

$$f(x) = -2 \sin x \cos x - x$$

$$= -\sin(2x) - x$$

Test #2. Find extrema  
in  $[0, 2\pi]$

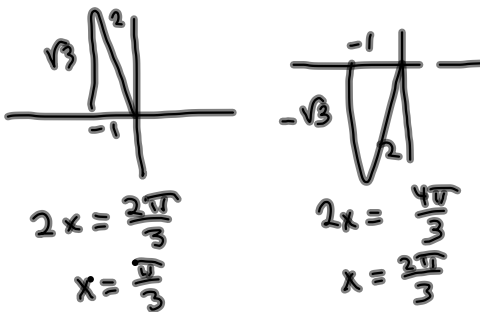
$$f(0) = 0$$

$$f(2\pi) = -2\pi$$

$$f'(x) = -2 \cos(2x) - 1 \stackrel{SE \neq 0}{=} 0$$

$$-2 \cos(2x) = 1$$

$$\cos(2x) = -\frac{1}{2}$$



$$2x = \frac{2\pi}{3}$$

$$x = \frac{\pi}{3}$$

$$2x = \frac{4\pi}{3}$$

$$x = \frac{2\pi}{3}$$

Solving for  $2x = \theta$ ,  
it's easy to miss  
the  $2x$ 's living in  
 $(0, 4\pi)$  that will  
put  $x \in (0, 2\pi)$

Spencer

$$\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3} = 2x$$

$$\Rightarrow \frac{4\pi}{3} = x$$

$$\frac{4\pi}{3} \in (0, 2\pi)$$

$$\frac{4\pi}{3} + 2\pi = \frac{10\pi}{3} = 2x$$

$$\Rightarrow x = \frac{5\pi}{3} \in (0, 2\pi)$$

$$f(x) = -2 \sin x \cos x - x \quad f(0) = 0, f(2\pi) = -2\pi$$

$$f'(x) = \overset{f'}{-2} \overset{g}{\cos x} \overset{g'}{\cos x} + \overset{f}{(-2 \sin x)} \overset{g'}{(-\sin x)} - 1$$

$$= -2 \cos^2 x + 2 \sin^2 x - 1$$

$$= -2 \cos^2 x + 2(1 - \cos^2 x) - 1$$

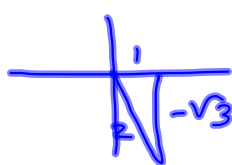
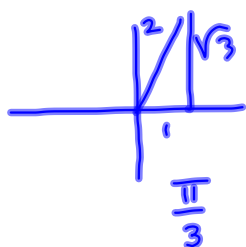
$$= -2 \cos^2 x + 2 - 2 \cos^2 x - 1$$

$$= -4 \cos^2 x + 1 \quad \text{SET } 0$$

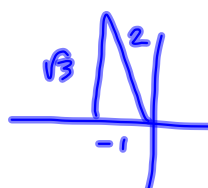
$$\cos^2 x = \frac{1}{4}$$

Spencer, Meg

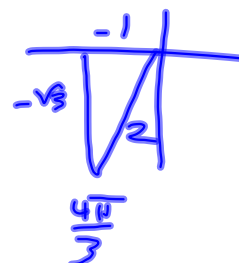
$$\cos x = \pm \frac{1}{2}$$



$$= \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$$



$$= \frac{2\pi}{3} = \pi - \frac{\pi}{3}$$



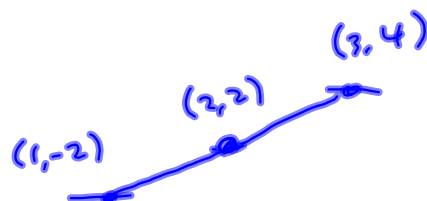
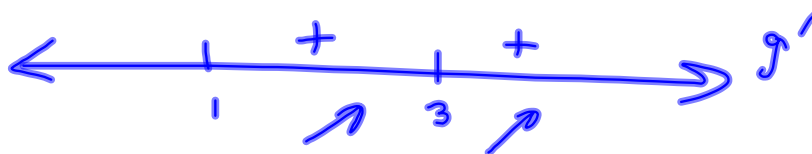
$$g(1) = -2, g(2) = 2, g(3) = 4$$



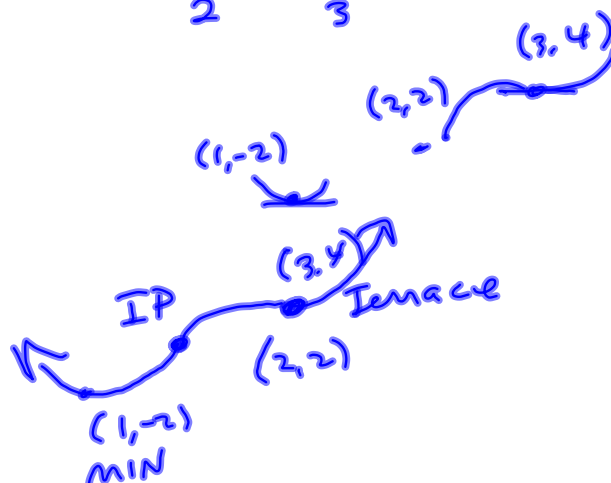
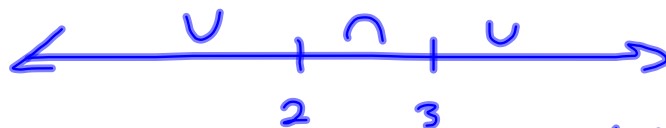
$$g'(1) = g'(3) = 0$$



$$g'(x) > 0 \text{ on } (1, 3) \cup (3, \infty)$$



$$g''(x) > 0 \text{ on } (-\infty, 2) \cup (3, \infty) \quad g''(x) < 0 \text{ on } (2, 3)$$



§ 5 - Volumes, areas between curves,  
MVT for Integrals.

MVT for derivatives ✓

$f$  cont<sup>d</sup> on  $[a, b]$

$f$  diff<sup>l</sup> on  $(a, b)$

$$\Rightarrow f'(c)(b-a) = \underbrace{f(b) - f(a)}_{\text{net change}}$$

$\exists c \in (a, b) \ni$

$$f'(c) = \frac{f(b) - f(a)}{b-a} = m_{AVG}$$

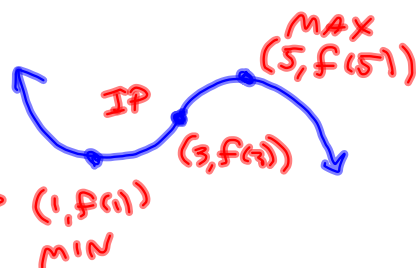
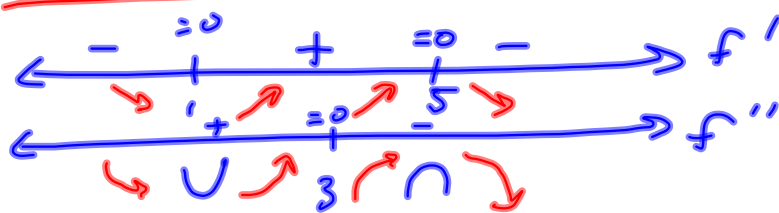
MVT for Integrals

$f$  cont<sup>d</sup> on  $[a, b] \Rightarrow$

$$\exists c \in (a, b) \ni f(c) = \frac{1}{b-a} \int_a^b f(x) dx = f_{AVG}$$

Likely to ask for a sketch of a cubic. Critical values, sign pattern for  $f'$ ,  $f''$ . Relative position important

Actual value of  $f$  not as important



x2

Newton's  
Show how  $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

Eq'n of tan. line @  $x_1$ :

§3.8



$$y = m(x - x_1) + y_1$$

$$y = f'(x_1)(x - x_1) + f(x_1)$$

↑  
 $x_2$  from  $y=0$

$$\frac{d}{dx} [\sqrt{x}] = ? \quad \text{by def'n of } \frac{dy}{dx}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \left( \frac{\sqrt{x+h} - \sqrt{x}}{h} \right) \left( \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\xrightarrow{h \rightarrow 0} \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

pass to limit after you've cleared out  
the 0 in denominator.



~~the~~  $f'(x) =$

How to factor  $a^3 \pm b^3$

$$x^3 + 8 = x^3 + 2^3 = (x+2)(x^2 - 2x + 4)$$

$x = -2$  makes it zero.

$$x^3 + 8 = 0$$

$$x^3 = -8$$

$$\sqrt[3]{x^3} = \sqrt[3]{-8}$$

$$\Rightarrow x = -2$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 8 \\ & & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & 0 \\ & x^2 & x & c & r \end{array} \quad (x+2)(x^2 - 2x + 4)$$

$$x+2 \overline{) \begin{array}{r} x^2 - 2x + 4 \\ x^3 + 0x^2 + 0x + 8 \end{array}}$$

$$- (x^3 + 2x^2)$$

$$-2x^2 + 0x + 8$$

$$- (-2x^2 - 4x)$$

$$4x + 8$$

$$- (4x + 8)$$

$$0 + 0$$

$$\frac{x^3}{x} = x^2$$

$$\frac{-2x^2}{x} = -2x$$

$$\frac{4x}{x} = 4$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

will never have real zeros!

Sum / Difference of cubes