201 Practice Test 1 FALL, 2013

(1) 10 pts $\dfrac{x^2-9}{x^3-27} = \dfrac{(x-3)(x+3)}{(x-3)(x^2+3x+9)} = \dfrac{x+3}{x^2+3x+9}$

$\xrightarrow{x \to 3} \dfrac{6}{9+9+9} = \dfrac{6}{27} = \boxed{\dfrac{2}{9}}$

(2) 5 pts (a) $\dfrac{2x^2+x-15}{x^2+5x+6} = \dfrac{(2x-5)(x+3)}{(x+2)(x+3)}$

$= \dfrac{2x-5}{x+2} \xrightarrow{x \to -3} \dfrac{-6-5}{-3+2} = \dfrac{-11}{-1} = \boxed{11}$

5 pts (b) $\dfrac{2x^2-x-15}{x^2+5x+6} = \dfrac{(2x+5)(x-3)}{(x+2)(x+3)} \xrightarrow{x \to -3} \nexists$

(3) Claim: $\lim\limits_{x \to 5} (2x-7) = 3$

Proof: Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{2}$.

Then any time $0 < |x-5| < \delta$, we have

$|(2x-7)-3| = |2x-10| = 2|x-5| < 2\delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon$ ∎

(4) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h)^2+5(x+h)+6-(x^2+5x+6)}{h}$

$= \dfrac{x^2+2xh+h^2+5x+5h+6-x^2-5x-6}{h} = \dfrac{2xh+h^2+5h}{h} = \dfrac{2x+h+5}{1}$

$\xrightarrow{h \to 0} \boxed{2x+5 = f'(x)}$

201 PT 1

(5) (a) $y=x^2+5x+6x^{-2} \Longrightarrow$

$\boxed{y'=2x+5-12x^{-3}}$

(b) $y=(x^2+5x)(7x-1) \Longrightarrow$

$y=(2x+5)(7x-1)+(x^2+5x)(7)$

(c) $y=\dfrac{x^2+5x}{7x-1} \Longrightarrow \boxed{y'=\dfrac{(2x+5)(7x-1)-(x^2+5x)(7)}{(7x-1)^2}}$

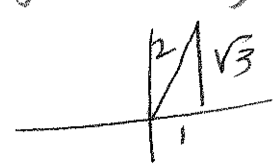
(d) $y=(x^2+5x)^3(7x-1)^5 \Longrightarrow$

$\boxed{y'=3(x^2+5x)^2(2x+5)(7x-1)^5+(x^2+5x)^3(5(7x-1)^4)(7)}$

(e) $y=\cot(\sec(x^2-5)) \Longrightarrow$

$\boxed{y'=-\csc^2(\sec(x^2-5))(\sec(x^2-5))(\tan(x^2-5))(2x)}$

(6) $y=\sin x \Longrightarrow y'=\cos x \longrightarrow y'(\frac{\pi}{3})=\cos\frac{\pi}{3}=\frac{1}{2}$



$\longrightarrow y=\frac{1}{2}(x-\frac{\pi}{3})+y(\frac{\pi}{3})$

since $\sin(\frac{\pi}{3})=f(\frac{\pi}{3})=\frac{\sqrt{3}}{2}$, this is y_1 in

$y=m(x-x_1)+y_1 \longrightarrow \boxed{y=\frac{1}{2}(x-\frac{\pi}{3})+\frac{\sqrt{3}}{2}}$

201 PT 1

(7) $\sin(65^\circ) \approx$

$65 = 60^\circ + 5^\circ$

$= \frac{\pi}{3} + \frac{5\pi}{180} = \frac{\pi}{3} + \frac{\pi}{36} = x_1 + \Delta x$

$\therefore \sin(x+\Delta x) \approx f'(x)(x-x_1) + y_1$

$= \frac{1}{2}(x - \frac{\pi}{3}) + f(\frac{\pi}{3})$

$= \frac{1}{2}(x - \frac{\pi}{3}) + \frac{\sqrt{3}}{2} \Longrightarrow$

$\sin(65^\circ) \approx \frac{1}{2}(\frac{\pi}{3} + \frac{\pi}{36} - \frac{\pi}{3}) + \frac{\sqrt{3}}{2}$

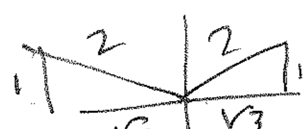
$= \frac{1}{2}(\frac{\pi}{36}) + \frac{\sqrt{3}}{2}$

$= \boxed{\frac{\pi}{72} + \frac{36\sqrt{3}}{72}} = \boxed{\frac{\pi + 36\sqrt{3}}{72}}$

(8) $f(x) = 1 + 2\cos x$

$f'(x) = 1 - 2\sin x \overset{\text{SET}}{=} 0$

$\Longrightarrow \sin x = \frac{1}{2}$



$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\quad \frac{\pi}{6}$

This gives $\boxed{x \in \{x \mid x = \frac{\pi}{6} + 2n\pi \text{ OR } x = \frac{5\pi}{6} + 2n\pi,\ n \in \mathbb{Z}\}}$

201 PT #1

(9) $\sec x + \sin y = 2xy - 3x^2y^2 \implies$

$$\sec x \tan x + (\cos y)y' = 2y + 2xy' - 6xy^2 - 6x^2yy'$$

$$\implies (\cos y - 2x - 6x^2y)y' = 2y - 6xy^2 - \sec x \tan x$$

$$\implies \boxed{y' = \frac{2y - 6xy^2 - \sec x \tan x}{\cos y - 2x - 6x^2y}}$$

B1 $\frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right] = \frac{1}{h}\left[\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right]$

$$= \frac{1}{h}\left(\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right)\left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}\right)$$

$$= \frac{1}{h}\left[\frac{x - (x+h)}{(\sqrt{x}\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}\right] = \frac{1}{h}\left[\frac{-h}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}\right]$$

$$= \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \xrightarrow{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x} + \sqrt{x})}$$

$$= \frac{-1}{x(2\sqrt{x})} = \boxed{-\frac{1}{2x\sqrt{x}}}$$

208 PJ # 1

$\boxed{B2}$ **Claim** $\lim_{x \to 3} (x^2+5x+2) = 26$

$9+15+2$

Scratch:

Want $|x^2+5x+2-26| < \varepsilon$

$|x^2+5x-24| < \varepsilon$

$|x+8||x-3| < \varepsilon$

$|x+8|\,\delta < \varepsilon$

Assume $\delta \leq 1$. Then

$2 \leq x \leq 4$

$10 \leq x+8 \leq 12$, i.e., $|x+8| \leq 12$!

$\frac{\varepsilon}{12}$ it is!

Proof Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{12}\}$.

Then $0 < |x-3| < \delta \Longrightarrow |x^2+5x+2-26|$

$= |x^2+5x-24| = |x+8||x-3| < |x+8|\,\delta \leq 12\,\delta \leq 12\frac{\varepsilon}{12} = \varepsilon$

$\blacksquare$