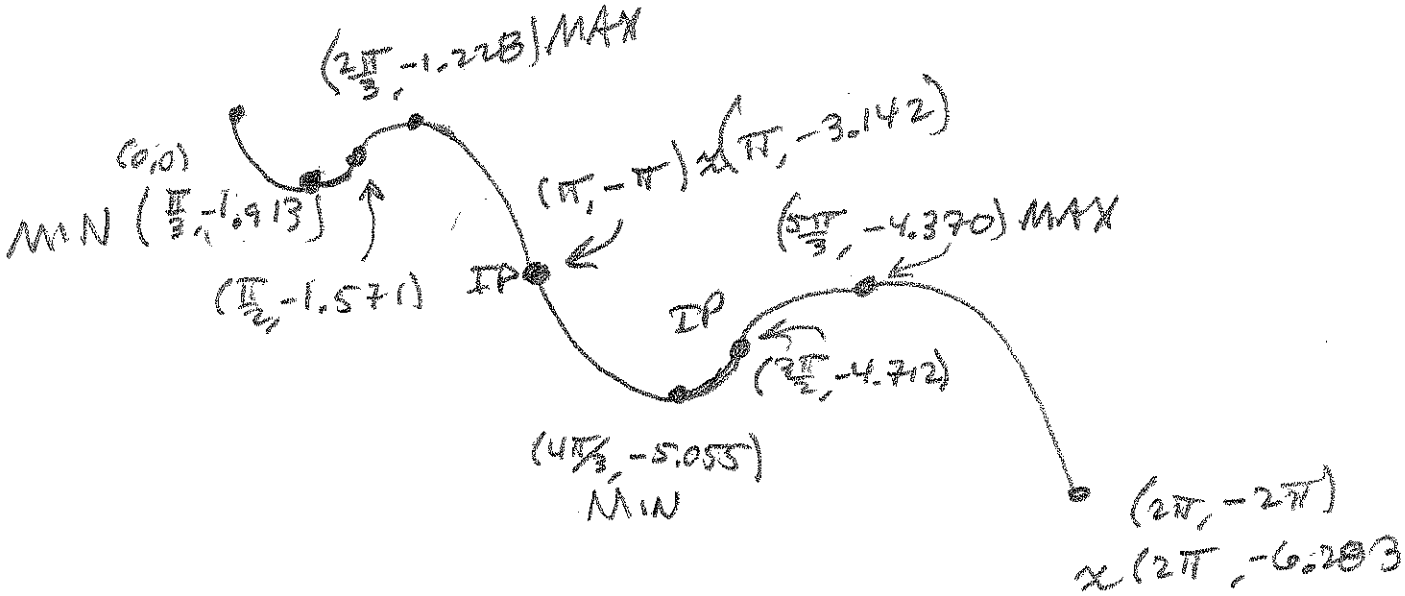
(1) $f(\frac{\pi}{2}) = -\frac{\pi}{2} \rightarrow (\frac{\pi}{2}, -1.571)$ $\quad -2\sin\frac{3\pi}{2}\cos\frac{3\pi}{2} - \frac{3\pi}{2}$

$f(\pi) = -\pi \longrightarrow (\pi, -3.142)$

$f(\frac{3\pi}{2}) = -\frac{3\pi}{2} \rightarrow (\frac{3\pi}{2}, -4.712)$

$f(2\pi) = -2\pi \rightarrow (2\pi, -2\pi)$

$f(0) = 0 \rightarrow (0,0)$



201 TEST 2 TAKE-HOME

(3) Sketch $f(x) = \frac{2x^2\ x-10}{x-2}$ $\mathcal{D} = \mathbb{R} \setminus \{2\}$

$b^2-4ac = (-1)^2-4(2)(-10)$
$= 1+80$
$= 81 \rightsquigarrow \sqrt{81} = 9$

$x = \frac{1 \pm 9}{2(2)} \to \frac{10}{4} = \frac{5}{2}$
$\to -\frac{8}{4} = -2$

$f(x) = \frac{2(x-\frac{5}{2})(x+2)}{x-2}$
$= \frac{(2x-5)(x+2)}{x-2}$

$\boxed{\text{V.A. } x=2}$

$\boxed{\text{O.A. } y = 2x+3}$

$\boxed{\text{x-ints } (\tfrac{5}{2},0), (-2,0)}$

$$
\begin{array}{r|rrr}
2 & 2 & -1 & -10 \\
 & & 4 & 6 \\
\hline
 & 2 & 3 & -4 \\
 & x & c & r
\end{array}
$$

$\boxed{2x+3}$

$f(x) = 2x+3 - \frac{4}{x-2}$, so easier for derivative

$f'(x) = \frac{(4x-1)(x-2) - (2x^2-x-10)}{(x-2)^2}$

Nah. $f(x) = 2x+3-\frac{4}{x-2} = 2x+3-4(x-2)^{-1}$

$\Rightarrow f'(x) = 2+4(x-2)^{-2} = \frac{2(x-2)^2}{(x-2)^2} + \frac{4}{(x-2)^2}$

$= \frac{2(x^2-4x+4)+4}{(x-2)^2} = \frac{2x^2-8x+8+4}{(x-2)^2}$ SET $0 \rightarrow$

$2x^2-8x+12 = 0$
$x^2-4x+6 = 0$
$(x-3)(x-1) = 0 \Longrightarrow x \in \{1,3\}$

$2(1)+3-\frac{4}{1-2} = 5+4 = 9$
$(1,9)$ max/min?
$2(3)+3-\frac{4}{3-2} = 9-4 = 5$
$\boxed{(3,5)}$ max/min?

201 TEST 2 Chapter 3

(2) cntd

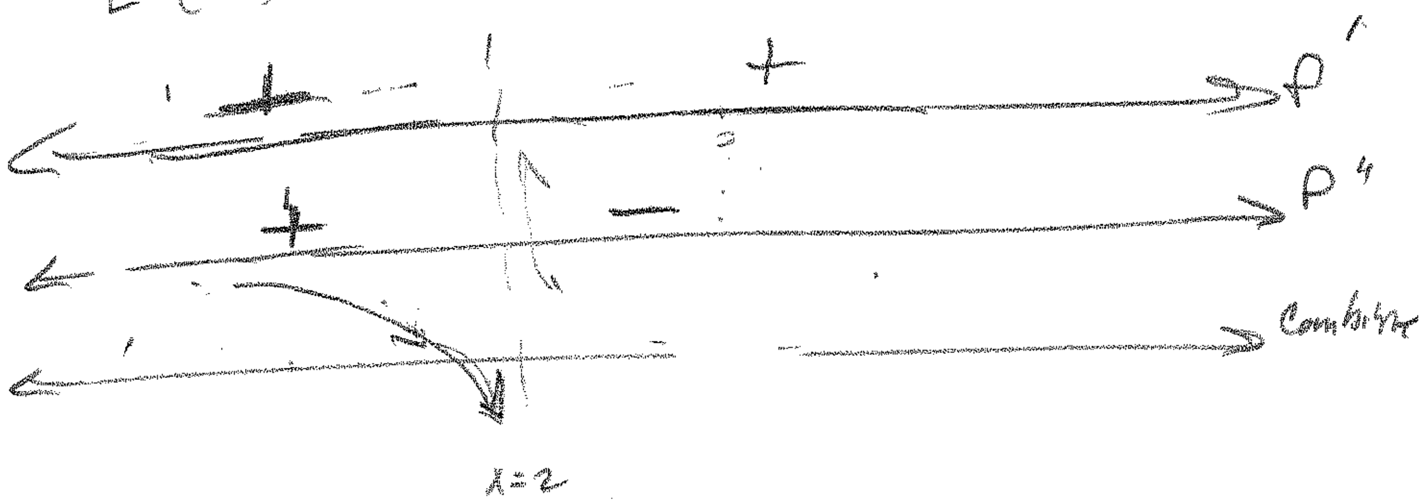
$$f''(x) = \frac{d}{dx}\left[\frac{2(x^2-4x+6)}{(x-2)^2}\right] = 2\,\frac{d}{dx}\left[\frac{x^2-4x+6}{(x-2)^2}\right]$$

$$= 2\left[\frac{(2x-4)(x-2)^2 - (x^2-4x+6)(2(x-2))}{(x-2)^4}\right] \quad \rightarrow 2(x-2)$$

$$= 2\left[\frac{(x-2)\left[(2x-4)(x-2) - (2)(x^2-4x+6)\right]}{(x-2)^2}\right]$$

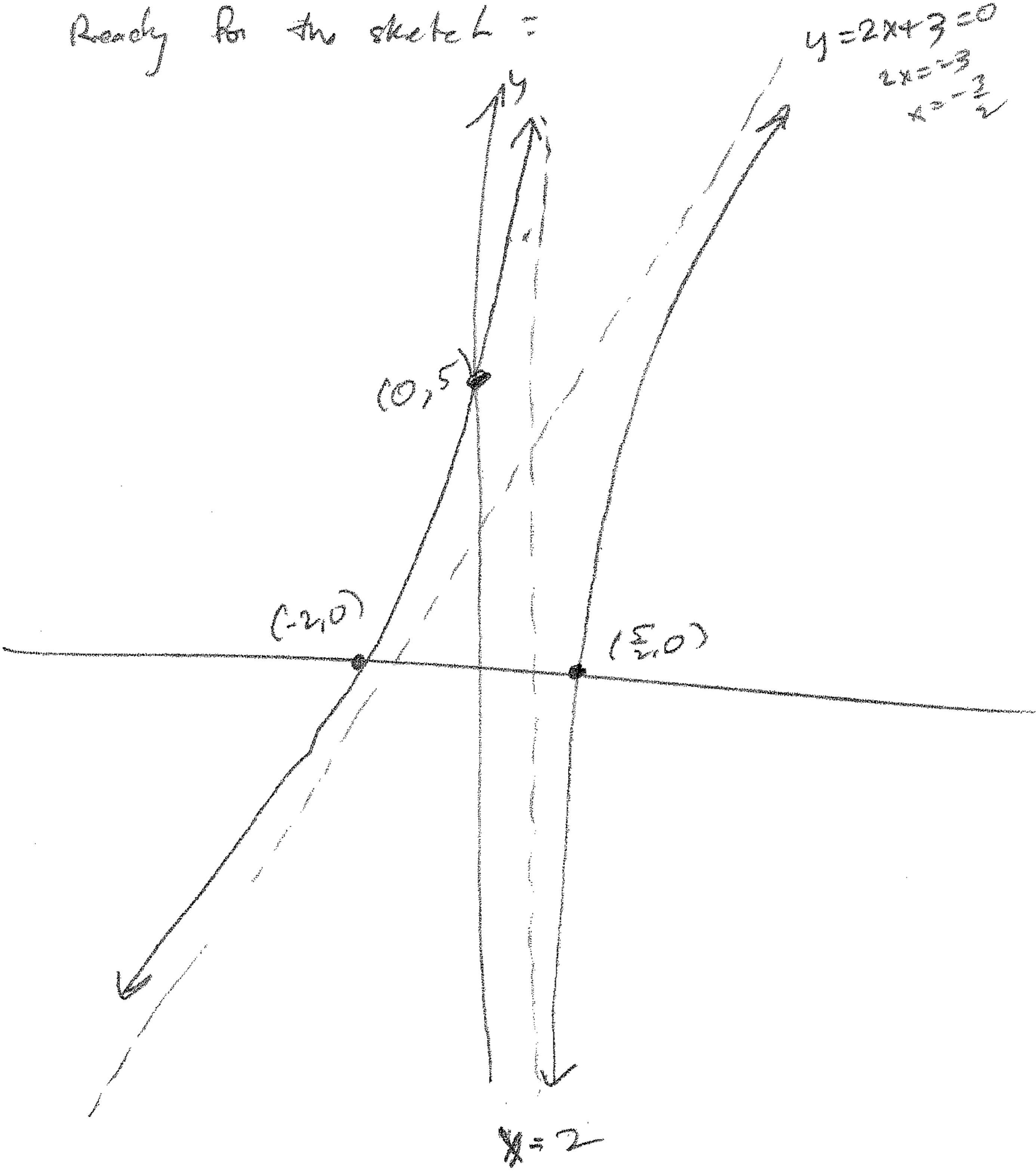
$$= 4\left[\frac{(x-2)^2 - (x^2-4x+6)}{(x-2)^3}\right] = 4\left[\frac{x^2-4x+4-x^2+4x-6}{(x-2)^3}\right]$$

$$= 4\left[\frac{-2}{(x-2)^3}\right] = \frac{-8}{(x-2)^3}$$



201 Test 2 Fall 2013 Take-home.

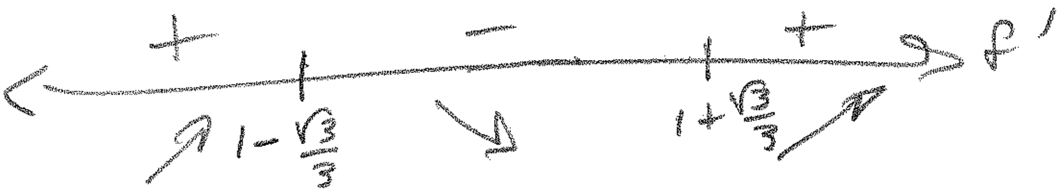
Ready for the sketch :

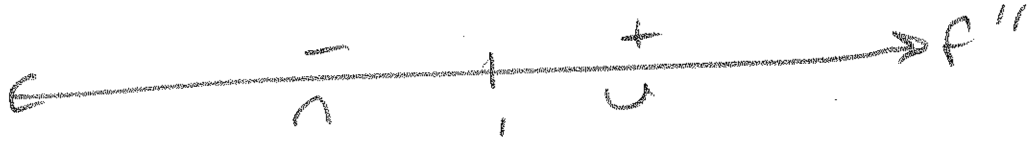


201 TEST 3 TAKE-HOME

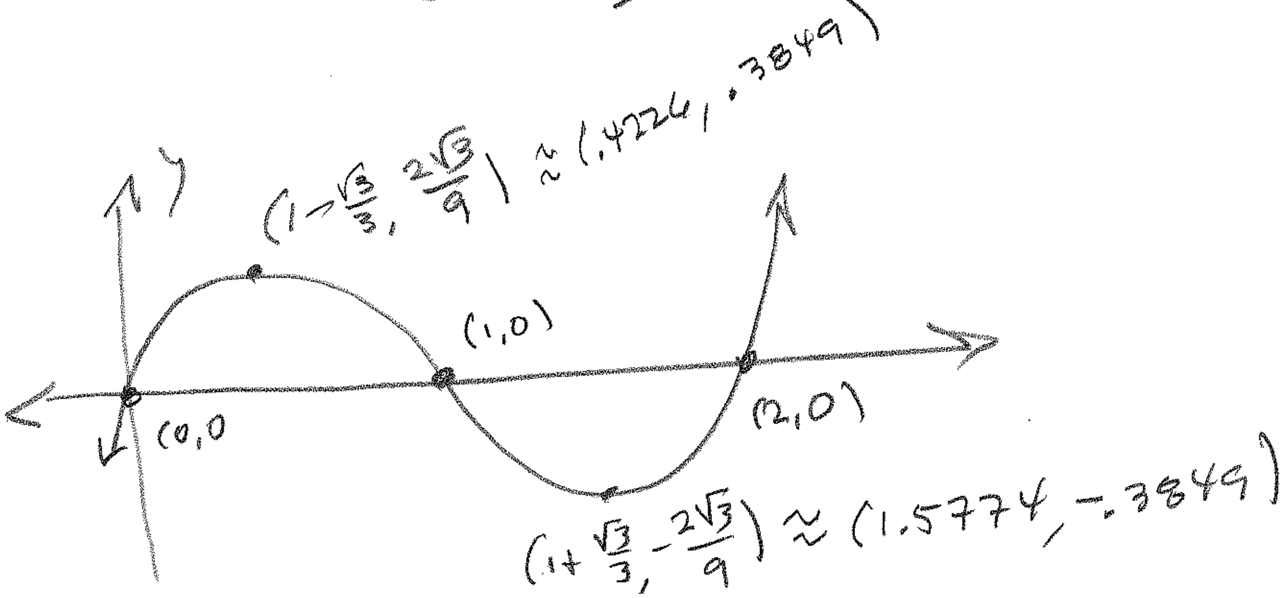
$f(1) = 1^3 - 3(1)^2 + 2(1) = 1 - 3 + 2 = 0$ ✓

IS an IP AND an x-intercept.





COMBINE



201 TEST 2 TAKE-HOME

(3) $f(x) = x^3 - 3x^2 + 2x \overset{SET}{=} 0 \qquad \mathcal{D} = \mathbb{R} = \mathbb{R}$

$x(x^2 - 3x + 2) = 0 \qquad$ x-ints: $(0,0), (1,0), (2,0)$

$x(x-1)(x-2) = 0 \qquad$ y-int: $(0,0)$

$f'(x) = 3x^2 - 6x + 2 \overset{SET}{=} 0$

$a = 3, b = -6, c = 2$

$b^2 - 4ac = (-6)^2 - 4(3)(2) = 36 - 24 = 12 \rightsquigarrow \sqrt{12} = 2\sqrt{3}$

$x = \dfrac{6 \pm 2\sqrt{3}}{2(3)} = \dfrac{3 \pm \sqrt{3}}{3}$

$\to 1 + \frac{\sqrt{3}}{3} \approx 1.577350269$

EXTREMA, HERE

$\to 1 - \frac{\sqrt{3}}{3} \approx .4226497308$

$f''(x) = 6x - 6 \overset{SET}{=} 0 \longrightarrow x = 1$ IP

$f\left(1 + \frac{\sqrt{3}}{3}\right) = \left(1 + \frac{\sqrt{3}}{3}\right)^3 - 3\left(1 + \frac{\sqrt{3}}{3}\right)^2 + 2\left(1 + \frac{\sqrt{3}}{3}\right)$

$= 1 + 3 \cdot \frac{\sqrt{3}}{3} + 3 \cdot \left(\frac{\sqrt{3}}{3}\right)^2 + \left(\frac{\sqrt{3}}{3}\right)^3 - 3\left[1 + 2 \cdot \frac{\sqrt{3}}{3} + \left(\frac{\sqrt{3}}{3}\right)^2\right] + 2 + \frac{2\sqrt{3}}{3}$

$= 1 + \sqrt{3} + 3\left(\frac{3}{9}\right) + \frac{3\sqrt{3}}{27} - 3 - \frac{6\sqrt{3}}{3} - \frac{9}{9} + 2 + \frac{2\sqrt{3}}{3}$

$= 1 + \sqrt{3} + 1 + \frac{\sqrt{3}}{9} - 3 - 2\sqrt{3} - 1 + 2 + \frac{2\sqrt{3}}{3}$

$= 0 + \sqrt{3} + \frac{\sqrt{3}}{9} - 2\sqrt{3} + \frac{2\sqrt{3}}{3} = \dfrac{9\sqrt{3} + \sqrt{3} - 18\sqrt{3} + 6\sqrt{3}}{9}$

$= -\frac{2\sqrt{3}}{9} \approx -.3849001795$

Similar work shows $f\left(1 - \frac{\sqrt{3}}{3}\right) = +\frac{2\sqrt{3}}{9} \approx .3849001795$