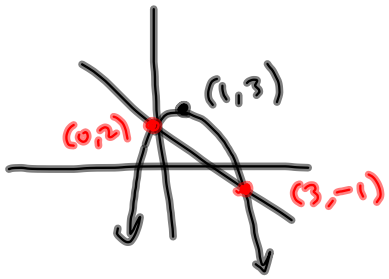
Volume



Build a parabola

$y = -\frac{7}{4}x^2 + \frac{7}{2}x + \frac{5}{4}$

Find volume of solid obtained by rotating the area bounded by the parabola and the line about :

(1) y-axis

(2) $y = 3$

(3) $y = -1$

Set up only. Shells and washers.

$m = \frac{-1-2}{3-0} = \frac{-3}{3} = -1$

$y = -x + 2$

$a(x-1)^2 + 3$

$a(3-1)^2 + 3 = -1$

$4a + 3 = -4$

$4a = -7$

$a = -\frac{7}{4}$

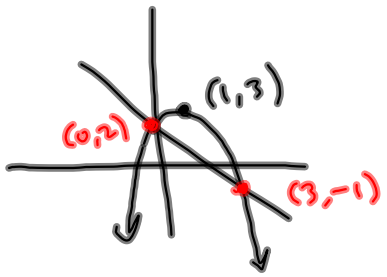
$y = -\frac{7}{4}(x-1)^2 + 3$

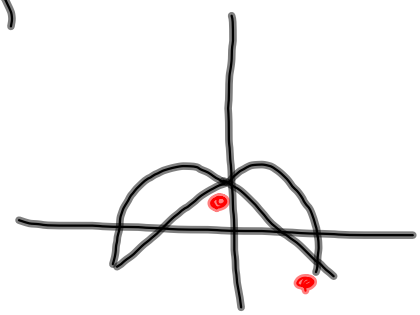
$-\frac{7}{4}(x^2 - 2x + 1) + 3$

$-\frac{7}{4}x^2 + \frac{7}{2}x - \frac{7}{4} + \frac{12}{4}$

=

(1)





$y = -\frac{3}{4}x^2 + \frac{3}{2}x + \frac{5}{4}$

$y = -x + 2$

Find where I screwed up.

Want $(0,2), (3,-1)$ on the parabola.

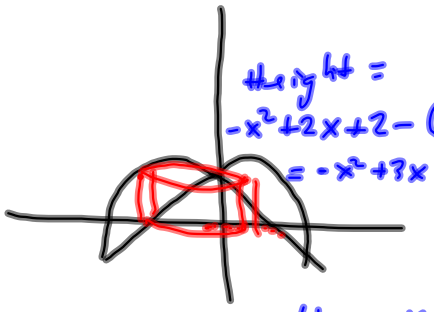
ax^2+bx+c

$= ax^2+bx+2$

$(3,-1)$ is on it

$9a + 3b + 2 = -1$

I wanted $(1,3)$ on it, which probably will not be the vertex.



$V = 2\pi \int_0^3 x(-x^2+3x)\,dx$

$a+b+2 = 3$

$9a+3b+2 = -1$

$a+b = 1$

$9a+3b = -3$, i.e.

$3a+b = -1$

$a+b = 1$

$2a = -2$

$a = -1$

$-1+b = 1$

$b = 2$

$y = -x^2+2x+2$

$\not\in$ $y = -x+2$

↳ Region bdd by these two.

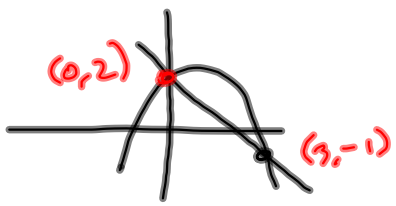
Find intersections:

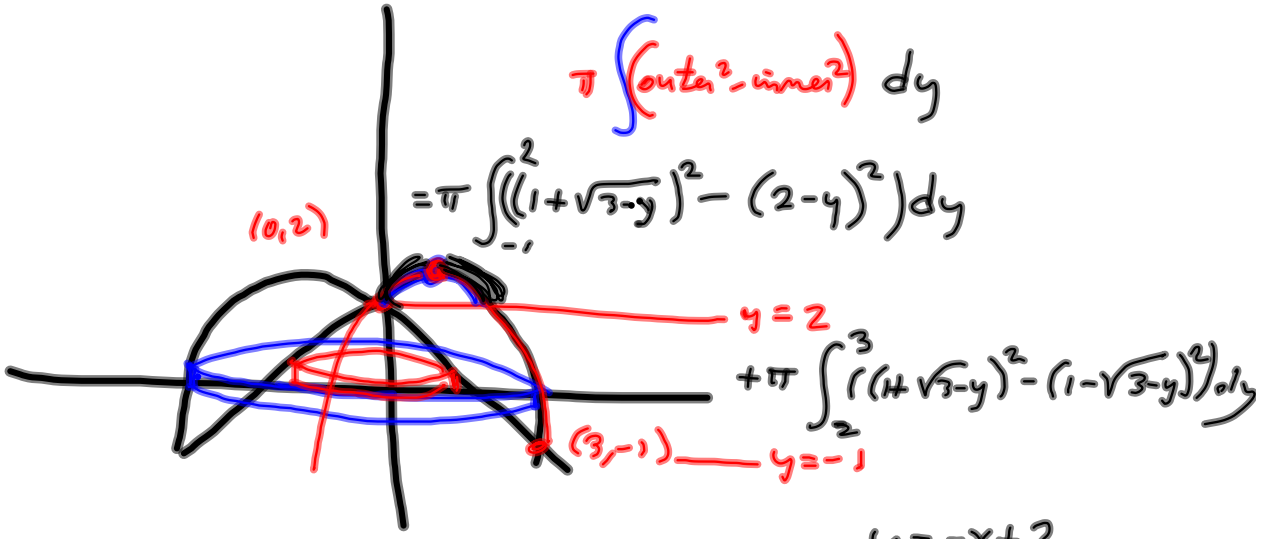
$-x^2+2x+2 = -x+2$

$-x^2+3x = 0$

$-x(x-3) = 0$

$x = 0, \quad x = 3$





$\pi \int (\text{outer}^2 - \text{inner}^2)\, dy$

$= \pi \int_{-1}^{2} \left((1+\sqrt{3-y})^2 - (2-y)^2 \right) dy$

$+ \pi \int_{2}^{3} \left((1+\sqrt{3-y})^2 - (1-\sqrt{3-y})^2 \right) dy$

Vertex: $y = -x^2+2x+2$

$= -(x^2-2x+1^2)+2+1$

$= -(x-1)^2+3$

$y = -x+2$

$-x = y-2$

$x = 2-y$

Tricky part: expressing this as $x = g_1(y)$, $x = g_2(y)$

$-x^2+2x+2 = y$

$-(x^2-2x+1)+2+1 = y$

$-(x-1)^2+3 = y$

$-(x-1)^2 = y-3$

$(x-1)^2 = 3-y$

$x-1 = \pm\sqrt{3-y}$

$x = 1 \pm \sqrt{3-y}$

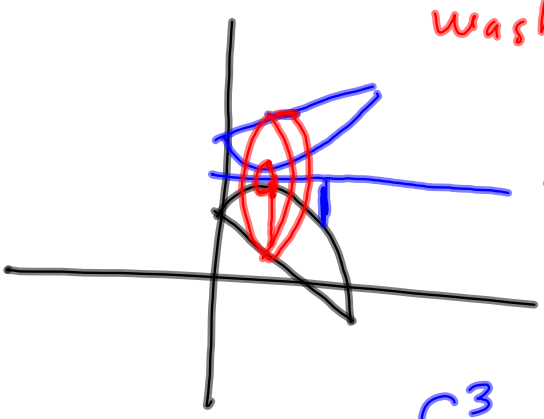
$f(x) = -x^2+2x+2$

$f^{-1}(x) = 1 \pm \sqrt{3-x}$

ain't a function.

(2) about $y=3$

Washers easier.

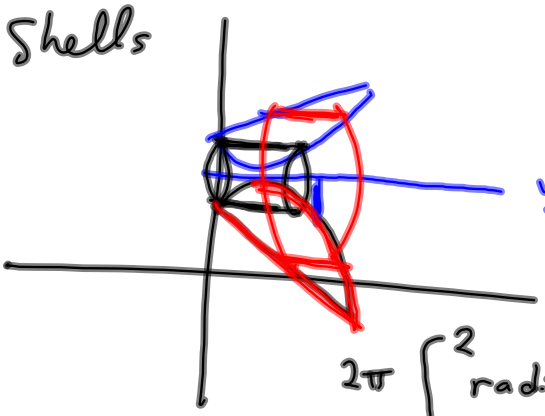


$y = -x^2+2x+2$

$y = -x+2$

$$\pi \int_0^3 \left((3-(-x+2))^2 - (3-(-x^2+2x+2))^2 \right) dx$$

Shells



$$2\pi \int_{-1}^{2} dy \quad \text{switch.}$$

$$+ \int_2^3 dy$$

$$2\pi \int_{-1}^{2} \text{radius} \cdot \text{height}\, dy$$

$$= 2\pi \int_{-1}^{2} (3-y)\left(1+\sqrt{3-y} - (-y+2)\right) dy$$

$$+2\pi \int_2^3 (3-y)\left(1+\sqrt{3-y} - (1-\sqrt{3-y})\right) dy$$

$-x^2+2x+2 = y$

$\therefore \quad x = 1+\sqrt{3-y}$

$$2\cdot\pi\cdot\int_0^3 x\cdot\left(-x^2+3\cdot x\right)\,dx$$

shells for (1)

$$\frac{27}{2}\pi$$

washers for (1)

$$\pi\cdot\int_{-1}^{2}\left(\left(1+\sqrt{3-y}\,\right)^2-(2-y)^2\right)dy+\pi\cdot\int_2^3\left(\left(1+\sqrt{3-y}\right)^2-\left(1-\sqrt{3-y}\right)^2\right)dy$$

$$\frac{27}{2}\pi$$

$$\mathrm{Pi}\cdot\int_0^3\left((3-(-x+2))^2-\left(3-\left(-x^2+2\cdot x+2\right)\right)^2\right)dx$$

washers for (2)

$$\frac{72}{5}\pi$$

$$2\cdot\mathrm{Pi}\left(\int_{-1}^{2}(3-y)\cdot(1+\mathrm{sqrt}(3-y)-(-y+2))\,dy+\int_2^3(3-y)\cdot(1+\mathrm{sqrt}(3-y)-(1-\mathrm{sqrt}(3-y)))\,dy\right)$$

shells for (2)

$$2\pi\left(\frac{36}{5}\right)$$