MAT 201 Test 2 – Fall, 2013 Name KEY

100 Points Covers Chapter 3 NO GRAPHING CALCULATORS!!!

1. (10 pts) Let $f(x) = x^3 - 3x^2 + 2x$. Find all absolute and local extremes of f on the interval [0, 3]. Final answers accurate to the 3rd decimal place are acceptable.

2. (10 pts) Confirm that $f(x) = x^3 - 3x^2 + 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval [0, 3]. Then find all values c in (0, 3) that satisfy the conclusion of the theorem.

3. (10 pts) Let $f(x) = -2\sin(x)\cos(x) - x$. Find all local extrema in the interval $[0, 2\pi]$.

4. (10 pts) Suppose a function g satisfies all of the following properties. Sketch a graph of g that incorporates all of the following properties into it:

$g(1) = -2 \quad g(2) = 2 \quad g(3) = 4$

$g'(1) = 0 \quad g'(3) = 0$

$g'(x) > 0$ on $(1,3) \cup (3, \infty)$, $\quad g'(x) < 0$ on $(-\infty, 1)$

$g''(x) > 0$ on $(-\infty, 2) \cup (3, \infty)$, $\quad g''(x) < 0$ on $(2,3)$

5. (5 pts each) Evaluate the limits:

   a. $\lim_{x \to \infty} \left(\sqrt{9x^2 + 3x + 7} - 3x \right)$

   b. $\lim_{x \to \infty} \left(\dfrac{3x^3 - 6x + 7}{2 - 5x^2 - 7x^3} \right)$

6. (10 pts) Find the equation of the oblique asymptote for $R(x) = \dfrac{3x^3 - 6x + 7}{x^2 - 5}$ This is *sort* of a limit at infinity.

7. (10 pts) Find the minimum vertical distance between $h(x) = 2x^2 - 5x + 12$ and $k(x) = 1 - 3x - x^2$.

8. (10 pts) Use the graph of the function $f(x)$, on the accompanying sheet, to show how x_2 would be found by Newton's Method, in an attempt to find a root. Derive the formula for x_2, and explain what's going on.

9. (10 pts) Suppose $f''(x) = 40x^3 - 24x^2 + 18x - 2$, and we have the initial conditions $f'(1) = f(1) = 3$. Find $f(x)$.

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(1) $f(x) = x^3 - 3x^2 + 2x$ on $[0, 3]$

$f(0) = 0 \rightsquigarrow (0,0)$

$f(3) = 27 - 27 + 6 = 6 \rightsquigarrow (3,6)$

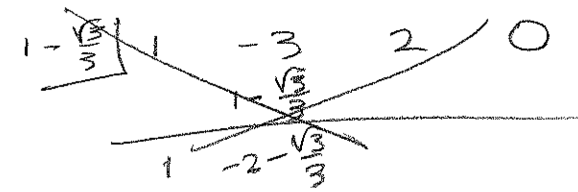
$f'(x) = 3x^2 - 6x + 2 \overset{SET}{=} 0$

$b^2 - 4ac = (-6)^2 - 4(3)(2)$

$= 36 - 24$

$= 12 \rightsquigarrow \sqrt{12} = 2\sqrt{3}$

$x = \dfrac{6 \pm 2\sqrt{3}}{2(3)} = \dfrac{3 \pm \sqrt{3}}{3} = 1 \pm \dfrac{\sqrt{3}}{3}$



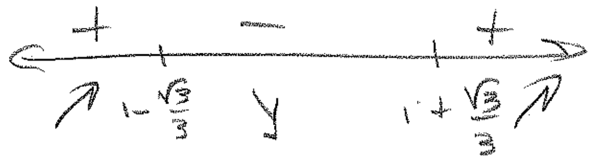
$f(1 - \frac{\sqrt{3}}{3}) \approx .3849001795$

$f(1 + \frac{\sqrt{3}}{3}) \approx -.3849001795$

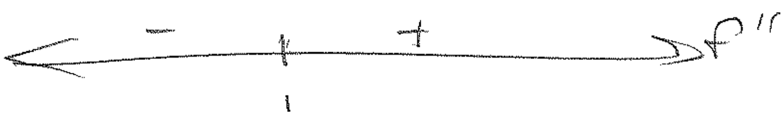
$(1 - \frac{\sqrt{3}}{3}, .385)$ MAX (LOCAL)

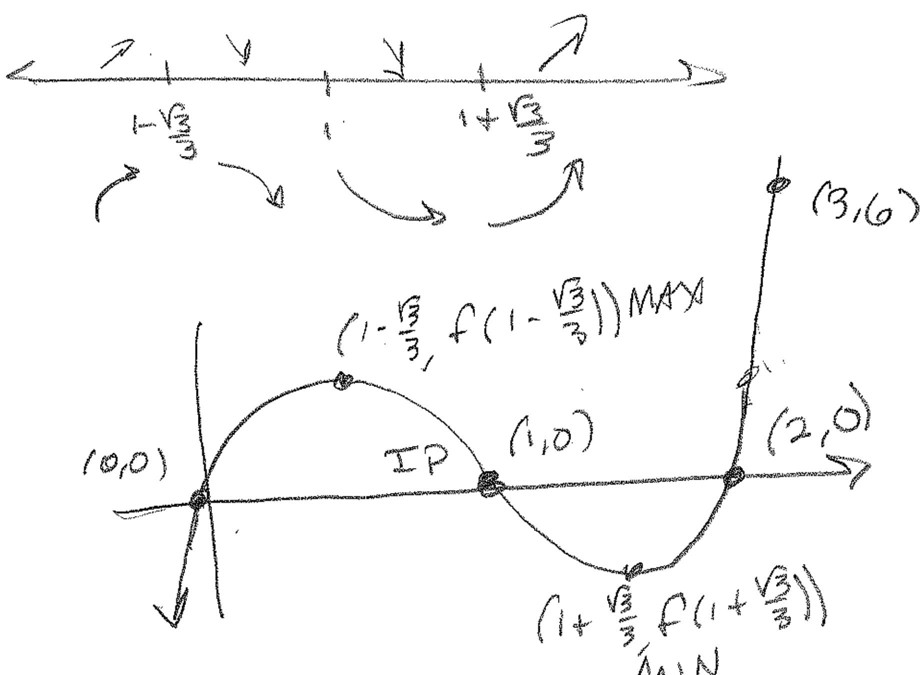
$(1 + \frac{\sqrt{3}}{3}, -.385)$ MIN (ABS)

$(3,6)$ MAX (ABSOLUTE



$f''(x) = 6x - 6 \overset{SET}{=} 0 \longrightarrow x = 1$





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(2) $f(x)$ is a polynomial, so it's cnts and difbl $\forall x \in \mathbb{R}$, so cnts on $[0,3]$ & $(0,3)$ in particular.

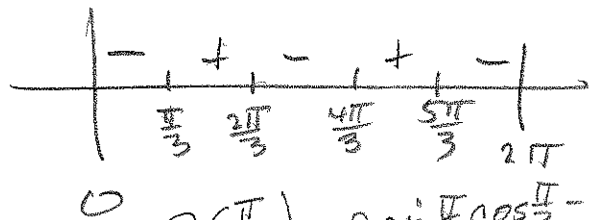
$$m_{AVG} = \frac{f(3)-f(0)}{3-0} = \frac{6-0}{3-0} = 2$$

$$f'(x) = 3x^2 - 6x + 2 \overset{SET}{=} 2 \longrightarrow$$

$$3x^2 - 6x = 0 \longrightarrow$$

$$3x(x-2) = 0 \longrightarrow$$

$$x = 0, \boxed{x = 2 = c}$$



(3) $f(x) = -2\sin x \cos x - x \longrightarrow$

$$f'(x) = -2\cos^2 x - 2(\sin x(-\sin x)) - 1$$

$$= -2\cos^2 x + 2\sin^2 x - 1$$

$$= -2\cos^2 x + 2(1-\cos^2 x) - 1$$

$$= -2\cos^2 x + 2 - 2\cos^2 x - 1$$

$$= -4\cos^2 x + 1 \overset{SET}{=} 0$$

$$\Rightarrow \cos^2 x = \frac{1}{4}$$

$$\Rightarrow \cos x = \pm\frac{1}{2}$$

$$f\left(\frac{\pi}{3}\right) = -2\sin\frac{\pi}{3}\cos\frac{\pi}{3} - \frac{\pi}{3}$$

$$= -\cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)\left(\frac{1}{2}\right) - \frac{\pi}{3}$$

$$= -\frac{\sqrt{3}}{2} - \frac{\pi}{3}$$

$$f\left(\frac{2\pi}{3}\right) = -2\left(\sin\frac{2\pi}{3}\right)\left(\cos\frac{2\pi}{3}\right) - \frac{2\pi}{3}$$

$$= -\cancel{2}\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\cancel{2}}\right) - \frac{2\pi}{3} = \frac{\sqrt{3}}{2} - \frac{2\pi}{3}$$

$$f\left(\frac{5\pi}{3}\right) = -2\sin\frac{5\pi}{3}\cos\frac{5\pi}{3} - \frac{5\pi}{3}$$

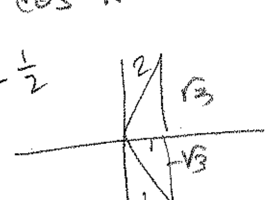
$$= -2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \frac{5\pi}{3}$$

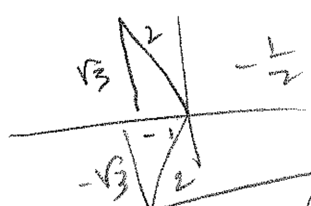
$$= \frac{\sqrt{3}}{2} - \frac{5\pi}{3}$$

$$f\left(\frac{4\pi}{3}\right) = -2\sin\left(\frac{4\pi}{3}\right)\cos\frac{4\pi}{3} - \frac{4\pi}{3}$$

$$= -2\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) - \frac{4\pi}{3}$$

$$= -\frac{\sqrt{3}}{2} - \frac{4\pi}{3}$$





$\frac{2\pi}{3}$, $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\frac{\pi}{3}$, $2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$

Local MINS: $\left(\frac{\pi}{3}, -\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right), \left(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2} - \frac{4\pi}{3}\right)$

Local Max: $\left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right), \left(\frac{5\pi}{3}, \frac{\sqrt{3}}{2} - \frac{5\pi}{3}\right)$

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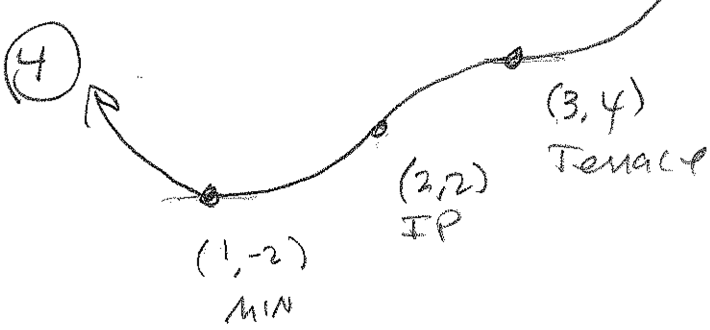

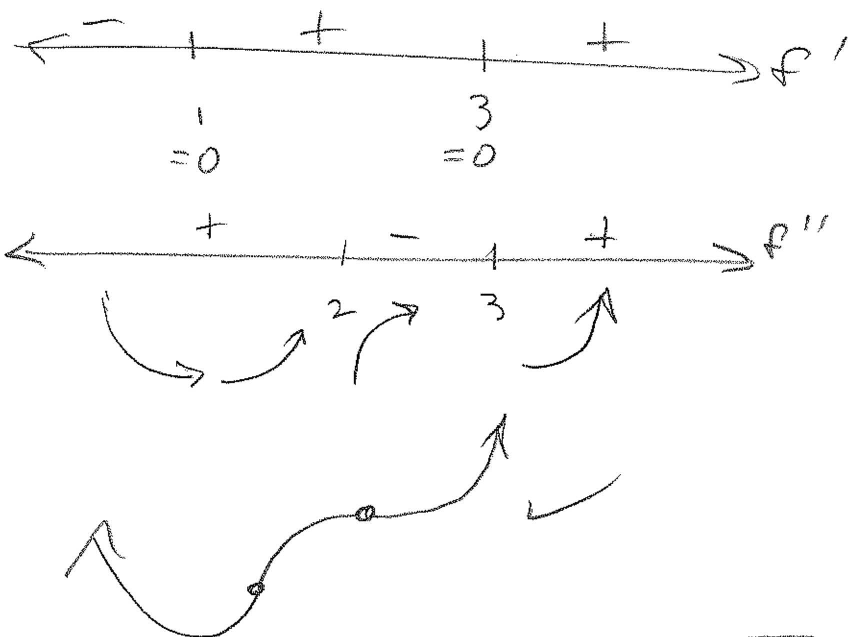

5 $\left(\sqrt{9x^2+3x+7}-3x\right)\left(\dfrac{\sqrt{9x^2+3x+7}+3x}{\sqrt{9x^2+3x+7}+3x}\right)$

$$= \frac{9x^2+3x+7-9x^2}{\sqrt{9x^2+3x+7}+3x} = \frac{3x+7}{\sqrt{9x^2\left(1+\frac{1}{3x}+\frac{7}{9x^2}\right)}+3x}$$

$$= \frac{3x\left(1+\frac{7}{3x}\right)}{3x\left[\sqrt{1+\frac{1}{3x}+\frac{7}{9x^2}}+1\right]} = \frac{1+\frac{7}{3x}}{\sqrt{1+\frac{1}{3x}+\frac{7}{9x^2}}+1} \xrightarrow{x\to\infty} \boxed{\frac{1}{2}}$$

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(5)(b) $\dfrac{3x^3-6x+7}{-7x^3-5x^2+2} \xrightarrow{x \to \infty} \boxed{-\dfrac{3}{7}}$

(6)

$$\begin{array}{r} 3x \\ x^2-5 \overline{\smash{)}\, 3x^3-6x+7} \\ -(3x^3-15x) \\ \hline 9x+7 \end{array}$$

$y = 3x$

(7) $g(x) =$
$|h(x) - k(x)| = 2x^2-5x+12-(-x^2-3x+1)$

$= 2x^2-5x+12+x^2+3x-1$

$= 3x^2-2x+11 > 0$ always, since

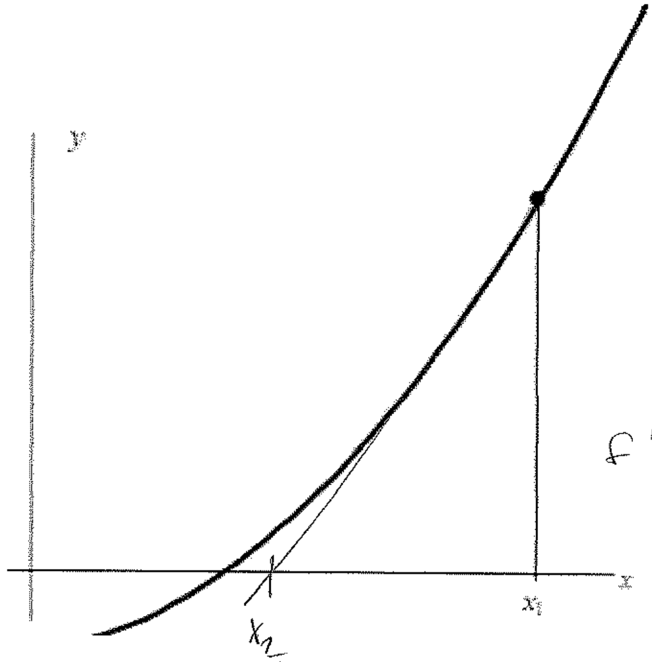
$b^2-4ac = (-2)^2-4(1)(11) < 0$ & @ 0, it is +11.

$g'(x) = 6x-2 \overset{\text{SET}}{=} 0 \implies 6x = 2 \implies x = \frac{1}{3}$

$g(\frac{1}{3}) = 3(\frac{1}{3})^2-2(\frac{1}{3})+11$

$= \frac{3}{9} - \frac{2}{3} + 11$

$= \dfrac{1-2+33}{3} = \boxed{\dfrac{32}{3}}$ is min distance.



x_2 is where tangent line hits the y-axis:

$y = f'(x_1)(x - x_1) + f(x_1)$ SET 0

$f'(x_1)x - f'(x_1)x_1 + f(x_1) = 0$

$f'(x_1)\,x = f'(x_1)x_1 - f(x_1)$

$$\boxed{x = x_2 = f'(x_1) - \frac{f(x_1)}{f'(x_1)}}$$

$$\text{Should be } x_2 = \frac{f'(x_1)x_1}{f'(x_1)} - \frac{f(x_1)}{f'(x_1)} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

MY ALGEBRA SUCKED ON THAT AND MATTHEW AND DANIEL WERE UNKIND ENOUGH TO RUB IT IN.

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(9) $$f''(x) = 40x^3 - 24x^2 + 18x - 2$$

$$f'(x) = 10x^4 - 8x^3 + 9x^2 - 2x + C$$

$$f'(1) = 3 \Longrightarrow$$

$$10 - 8 + 9 - 2 + C = 3$$

$$9 + C = 3$$

$$\boxed{C = -6}$$

$$f(x) = 2x^5 - 2x^4 + 3x^3 - x^2 - 6x + D$$

$$f(1) = 3 \Longrightarrow$$

$$2 - 2 + 3 - 1 - 6 + D = 3$$

$$5 - 9 + D = 3$$

$$-4 + D = 3$$

$$\boxed{D = 7}$$

$$\boxed{f(x) = 2x^5 - 2x^4 + 3x^3 - x^2 - 6x + 7}$$